



## Word and phrase index

I have tried to include in this index not only the location where each term is defined, but also all significant occurrences of the concepts in question; but it has not been easy to decide which occurrences are significant. I would welcome your observations on the types of cases you would find it useful to have in the index, and on any entries that are erroneous, unnecessary, or missing.

Pages where terms are defined or where conventions relating to them are set are shown with boldface page numbers. (Sometimes a formal definition occurs after the first page of discussion of a topic, and sometimes more than one version of a concept is defined, leading to occasional entries such as 100-**101**-115, **130**-140.) At some future time, I may try to provide other information in similar ways: e.g., perhaps small type for brief tangential references, a raised dot after a page-number to signal the approximate height on the page at which a term occurs, etc.. I would be interested in your thoughts as to what information of this sort would be useful.

I have generally grouped specific concepts under more general ones. Thus, “dihedral group” appears under “group”, not “dihedral”. But when a subtopic would have a large number of entries relating to it, it is often made a separate heading. E.g., “abelian groups” has its own heading; on the other hand, items relating to “commutative rings” are shown under “rings”, since we don’t discuss as many facets of that subtopic.

When two broad concepts intersect, it is hard to give a rule as to where the intersection is indexed. Concepts studied in this course that cut across various types of algebras outrank the specific types of algebras; so, “free group” appears only under “free”. Occasionally, I reference the same topic under more than one heading.

The great majority of the entries in this index concern algebraic topics. Most of the remainder concern foundations and logic, topology, or meta-topics such as “heuristics” and “open questions”. For convenience, the handful of entries relating to still other subjects are grouped under the heading “miscellaneous areas”.

Terms used by other authors for which different words are used in this work are, if referenced, put in single quotes; e.g., ‘free product’, for what we call “coproduct”.

In secondary headings, the words of the main heading are abbreviated “–”. In cross-references, the form “*see* main-heading: subheading” is used; in particular, “*see* –: subheading” points to another subheading under the same main heading. Cross-references are often abbreviated by replacing words after the first one or two by “...”; on the other hand, “etc.” means “and similar topics”. “...” without immediately preceding words, as in “*see* different-heading: ...”, indicates a phrase similar to the present subheading.

- |  |   |
|--|---|
| abelian group(s), <b>43-45</b> , 123, 130, 252, <i>see</i><br><i>also</i> group, module, tensor product,<br>bilinear map | duality of – via $\mathbb{Q}/\mathbb{Z}$ , $\mathbb{R}/\mathbb{Z}$ , 182, 375 |
| additive – structure of rings, 67, 175, 225,<br>228, 343   | endomorphism rings of –, 150  |
| – as $\mathbb{Z}$ -modules, 57, 358, 363   | – of $\mathbb{Z}$ -valued functions, 44, 180                                  |
| category of –, 160   | representability question for some functors<br>on –, 297, 345                 |
| divisible –, <b>186</b> , 233  | representable functors on –, 343, 357-358,<br>365, 375                        |
|  | solenoids, <b>240-241</b>   |

- structure of the product and squaring
  - functors on  $-$ , 326
  - $-$  structure on hom-sets of  $-$ , 53, 59, 227, 357, 358, *see also* modules: ...
  - subgroup lattices of  $-$ , 130, 143
  - topological  $-$ , Pontryagin duality of, 182, 240, 375
  - torsion  $-$ , **195**, 345
- abelianization, *see* group(s): ...
- above and below, *see* constructions: from ...
- abuse of notation, *see* loose usage
- ACC, DCC, *see* chain: conditions
- actions, *see* representations *and*  $G$ -sets
- adjunctions, 220-**224**-229, 244, 267, 330-331
  - $-$  and empty algebras, 286
  - $-$  between varieties, 294, 339-383, *see also* *Table of Contents for details*
  - composition of  $-$ , 228-229, 341-342, 345, 354, 362-364, 376-377, *see also* composition: representing object ...
  - conjugate morphisms between adjoint functors, **271**
  - contravariant left  $-$ , **272**
  - contravariant right  $-$ , 271-**272**-273, 325, 372-378
  - Freyd's adjoint functor theorem, **265**, 285, 287, 293, 345
  - Freyd's special adjoint functor theorem, 266
  - functors with adjoints on both sides, 225, 226, 343, 368
  - morphisms among adjoint functors, 267-271
  - relations of  $-$  with other universal constructions, 225, 244, 247-251
  - "Structure is adjoint to Semantics", *see* varieties ...: ...
  - underlying-set-preserving functors have adjoints, 321, 342-343, 365
  - unit and counit of  $-$ , 223-**224**, 228, 248, 267-270, 351-352, 355, 381
  - universal constructions which are not adjoints, 227
- algebra(s), *see also* rings ...
- ambiguity of the word  $-$ , 275
- $\mathbf{C}$ -based  $-$ , *see*  $-$ : (co)- objects ...
- (co)- objects in a category, 332-**334**-**337**-383
  - $-$  defined by an arbitrary binary operation, 365, 371, 380
  - empty  $-$ , 276-277, 286, 289, 306, 314
  - finite and profinite  $-$ , 240, 299, 375-376
  - finitely generated  $-$ , 375
  - font-convention (not used here) on  $-$  and underlying sets, 11
  - "General  $-$ " or "Universal  $-$ "?, 8, 275
  - generating subsets of  $-$ , 20-22, 276, 280-282
  - generators and relations for  $-$ , *see* presentations
  - homomorphic images of  $-$ , **276**, 291, 300-303, *see also* congruences
  - many-sorted  $-$ , 316
  - $-$  means "set-based  $-$ " if contrary not stated, **334**
  - $\Omega$ - $-$ , 18, **274**-299, 315, *see also* (co)- objects *above*
  - origin of the word  $-$ , 275
  - quotient or factor  $-$ , *see* congruences
  - residually finite  $-$ , **47**, **287**
  - simple  $-$ , **346**
  - sub-, **276**-277, 280-282, 285, 291-303, *see also* lattices: of ..., *and* "sub-" *under specific sorts of algebras* (groups *etc.*)
  - topological  $-$ , 375
  - trivial  $-$ , 291, 366
  - type of an  $-$ , **274**-275, 333, *see also*  $-$ :  $\Omega$ - $-$
  - underlying sets of  $-$ , *see* functors: forgetful *and*  $|A|$  *in symbol index*
  - varieties of  $-$ , *see* varieties ...
- algorithm, *see* normal forms
- origin of the word  $-$ , 275
- arity, **13**-14, 17, **274**-275, *see mainly* algebras:
  - type of
  - finite and infinite  $-$ , 16, 19, 132, 137, 143, 274, 278, 280-282, 285, 302, 314, 327
  - $G$ -set as structure with unary operations, 38
  - $-$  of a relation, **88**
- associative algebras, *see* rings ...
- associativity, 15, 17, 125, 126, 156, 335, 373
  - $-$  allows parenthesis-free notation, 29, 32
- analog of  $-$  for clone of operations, 315
- co-, 347-350, 365
  - $-$  in varieties  $\mathbf{V} \circ \mathbf{W}$ , 379

- of  $n$ -fold products, 147
- of product and coproduct constructions, 51, 78
- automorphism
  - groups, 148, 150, 154, 175-176, **185**, *see also* representations: of groups ...
  - s of complex numbers with “exp”, 18
- axioms of set theory, *see* foundations ...
- bilinear map, **56-60**, 226, **363**, *see also* tensor product *and* categories: **Ab**-based
  - as part of ring structure, 57, 67
  - ‘balanced’ –, 363
  - category of –, 228
  - image of a –, 58
  - is not a homomorphism on product, 56
- bimodules, 227, 312, **360-365**, 368, 381
  - category with – for morphisms, **165**, 211
- bimonoid (temporary terminology), 155-157, 160, 161
- binary, *see* arity *and* algebras: defined by ...
- Boolean algebras and Boolean rings, **76-78**, 180, 182, 205, 294, 305
  - analogs of – using other primes than 2, 374
  - duality between finite – and finite sets, 206, 375-376
  - lattice-structures of –, 127, 130, 320
  - of idempotents in rings, **77**, 225, 343, 374
  - subring-lattices of –, 130
- cardinality, *see* ordinals and cardinals
- categories, 8, **157-170-273**
  - Ab**-based, **Cat**-based,  $k$ -linear –, **210**, 271
  - attitudes regarding –, 168, 187
  - auxiliary – where universal objects become initial, terminal, 213-214, 263-264
  - category of –, 177, 181, 209, 210-212, *see mainly* functors
  - ‘(co)complete –’, *see* limits and colimits: categories having ...
  - comma –, **199**, 215, 227, 264, 317, 371, *see also* –: pointed and augmented objects of commutative squares in –, 162, 201
  - concrete –, **178-179**, 185-189, 192, 324-327, *see also* free: objects in ...
  - constructed from monoids and ordered sets, *see* monoid(s): groups ... *and* ordered sets: made ...
  - contravariant equivalences of –, 375-376
  - diagram –, 162-163, 175, 200-201
  - directed (inversely directed) systems in –, **234**
  - diversity of conventions on –, 166-168
  - empty –, **173**, 253, 259
  - equivalence of –, **206-207**, 210, 271, 273, 318, 326-328, 356, 366, 368
  - functor –, **201**, 204, 207-210, 212, *see also* functor(s): morphisms of large, small, legitimate –, 169-**170-173**, **177**, 178-179, 201, 242, 249-250, 318, 326
  - morphisms (‘arrows’) in –, **157**
  - no “element chasing” in –, 168, 257, 334
  - objects of –, **157**
  - opposite –, dualization of results on –, **180-181**, 186, 213, 267, 271-273, 339, 372-377, *see also* functors, contravariant pointed –, **366-367**
  - pointed and augmented objects of –, 192, 199, 306, 356, **366-367**, 379
  - “Should hom-sets be disjoint?”, 167-168
  - skeleta of –, **207**
  - sources of confusion regarding –, 158, 161, 168
  - sub–, full sub–, **173**, 175, **177**, 190, 203, 207, 246
  - subobject, 190-191, **204-205**, 262, *see also* monomorphisms: distinguished ...
  - $\cup$ -small etc., *see* –: large, small ...
  - with finitely generated hom-sets, 258, 260, 327
  - “– without objects”, 167, 211
  - zero objects of –, *see* initial: ...
- chain, *see mainly* ordered sets: –
  - conditions (ACC, DCC), 98-**99-104**, 107, 108, 112, 121-123, 133, 138, *see also* ordering ...: well-–ed set non-order-theoretic sense of –, 89
- class, *see* foundations ...: universes ...
- clones, clonal theories, *see* operations: derived *and* varieties ...: ...
- closure operators, 23, **140-146**, 153, 171
  - and Galois connections ( $q.v.$ ), 148, 149
  - determined by subsets of  $\mathbf{P}(S) \times S$ , **140-145**

- dual concept to  $-$  (“interior operators”), 145
- equivalent concepts: closure systems, closure families, 144-145
- exchange property for  $-$ , *see*  $-$ : matroids
- finitary (‘algebraic’)  $-$ , **143-144**, 327
- matroids, **148**
- on classes of algebras, *see* varieties ... : Birkhoff ...
- Peter Frankl’s question on  $-$ , 145
- coequalizer, 162, **195-196**, 278, 292-293, 327
  - made into functor, 200
  - maps, surjectivity, and epimorphicity, 196, 243, 293
  - of monoid maps, **64-65**
  - s and general colimits, 245-246, 247, 252, 254
- commutativity, *see also* abelian groups, rings ... , lattices, categories
  - and morphisms of composite functors, 209
  - between operations of arbitrary arities, **373-374**, 377, 378-382
- co $-$ , 367
  - of coproduct construction, 51
  - of  $\text{End}(\text{Id}_{\mathbf{C}})$ , 203, 382
- partial  $-$ , 52
- commutator(s)
  - brackets in rings, **307-308**, 309, 311, 312, 320
  - in groups, 17, 33, **44**, 237
  - in rings, 225
  - subgroup and  $-$ -factor group of a group, **45**, *see mainly* group(s): abelianization
- composition, *see also under* adjunctions, functors
  - “Don’t call composites  $-$ s”, 169
  - in enriched categories, 210-211
  - of morphisms, 157-158, 160, 166, 167, 169, 180-181
  - of morphisms of functors (in several senses), **201**, **208-210**
  - of operations of arbitrary arity, 314
  - of relations, **164**
- order of writing  $-$ , 11, 166, 190, 318, 359, 360, 364
- representing object for composite of representable functors, 342, 354, 362-364
- congruences, **62-64**, **277-278**, 287, *see also* equivalence relations
  - algebras without nontrivial  $-$ , 346
  - as subalgebras of products, 64, 278, 293
  - factor-algebras (quotients) of algebras by  $-$ , **62**, **277-278**
  - generated by sets of pairs, **278**
  - invariant under all endomorphisms, 303-304
- lattice of  $-$ , **278**
  - on factor-algebras, 62
  - on groups come from normal subgroups, 63
  - on monoids, **62-64**
  - on rings come from ideals, 63
- conjunction, 11
  - as intersection, 23, 152
- constant, *see* operations: zeroary *and* functors: diagonal
- constructions, *see also* recursion, Zorn’s Lemma, functors
  - completing partial  $-$ , 118-119
  - from above and below, 23, 24, 35, 125, 141, 153, 280, 300
  - made into functors, 219-226
  - of the natural numbers, ordinals, cardinals, 104-105, 109-110
- coproduct(s), **50**, **193-194**, 245, 247, 254, *see also* presentations: of  $-$ 
  - as (adjoint) functor, 183, 225-226, 252
  - as codomains of co-operations, 332-333, **337**, *see mainly* algebra: (co) $-$  objects
  - as initial objects of auxiliary categories, 213
  - as representing objects, 216
- associativity and commutativity of  $-$ 
  - construction, 51, 78
- codiagonal maps from  $-$ , 52
  - depends on category in which taken, 53, 246, 247
  - of abelian groups, 52-53, 59, 226, 357
  - of categories, **184**, 208
  - of empty family of objects, 72, 193, 332

- of groups, 47-52, 369-370
- of monoids, 63-64, 347
- of partially ordered sets, 91
- of rings, 73-76
- of sets (disjoint union), **48**, **78**, 247, 250, 259-261
- coprojection maps
  - to colimits, 234, **242**, 252
  - to coproducts, **193**, 338, 347
  - to pushouts, **197**
- countability, *see* ordinals and cardinals: ...
- course taught from these notes, 5-8, *see also* these notes *and* pedagogy
- exercises, homework, proofs, 6-7
- prerequisites for –, 5
- “pro forma” questions, **6**
- studying for –, 5
- ‘direct sum’, 53, *see mainly* coproduct *and* retraction
- directed, *see* ordered sets: ... *and* categories: ...
- disjoint union, *see* coproduct: of sets
- disjunction, 11
  - of propositions corresponds to union of sets of models, 152
- embedding, embeddability, *see*
  - monomorphisms: distinguished ... *and* Yoneda ...
- of algebras, 65, 77, 96, 136, 299, 308, 370, *see also* representations ...: Cayley’s theorem
- of ordinals, partially ordered sets, (semi)lattices, 112, 136, 138-140, 142
- of topological spaces, **82**, 83, 84
- empty set
  - and zeroary operations, 13-14, 347, *see also* algebras: empty
  - and Zorn’s Lemma, 120-121
  - as starting-point for set theory, 105, 106
- closure of – under a closure operator, 140
- inverse limits of non–s, 236, 354
- meets and joins of –, 132-134, 135
- endomorphism, *see* monoid(s): of ... *and* rings ...: of ...
- enveloping
  - universal – algebra of a Lie algebra, *see* Lie algebras
  - universal – group of a monoid, **65**, 352, *see mainly* group(s): constructions ...
- epimorphisms, **186**-190, 196, 269, 345, *see also* coequalizer ...: maps ...
  - and pushouts, 198
- conflicting meanings of “–”, 188
- epimorphs of initial objects, 297, 382
- idea of –, 187
  - of various familiar structures, 186-187
- relation between – and surjective maps, 186-189
- equalizer, 162, **195**-196, 197, 250, 252, 277, 292, 302
  - made into functor, 200
  - maps, one-one-ness, and monomorphicity, 196, 243, 263, 293
  - of monoid maps, **64**-65
  - of representable functors, 369
  - s and general limits, 241, 245-246, 247, 254, 262-263
- equivalence relation(s), 79, 95, 104, 145-147, 165, 185, 206, *see also* congruences, terms: construction ..., *and* categories: equivalence ...
  - as closed sets of a closure operator, 141
- lattice (semilattice, set) of –, 138-140, 180, 182, 216, 268
- union of directed system of –, 236
- existential quantification ( $\exists$ ), **11**
  - as disjunction, 153
- expressions, *see* terms
- family, *see* tuple
- field
  - algebraic closure of a –, 176
  - compositum of – extensions, **74**
  - Galois theory, Galois extensions of –s, 74, 148, 150, 153
  - Galois theory of infinite – extensions, 238
  - matrix group over a –, 45, 46
    - of complex numbers, 18, 147, 150
    - of fractions, 28, 66, 186, 188, 237
    - of  $p$ -adic rationals, **239**

- of rational functions, 28
- skew –, *see* rings ... : division
- transcendence degree of a – extension, 98, 148
- foundations, set-theoretic, 104-108, *see also* heuristics ... : motivation ...
- axiom of abstraction (rejected), **107**
- axiom of choice, **106-108**, 114-115, 118-120, 123, 124, 134, 176, 207, *see also* Zorn's Lemma
- axiom of projective determinacy, 124
- axiom of regularity, 101-102, **106-107**, 110, 111
- axiom of replacement, **106**, 113
- axioms of ZFC (Zermelo-Fraenkel set theory with Choice), **105-107**, 169-172
- comparability of cardinals, **119**
- inaccessible cardinals, continuum hypothesis, 172, *see also* –: universes ...
- independence and consistency results, 123, 172
- intuitionism, 123, 124
- large and small sets, 169-**170-173**, **290**, 298, 299, *see also* categories: ... *and* solution-set ...
- quasi-small sets, **173**, 298, 299, 324, 326
- Russell's Paradox, 107
- “set vs. class” questions, 25-26, 29, 108, 110, 111, 113, 158, 169-170, 172, *see also* –: universes ...
- universes, axiom of universes, **169-170-173**, 177, 201, 298, 346
- Vopěnka's principle, 299
- well-ordering principle, **119**
- free
  - abelian groups, **43-44**, 45, 57, 59, 68-69, 201, 208, 228
  - algebras in a variety, 294-299, 301, 303, 316-318, 324-327, 341, 343, *see also* –:  $\Omega$ -algebras
  - algebras on the empty generating set, 72, 331
  - algebras on unspecified generating sets, 22, 285
  - Boolean rings, 77, 374
  - Boolean rings, complete, 266
  - cases of nonexistence of – objects, 22, 28, 266
  - commutative rings (*i.e.*, polynomial rings), 28, 67-68, 150, 183, 216, 227, 232, *see also* polynomial
  - “– fields, – division rings”, 28
  - groups, 20-**21-34**, 36, 39, 40, 41, 43, 51, 56, 192, 201-202, 203, 208, 213-214, 215, 220, 221, 223, 224, 228, 237, 247, 261, 263, 268, 270, 285, 300
  - $G$ -sets, 217
  - lattices, semilattices, 34, 128, 129, 266, 285
  - Lie algebras, 33, 310
  - monoids, 61, 68-69, 96, 228, 285, 353
  - object constructions as functors, 174
  - objects in a concrete category, **192-193**, 195, 215, 221, 225, 250
  - $\Omega$ -algebras, 283-290, 293-301, 303-304
  - ‘– product’, *see* coproduct
  - relatively – algebras, 303-**304-305**
  - rings, 68-69, 228, 285, 305
  - set-representations of a category, 217-218
  - subalgebras of – algebras, 285
- function
  - algebras of –s and algebra-valued representable functors, 333
  - conventions on –al notation, 11, 166, 360
  - germ of a –, 164, 233
  - indexed sets (families) as –s, **13**
  - restriction of a –, **100**
  - rings of almost periodic –s, 85
  - rings of bounded –s, 84-85
  - s as binary relations, 188
  - “Should a – determine its codomain?”, 166-168
  - support of a –, **44**
- functor(s), **173-185**, *see also* subheading “... as functors” *under various constructions* (tensor products, coproducts *etc.*) *or* *entities* ( $G$ -sets, representations, *etc.*)
- adjoint –, *see* adjunctions
- bi–, **184-185**, 207, 211, 218-228, 267-270
- comparison –, **326-327**

- composition of –, **177**, 180, 183, 209, 228-229
- concretization –, *see* categories: concrete
- constructions that are not –, 175-176
- contravariant –, 91, **180**-183, 184, 189, 202, 206, 360, *see also* adjunctions: contravariant ...
- covariant –, **180**, 184
- diagonal (constant) –, **226**, 228, 244-246, 250, 253
- faithful –, **177**-179, 183, 186, 203, 206
- forgetful –, 175, 178, 179, 195, 215-216, 221, 223, 224-225, 228, 247, 248, 250, 268, 270, 285, 295, 314, 316, 318, 320, 325, 339, 340, 345, 355, 371
- full –, **177**-178, 203, 206
- hom –, **179**, **182**-183, **184**, 185-186, 189, 202, 214-215, 217-220, 222, *see also* –, representable, *and* Yoneda's Lemma
- ial operations, **29**, 84, 203, 270, 371
- identity –, 175, 177, 202, 203, 204, 205, 206, 209-210, 246, 263, 382
- morphisms of –, 200-**201**-210, *see also* –: sub–, *and* morphisms: of (co)algebra objects ...
- non-representable –, 216, 253, 265, 354
- of several variables, **184**, *see also* –: bi–
- on diagram categories, 175, 200-201, 212, 243, 246, 257
- power (product of copies) of –, *see* power
- power set –, 77, 91, 129, **181**, 182, 202, 204, 216, 326
- representable algebra-valued –, **336**-**339**-383, *see also* algebra: (co)– objects
- representable set-valued –, 214-**215**-223, 244, 248-256, 263-266, 296-297, 324, 326, 331, *see also* Yoneda's Lemma
- sub–, 204-**205**, **292**, 344, 352, 353, 354
- underlying-set-preserving –, 320-321, 324, 342-343, *see also* –: forgetful *and* adjunctions: ...
- Galois connections, 148-**149**-153, 273
  - and pushouts, pullbacks, 198
  - between algebras and identities, 290-291, 300
  - measuring “what respects what”, 254, 261
- Gel'fand-Kirillov dimension, *see* growth rates
- Golod-Shafarevich construction, *see* rings ... : ...
- group(s), 96-97, 130, 264, *see also* abelian –, automorphism: ..., matrices
- abelianization of –, **45**, 53, 56, 58, 174, 178, 213, 221, 224, 228, 237, 261
- acting on a set, *see* *G*-sets *and* permutation
- acting on object of a category, **200**, 201, 242
- adjunctions between category of – and category of – maps, 226
- affine algebraic –, **346**
- Bohr compactification of topological –, 83, 85
- Burnside problem, *n*-Burnside –, **45**-47
- category of –, 160, 207, 211
- center of a –, **175**
- centralizer subgroups of –, **152**
- co–, 332, 339, 344
- cokernel of a homomorphism of –, **55**
- concept examined, 11-12, 145-147
- conjugation in –, 17, **33**
- constructions relating – and monoids, 65-66, 161, 173, 204, 205, 221, 224, 321, 343-344, 353, 355-356, 357-358, 365
- cyclic –, 42, 51, 59, 154, 162, 179, 232, 343
- derived operations in –, 17-18, 306
- ‘derived sub–’, *see* commutators: subgroup ...
- dihedral –, 17, 40
- divisible –, **186**
- elements of exponent 2 in –, 369-370
- exponents of – elements, **42**, 175, 179, 204, 238, 343, *see also* –: Burnside ...
- finite –, 29, 34, 45-47, 51
- finitely and non-finitely generated –, 26, 257, 259
- fully invariant sub–, **205**
- inner automorphisms of –, 370
- Lie –, 186, 216, 240, **310**-311
- made into categories, *see* monoids: groups *and* ...

- normal sub–, kernels of maps, 37, 39, 55
  - of automorphisms, 148, 150
- order of a – element, 42, 45, 51, 175
- quotient –, 36-38, 39, 63
  - redefined, without zeroary operation, 314
- simple –, 346
- solvable –, 45
- subgroup lattices of –, 127, 130, 141, 182,
  - see mainly* lattices: of subalgebras
- sub– of finite index, 165
- symmetric –, 17, 27, 103
- symmetry –, 41
- ternary operation  $xy^{-1}z$  on –, *see* heaps
- three sub– theorem, **33**
- topological –, 83, 85, 182, *see also* –: Lie
- torsion (‘periodic’) –, **195**, 238
- trivial homomorphism of –, **51**, 367
- universal property of (normal) sub–
  - generated by a set, 35-36
- usefulness of – theory, 168
- various universal constructions for –, 35-60
- growth rates (in groups,  $k$ -algebras, etc.), 96-98
- $G$ -sets ( $G$  a group or monoid), **37-38**, 55, 146,
  - see also* permutation and group(s): acting on object ...
- as functors, 241-242
- category of –, 160
- fixed-point sets of –, 241, 243, 257, 259-260, 343
  - made into categories, 165
  - made into functors, 200
- orbit sets of –, 242, 243, 261, 273, 343
- heaps, **306**, 320-321, 322, 357, 381
  - of isomorphisms, 306
- heuristics, intuition, general techniques
  - ‘‘above and below’’, *see* constructions: from ...
- derivations viewed as infinitesimal
  - automorphisms, 311-312
  - for characterizing closed sets under a Galois connection, 153, 300-301
  - for finding left-universal objects, 24, 33, 35, 37, 48, 54-55, 330, 361
  - for finding right-universal objects, 54-55, 85
- generalizing from **Set** to other categories, 210
- how to view functor categories, 210, 267
- how to view general categories, 168-169
- idea of Yoneda’s Lemma, 217, 218
- module case as model for thinking of representable functors, 363
- motivation for set-theoretic foundations, 104-105, 123-124
- motivation for van der Waerden’s trick, 32, 69
  - on left and right functional notation, 360
- homomorphism, 11, **275**, *see also* lattices, categories, morphisms
- ideal, 71
  - Nullstellensatz, 150
  - prime –, 74, 84, 180
- idempotent
  - Boolean ring of –s in a ring, **77**, 225, 343, 374
  - element with respect to an operation, **77**, **125**, 356
  - endomorphism, 190, *see also* retraction
  - operation, **125**, 128, 146, 322
  - operator (closure etc.), 140, 149, 302
- identities, 12, 33, **289-292**, 293, 300, 303-305, *see also* associativity, etc.
- (co)– in (co)algebra objects of a category, 334-**336**, **338**, 372
  - for  $G$ -sets, 38
- group –, 31, 40
- hyper–, **322**
- imposing – on an algebra, **294**, 343, *see also* group(s): abelianization
  - in finite groups, 34
  - in lattices, 125, 126, 130-131
- Mal’cev conditions, 323
  - of Boolean rings and algebras, 76-77
- origin of group –, 32
- Phillip Hall’s – (for groups), 33
- polynomial –, *see* rings ...: with ...
  - $x^n = 1$ , *see* group(s): Burnside problem
- imposing relations, *see* relations (equations)
  - ...: imposing ...

- indexed family, *see* tuple
- induction, 98-104, 132, *see also* recursion and Zorn's Lemma
- initial and terminal objects, **191-192**, 196
- as (co)domains of zeroary (co-)operations, 13-14, 332, 344, 347
  - categories with (co)limits but no –, 266
  - classes of functors respecting –, 253
  - existence results for –, 262-263
  - expressed in terms of other constructions, 193, 217, 246
  - of categories **Rep(V, W)**, 371-372
  - other constructions expressed as –, 213-214, 218, 331, 332, *see also* categories: auxiliary
  - represent trivial functors, 217, 366
  - zero objects, **191-192**, 194, 196, 366-367
- integers, 12, *see also* natural numbers
- Fermat's Last Theorem, 123
  - Fibonacci numbers, **100**
  - Gaussian –, **75**
  - $p$ -adic –, 229-**230-232**, 233, 238-239
- intersections: families closed under, *see* closure operators
- invertible, *see also* unicity: of two-sided ... and matrices: group ...
- elements in monoids, 66, 173, 178, 204, 216, 221, 291, 296, 344-345
  - elements in rings, 231, 271
  - morphisms in categories, *see* isomorphisms
  - one-sided – elements, 63, 185, 189-190, 196, 216, 352, 353-354
  - one-sided – morphisms in categories, *see* retractions
- isomorphisms
- between two objects form a heap, 306
  - classes of algebras closed under – ('abstract classes'), 301
  - in a category, **185**, 190, 191, 206
  - of bifunctors, 219-228, 267-270, 272
  - of categories, 205, 207, *see also* categories: equivalence of
  - of functors, 202, 203, 206, 210, 214-216, 219-225, 325, 351-352, 353, 355, 368, *see also* functor(s): representable ... and categories: equivalence of
- $K$ -theory, 66
- lattices, 8, **126-140**, 147, 225, 322
- and closure operators, 141-143, 144-145
  - antiisomorphisms of –, **148**
  - ( $< \alpha$ -)(semi)complete (semi)–, 132-**133-136**, 137-140, 142, 143, 194, 266, 276, 291, 326, 327
  - Brouwerian –, 124
  - cofinal sub–, 129
  - compact elements in –, **138**
  - compatibility identities, **126**, 127, 137
  - concrete –, *see* closure operators
  - distributive –, **131**, 368
  - duality between distributive – and partially ordered sets, 183, 272, 374
  - fixed point theorem for complete –, 134
  - greatest and least elements in (semi)–, 128, 374
  - homomorphisms of ( $< \alpha$ -complete) (semi)–, 128-129, 136, 138-140
  - ideals and principal ideals in –, **142**
  - made into categories, 257
  - modular –, 34, **130-131**
  - of equivalence relations, congruences, 138-140, 180, 182, 216, 268, **278**
  - of subalgebras of an algebra, 127, 132, 137, **276-277**
  - of sub(semi)– of (semi)–, 130
  - of varieties of algebras, 290
  - power sets as –, 127, 130-131
  - representable functors to or from (semi)–, 367, 381
  - ring-theoretic notation for – (not used here), 127
  - semi–, **125-126**, 127-129, 145-147, 225, 268, 322
  - sub(semi)–, **128**
  - underlying partially ordered sets of (semi)–, **125**, **127**, 129, 225, 368
  - universal constructions for (semi)–, 128-129
  - upward and downward generating numbers of –, **136**
  - with group action, 242

- Lie algebras, 147, 307-**308**-312, *see*  
*also* group(s): Lie  
 – and Lie groups, 310-311  
 – of vector fields, 309-311  
 $p$ - , 312  
 universal enveloping algebras of –, 225, 308, 343
- limits and colimits, 241-**242**-267, 321  
 – are adjoints to diagonal functors, 244  
 – as objects with universal cones, **245**, 249, 250, 251, 255  
 – as representing objects, 244  
 categories having small –, **242**, 255-259, 262-267  
 comparison morphisms for –, **255**, 256-264  
 direct and inverse limits, 232-**234**-243, 245-247, 252, 254, 258-261, 278-282, 291, 299, 302, 354  
 functors respecting –, 247-248, **249**-261, 345-346, *see also* –: comparison morphisms  
 – in functor categories, 204, 368-369  
 – in varieties of algebras, 277, 287, 291-292, 294  
 ‘inductive’ and ‘projective’ (= direct and inverse) limits, 234, 240  
 – of –, 252, 256-261  
 – of identity functors, empty functors, 246, 250, 254, 263  
 – of partially ordered sets, 375  
 – of systems of representable functors, 354  
 – used in getting other universal constructions, 262-265, 340-341, *see also* adjunctions: Freyd’s ...
- logic and model theory  
 axiomatic model classes, theories, **151**, 153, *see mainly* varieties ...  
 compactness arguments, 239  
 Galois connection between propositions and models, 151, 152  
 propositions, 95, 155  
 rules of inference for a theory, 153  
 word problems, *see* normal forms
- loose usage, 11, 12, 21, 29, 36, 40, 78, 88, 125, 167, 211, 234, 278, 285, 289
- maps, **11**, **166**, *see mainly* morphisms and functions
- matrices  
 category with – for morphisms, 156, 163, 206  
 determinants of –, 34, 271, 275, 330-331, 338, 341, 369  
 free groups of –, 34  
 group of invertible –, 45, 46, 192, 271, 330-332, 338, 341, 368  
 identities satisfied by rings of –, Amitsur-Levitzki theorem, 305  
 rings of –, 206, 305, 343, 345, 368
- metric spaces, 82, 145-146  
 completion of –, 230-231  
 lattice-valued –, **139**
- minimal and maximal elements, *see* ordered sets ... , chain conditions, lattices
- miscellaneous areas  
 adjoint operators on Hilbert space, 224  
 affine geometry, affine subspaces of a vector space, **128**, 129, 131  
 algebraic geometry, 150, 346  
 base- $p$  expansions of real numbers, of  $p$ -adics, etc., 231, 239, 240  
 coloring problems, 236-237  
 continued fractions, 19  
 convex sets, polyhedra, 150  
 cross product of vectors, 311  
 differential equations, 101, 232  
 electrical circuits, 319-320  
 graphs, 95, 319, *see also* monoids: and  $E$ -systems  
 projective geometry, **128**, 131  
 quantum mechanics, 72  
 vector fields, 309-311
- modules, 56, 59, 150, 227, **358**, *see also* abelian groups, vector spaces, bimodules, tensor products  
 categories of –, 160, 211, *see also* rings: Morita ...  
 clones with **Ab** in center are theories of varieties of –, 380

- group structure on hom-sets of  $-$ , 210, 362,  
*see also* abelian groups: ...
- homomorphisms of  $-$  written on opposite  
 side to actions of ring-elements, 360
- lattices of sub- of  $-$ , 130
- $-$  over matrix rings, 368, *see also* rings:  
 Morita...
- $-$  over semirings with 0 and 1, 380, 382
- projective  $-$ , **187-188**
- representable functors among varieties of  $-$ ,  
 358-365, 368, 381
- restriction and extension of scalars, **365**
- monoid(s), **60-66**, 96-98, *see also* congruences:  
 on  $-$   
 abelian  $-$ , 65, 379  
 $-$  and  $E$ -systems, **351-357**  
 categories of  $-$ , 160  
 clones as generalization of  $-$ , 314, 382  
 constructions relating  $-$  and groups, 65-66,  
 161, 173, 204, 205, 221, 224, 303, 321,  
 343-344, 353, 355-356, 357-358, 365  
 constructions relating  $-$  and semigroups, 61,  
 66  
 functors from  $-$  to  $-$ , 346-357, 369  
 Grothendieck group of an abelian  $-$ , **66**  
 groups and  $-$  made into categories, 161, 178,  
 181, 185, 190, 194, 200, 201, 203, 211,  
 217, *see also*  $G$ -sets: as functors  
 kernel, cokernel of a  $-$  homomorphism, 64  
 left congruences on  $-$ , **260**, 279  
 multiplicative  $-$  structures of rings, 66,  
 67-69, 70, 155, 175, 225, 228, 343, 345  
 $-$  of endomorphisms of an object, 41,  
 154-155, 165, 285  
 $-$  of endomorphisms of  $\text{Id}_{\mathbb{C}}$ , 203, 382  
 opposite  $-$ , **175**, 352-353  
 partially ordered  $-$  of operators on classes of  
 algebras, **302-303**  
 $-$  rings, **70**, 75, 225, 228, 248, 343  
 ‘‘Should they be called ‘ $-$ ’ or ‘semigroups  
 with  $e$ ?’’’, 60  
 trivial  $-$ , 72  
 $-$  with cancellation, 65
- monomorphisms, **185-191**, 192, 204, 269, *see*  
*also* equalizer ...: maps ...  
 $-$  and pullbacks, 198  
 $-$  are usually the one-to-one maps, 186, 190,  
 293  
 conflicting meanings of ‘‘ $-$ ’’, 188  
 distinguished classes of  $-$  called  
 embeddings, inclusions, 190, 204-205  
 non-one-to-one  $-$ , 186, 189
- morphisms, **157**, *see also* category: ... *and*  
 functor(s): ...  
 domain and codomain of  $-$ , **158**  
 embedding or inclusion  $-$ , *see*  
 monomorphisms: distinguished ...  
 identity  $-$ , **158**, 166, 211, 214-218, 222, 223,  
 248, *see also* Yoneda’s Lemma  
 $-$  of (co)algebra objects in a category, and  
 of representable functors, **334**, **337**, 352,  
 354, 356
- natural numbers, **98**, 106, 123, *see also* growth  
 rates  
 category of  $-$ , 317, 319, *see also* varieties  
 ...: clones ...  
 functors on finite sets yield functions on  $-$ ,  
 177, 183, 185  
 semiring of  $-$ , 382  
 von Neumann construction of  $-$ , 104-105,  
 109
- neutral element, 11, 60, 352, 373, *see mainly*  
 group(s), monoid(s), rings ...  
 co- $-$ , 346-348, 357, 365-366  
 $-$  laws and definition of clonal category, 315  
 one-sided  $-$ , 356
- normal forms  
 $-$  in Boolean rings, 77  
 $-$  in coproducts of groups and monoids,  
 48-50, 51, 64, 347  
 $-$  in free abelian groups, 43-44  
 $-$  in free groups, **33**  
 $-$  in free lattices, semilattices, 34, 129  
 $-$  in monoids, 61, 63, 64, 66, 352  
 $-$  in objects with particular presentations, 41,  
 52, 63, 66, 71  
 $-$  in rings, 67-69, 71-75  
 problem of obtaining  $-$ , 33-34, 46

- unsolvable word problems, 34, 47
- van der Waerden's trick, **32-33**, 34, 49, 198, 352
- $n$ -tuples, *see* tuples
- one-one-ness, *see* monomorphisms *and*
  - equalizer ... : maps ...
  - and functors, 286
  - is not characterizable category-theoretically, 185
  - of map from  $X$  to free object on  $X$ , 21
- open questions, 34, 46, 93, 94, 130, 145, 238, 280, 295, 300, 357
  - list of – in group theory, 43
  - questions I don't know answer to, don't know whether they've been studied, 18, 130, 166, 254, 261, 267, 320, 367, 372, 382
- operads, 323
- operations, **274**, **333**, *see also* arity
  - associative – and the empty string, 48, 75, 132-134, 147
  - co–, 288, 332, **337**, *see also* algebras: (co)...
  - depending on only a subset of their indices, **313-314**
  - derivations, **307**, 311-312
  - derived –, **17-18**, 140, 163, **298-320**, **334-336**, 338, 378-380
  - distinguished elements as –, *see* –: zeroary
  - formally infinite expressions in finitary –, 44
  - generalized – (functorial, derived), **29**, 270, 298-299, *see mainly* functor(s): -ial ...
  - majority vote –, **18**, 376
  - Mal'cev conditions, 323
  - mutually commuting –, *see* commutativity:
    - between ...
    - on an object of a category, **333**, *see mainly* algebra(s): object
    - on functors, 181-182, 298-299, 316-328, 333-339, 373
    - on quotient-sets, 23
    - on  $\mathbb{R}$  (e.g.,  $x^p$ ,  $x^2 + y^3$ ), 300
  - pointwise –, 333, 373, *see also* product
  - primitive –, 17, **298**, 315, 319, 320-326
  - projection –, 17, 314-316, 324, 335, 377-380, *see also* projection ...: from product ...
  - zeroary –, 13-14, 18, 276-277, 291, 313-314, 356, 364, 366-367, 371, 373, 379, *see also* neutral element
- ordered sets (partial and total), **88-104**, 129, 145-146, 253, 254, *see also* ordering ... *and* lattices
  - $< \alpha$ -directed –, **260**, 279-280, 282, 291
  - antichains in –, **92**, 122
  - categories of –, 161, 191
  - chains in –, **89**, 93, 119-122, 129, 132-135, 143, *see also* chain conditions *and* ordering ... : total
  - cofinal subsets of –, cofinality of –, **92**, **117**, 121, 129, 135, 235
  - 'covering' relation in –, **90**
  - directed and inversely directed –, **234-236**, *see mainly* limits and colimits: direct and inverse limits
  - duality between – and distributive lattices, 183, 374
  - 'filtered' –, *see* –: directed
  - Fredman's conjecture on –, **93-94**
  - Galois connection between two –, **149**, 273
  - group actions on –, 242
  - height and width of –, **93**, **112**, 123
  - incomparable elements in –, **92**
  - inductive –, **119-121**, 122
  - initial segments of –, **109-111**, 114-115
  - interpolation properties for –, **135-136**, 234
  - intervals in –, **92**
  - isomorphisms of –, **89**
  - isotone maps of –, **89**, **90**, 128-129, 302
  - l.u.b.'s and g.l.b.'s in –, 125-127, 132-134, 135, *see mainly* lattices
  - made into categories, 161-162, 178, 181, 185, 190, 191, 193, 194, 234, 254, 258, 259
  - minimal, maximal, least, greatest elements in –, **91**, 98, 99, 128, 235, 375, *see also* lattices: ( $< \alpha$ )-(semi)complete
  - pictures ('Hasse diagrams') of finite –, **92**
  - reconstruction problem for finite –, **94**
  - underlying – of (semi)lattices, *see* lattices: ...
  - ways that – arise, 155
  - well- –, **99**, 103, 108-115, 119-123, *see also* ordinals

- ordering (partial and total), **88-104**, *see*  
*also* ordered sets ... *and* chain conditions  
 divisibility –, 89, 95, 127  
 duality in the theory of –s, **89**, 102, 126,  
 175, 181  
 extension (including linearization) of –,  
**93-94**, 121-122  
 induced – on a subset, **88**, 128  
 lexicographic –, **103-104**  
 majorize an element or set under an –, to, **92**  
 opposite (‘dual’) –, *see* –: duality  
 pre–, **95-98**, 108, 146-147  
 product –, **90**  
 strict – (e.g., ‘<’), strict isotone map, **90**,  
 161, 242  
 total (or ‘linear’) –, **88**, 93-94, 103, 108,  
 119, 122, 127, *see also* ordered sets ... :  
 chains ...  
 when morphisms in **RelSet** are –s, 188
- ordinals and cardinals, 109-**110-115**-120, 122  
 – and lattice operations, *see* lattices:  
 ( $<\alpha$ -)(semi)complete  
 – and the process of generating an algebra,  
 26, 27, 81, 144, 266, 280-282, 284, 290,  
*see also* solution-set  
 arithmetic of –, **112-114**, **115-117**  
 cardinality, **115**  
 countability, uncountability, 26-27, 44, 107,  
 115, 116, 117, 122, 129, 134, 179, 237,  
 261, 266, 280-281, 290, 299  
 decomposition of ordinal as ‘limit ordinal +  
 finite ordinal’, 266  
 inaccessible cardinals, **172**  
 limit – and successor –, **112-114**, 117-119  
 regular –, singular –, cofinality of –,  
**117-118**, 121, 172, 281-282, 290, 333, 337  
 von Neumann construction of –, 109-110
- pair, *see* tuple
- parentheses  
 – can be dropped for associative operation,  
 29, 32  
 need for – in defining group-theoretic terms,  
 16
- pedagogy and philosophy, *see also* course ...  
*and* heuristics ...  
 anthropology of mathematics, 146-147  
 approach to teaching, 5-6  
 ‘‘Are mathematical objects real?’’, 123-124  
 handwaving, 6  
 ‘‘or’’ vs. ‘‘and’’ in freshman math, 152  
 petty details, 5  
 threefold structures of many mathematical  
 concepts, 145-148
- permutation, 38, *see also* group(s): symmetric  
 group concept comes from properties of –s,  
 32, 49, 146, 154  
 sign of a –, 305
- polynomial, *see mainly* free: commutative  
 rings, *and* symmetric: ring ...  
 differentiation of –s, 72, 151  
 – functions on  $\mathbb{N}$  and  $\mathbb{R}$ , 108, 127  
 zero-sets of –s, 150, 182
- power, *see mainly* sets: power *and references*  
*there*  
 object with finite –s yields clonal category,  
 324  
 – of an object or a functor (i.e., product of  
 copies), **195**, 203, 296, 355, *see*  
*also* operation
- preorder, *see* ordering ... : pre-  
 presentation(s), **40**, 213, 215, **286-287**,  
 294-296, 331, 340, 341, 342, 350, 354  
 bounds on numbers of generators, relations  
 needed in –, 41, 45  
 canonical – for an algebra, 222  
 every algebra is direct limit of finitely  
 presented algebras, 237  
 finitely presented object, **41**  
 – of coproducts, 47-48  
 – of groups, 38-**40-47**, 59, 65  
 – of lattices and semilattices, 128-129  
 – of monoids, 61-66, 70  
 – of partially ordered sets, 91  
 – of representing algebras for functors, 331,  
 337-338, 343-344  
 – of rings, 70, 71-73, 75  
 – of sets, 78  
 – of tensor products, 57-58, 361-363

- relators in  $-$ , **40**, 57
  - with empty set of relations or generators, 41, 296, 343
- pretty pictures, 27, 50, 79-80, 92, 196, 208, 341, 353
- product (of sets, algebras, objects in a category), 50, 56, **193**-195, 197, 259, **277**, 333
  - as adjoint, 225-226
  - as domain-object of operation, **333**
  - as functor, 183, 204, 211
  - as limit, 241, 254
  - as representing object, 216
  - as terminal object of auxiliary category, 213
- associativity and commutativity of  $-$  construction, 78
- category having  $-$ s, **193**, 210, *see also* varieties ... : clones ...
- diagonal map into  $-$ , 52
- ‘direct  $-$ ’ (classical term), **53**, 277
- example of  $-$  not based on  $-$  of underlying sets, 195
- functors respecting  $-$ s, 195, 279, 333, *see also* varieties ... : clones ...
  - of algebras doesn’t depend on variety, 53, 277
  - of categories, **184**, 207-209
  - of chains, as lattice, 127
  - of empty family, 72, 78, 193, 366
  - of functors, 353-354
  - of groups, 47, 52-54
  - of monoids, 63-64, 97
  - of partially ordered sets, 90, 103, 375
  - of rings, 72, 74, 76-77
  - of  $-$ s, 193
  - of sets, 78-79, 105
  - of too many objects, 194
- second universal property of  $-$  groups, 51-52, 74
- subobject-of-  $-$  constructions, 25-29, 37, 39, 43, 61, 81-82, 83, 245-246, 262-263, 285, 293, *see also* solution-set
- topology on  $-$ , 81, 239, 287
- varieties are closed under  $-$ , 292, 293, 300-303
- projection maps
  - from limit objects, 231, 234, **242**
  - from product objects, 47, 184, **193**, 235, 298, 314-316, 318-319, 335, 377-380
  - from pullback objects, **197**
- projective (modules and other objects), 187-188
- pullbacks and pushouts, **196**-198
  - and general limits, colimits, 241, 257
  - and monomorphisms, epimorphisms, 198
  - as (co)products in auxiliary categories, 197
  - as functors, 201
  - as initial and terminal objects in auxiliary categories, 213
  - obtained from (co)products and (co)equalizers, 197
- recursion, 16, 23, **100**-102, 109, 112-**113**-114, 115, 118-119, 120, 280, 283-284, *see also* foundations ... , *and* Zorn’s Lemma
  - using DONE, **109**, 115
- relation (on a set or family of sets), 23, **88**, *see also* ordering ... , *and* equivalence relation
  - antireflexive  $-$ , **90**
  - antisymmetric  $-$ , **88**
  - as generalization of function, 164, *see also* **RelSet** in *symbol index*
  - composition of  $-$ s, 164
  - induces Galois connection, **148**, 149, 198, 254, 261
  - reflexive  $-$ , **88**
  - restriction of a  $-$  to a subset, **88**
  - symmetric  $-$ , 95
  - transitive  $-$ , **88**
- relations (equations) in an algebra, 14, 20-26, **286**, *see also* term(s)
  - and identities, 289
  - and representable functors, 296
  - expressed using elements of free algebras, 40, 296
- imposing  $-$  on an object, 36-38, 61-63, 64, 71-73, 91, **278**, 341, *see also* presentations, coequalizers, *and* identities: ...
  - in fields, 28

- in  $G$  are identities of  $G$ -sets, 38, 305
- relators, *see* presentations
- representations (of algebraic structures), 34
  - Cayley's theorem and analogs, **154-160**, 178, 217
  - of Boolean rings by subsets of sets, 77, 78
  - of categories by maps among sets, 158-160, 178, 217
  - of clonal categories by algebra and coalgebra objects, 319, 335, 338, 378
  - of groups, monoids by set-maps, algebra automorphisms etc., 34, **70**, 154, 155, 217, *see also* permutations,  $G$ -sets
  - of partially ordered sets in power sets, 91, 155
  - of rings by linear maps, 69, 343
  - of (semi)lattices, 127, 138-140, 141-143
  - of various structures, made into functors, 200, 319
  - universal –, 343, *see also* –: Cayley .. and normal ...: van der Waerden
- representing objects, *see* functor(s), representable
- retract(ion)s, 189-190, 194
  - of (abelian) groups, 189
  - of categories, 355
- rings and  $k$ -algebras, **66-67-78**, 84-85, 97-98, 228, 242, 248, 253, 271, *see also* field, integers, Lie algebras, matrices, modules and subheadings under abelian groups, monoids, tensor products
  - associative if contrary not stated, **67**
  - (bi)commutants of – of endomorphisms, **150**, 151
  - categories of –, 160
  - chain conditions (*q.v.*) on – (Artinian and Noetherian conditions), 99, 102
  - characteristic of –, 305
  - commutative –, **67**, 103-104, 147, 182, 330-332, 338, 341, 346, 364
  - derivations on –, **307**, 311-312
  - division –, **28**, 147
  - factor –, 63
  - factoring elements of – into irreducibles, 99
  - functor from – to Lie algebras, 225, 308, **320**, 343
  - Golod-Shafarevich construction, 46
  - Hensel's Lemma, 231
  - ideal and subring lattices of –, 130
  - integral domains, 71, 238, 291, 377
  - Jordan –, 312
  - localization of commutative –, 66, 231
  - Morita contexts, Morita equivalent –, 156-157, 165, 207, 368
  - nonassociative –, **67**, *see also* Lie algebras
    - of endomorphisms, 69, 150
    - of formal power series, 216, 227, 232, 238, 343
  - '– of noncommuting polynomials', *see* free: rings
  - opposite –, **175**, 364
  - $p$ -adic numbers, *see* integers: ...
  - "pointed" – are trivial, 367
  - principal ideal domains, 237
  - restriction and extension of scalars, **365**
  - semirings or half-rings, **380**, 382
  - symmetric – on abelian groups and  $k$ -modules, **71**
  - Weyl algebra, **72**, 151
    - with and without "1", 67, 358
    - with involution, **147**
    - with polynomial identity ("PI"), 187, 305
- semigroup(s), 73, 321, 356-357, 365-366, 371, *see mainly* monoids
  - categories of –, 160
  - zero and one-sided zero elements and multiplications in –, **356**
- semilattices, *see* lattices: semi-
- set(s)
  - category of –, 160, 170, 173, 175, 177, 178, 179, 243, 244, 255, 256-261, 273, 279, 286, 291, 302, 321, 328, 371, 374, 380
  - category of finite –, 177, 183, 185, 203, 206
  - cofinite sub–, **78**, 165
  - disjoint union of –, *see* coproduct: of sets
  - duality between finite – and finite Boolean rings, 206, 375-376
  - large and small –, *see* foundations ...: ... and solution-set ...
  - opposite of category of –, 356
  - pointed –, 356

- power  $-$ , **76-78**, 106, 180, 181, *see*  
*also* lattices: ..., functors: ..., topological  
spaces: ...
- structure of the product and squaring  
functors on  $-$ , 326
- $-$  theory, *see* foundations ...
- underlying  $-$ , **11**, *see mainly* functors:  
forgetful
- sfield, skew field, *see* rings ...: division
- solution-set condition, 262-**263**-266, 282, 293,  
324, 345-346
- $S$ -sets ( $S$  a monoid), *see*  $G$ -sets
- Structure and Semantics, *see* varieties ...: ...
- subobjects, *see* algebras: sub- *and*  
monomorphisms: distinguished classes
- substitution into terms, *see* operations, derived
- surjectivity, *see* coequalizer ...: maps ... *and*  
epimorphisms
- $-$  and functors, 286, 327
- $-$  is not characterizable category-  
theoretically, 185
- symmetric
- $-$  elements in free algebras, coproduct  
algebras, 70, 103, 367
- $-$  ring on an abelian group or module, **71**
- tensor product(s)
- $-$  and adjunctions, 226-227, 228, 361, 362
- $-$  and the Hom functor, 59, 227, 362
- $-$  as functor, 183, 211, 345
- commutativity of  $-$  functor, 202
- “nonabelian  $-$ ”, 58
- $-$  of abelian groups, **57-60**
- $-$  of modules and bimodules, 57, 165,  
**361-363-365**
- $-$  of rings and  $k$ -algebras, **73-75**, 97, 364
- tensor powers of a vector space, 205
- tensor rings and algebras, **71**, 225, 228, 343
- term(s), **15**, 18, 20, 22-25, 28, 29-34, 38, 40,  
**283**
- $-$  algebra as free  $\Omega$ -algebra, 282-284
- constructions by  $-$  modulo equivalence  
relation, 22-25, 37, 39, 43, 293
- evaluation of  $-$ , 16-17, 20, 24, 32, **297-299**
- group theoretic  $-$ , **14-17**
- ternary, *see* arity, heaps
- theory, *see* logic ... *and* varieties ...
- these notes, 5-9, *see also* course ...
- exercises in  $-$ , 8
- numbering of statements in  $-$ , 8
- topics to be added to  $-$ , 3, 34, 207, 225,  
240, 262, 308, 316, 368, 375, 377, 383
- typographical errors in  $-$ , 9
- topological spaces, 92, 98, *see also* metric  
spaces, vector spaces: linearly compact,  
abelian groups: ... *and* algebras: ...
- Cantor set, 287-288
- cohomotopy groups of  $-$ , 183
- compact  $-$ , 104, 138, 236, 327-328, *see*  
*also* logic ...: ...
- connected components of  $-$ , 82
- embedding of  $-$ , **82**
- fundamental groups of  $-$ , 154, 156, 163,  
179, 207, 339, 381
- homotopy of maps between  $-$ , 85-86, 164,  
175, 178, 179, 183, 339
- lattices and closure operators associated with  
 $-$ , 130, 134, 140, 141, 144
- order-of-limit questions in  $-$ , 253
- path-lifting property for maps of  $-$ , **85**
- power sets as  $-$ , 182
- Stone-Ćech compactification of  $-$ , 79-**80-85**,  
225, 266, 327
- topological groups, 83, 85, 182, 381, *see*  
*also* group(s): Lie *and* abelian groups:  
solenoids
- (universal) covering spaces of  $-$ , **85-86**
- Urysohn’s Lemma, 83, 266
- vector bundles on  $-$ , 66
- $-$  with ACC on open sets, 104
- tuple, **13**, 16
- conventions on  $n$ --s, **13**, **112**
- “pair” vs. “2--” in foundational context,  
105
- ultrafilter, 80
- unary, *see* arity
- unicity, *see also* monomorphisms,  
epimorphisms
- $-$  in recursive constructions, 100, 113
- lack of  $-$  in axiom of choice, 107

- of components of ordered pairs, 105
- of free generating set for free algebras in some varieties, 285, 288
- of homomorphism specified on generating set, 20, 26, 39, 262
- of partial isomorphisms between well-ordered sets, 108-111
- of reduced expressions for elements of free group, 31
- of set of  $\Omega$ -terms in  $X$ , 15
- of two-sided inverses, 40, 186, 344, 353
- of universal objects, and their canonical isomorphisms, 21, 39, 62, 80, 191, 203, 213, 225
- units, *see* invertible elements *and* adjunctions: ...
- universal elements, 21, 214-216, *see also* universal properties
  - for representable functors, **215**, 217, 223
- universal properties, 21-85, 184, 241-242, 254, 262, *see mainly specific universal constructions*
- existence theorems (general) for objects with  $-$ , 261-267, *see also* product ... : subobject-of- ...
- nonexistence of objects with some  $-$ , 22, 28, 56, 253, 265-266, 346
- of free groups, **21-22**, 26, 27, 31
- right and left  $-$ , 53-56, 213, 243, 248, 272-273, 277, 282, *see also* heuristics ... : for ...
- universal quantification ( $\forall$ ), **11**
  - as conjunction, 153
  - classes of algebras defined using only  $-$ s, 12
- universe, *see* foundations ... : ...
- $\mathbb{U}$ -small etc., *see* foundations ... : large .... *and* categories: large ...
- van der Waerden's trick, *see* normal forms: ...
- varieties and equational theories, **290-307**
  - as categories of representations of a clonal theory **X**, **318**
  - Birkhoff's **HSP** Theorem and related results, 301-303
  - categories that are not varieties, 291, 293, 327
  - $\circ$ -idempotent  $-$ , **382**
  - clones, clonal theories, **314-317-318-326**, 327, 335, 336, 338, 374-375, 376, 377-382, *see also* functors: underlying-set-preserving  $-$
  - equational theory generated by a set of identities, **291**
  - Lawvere's definitions of  $-$ , 316, 317, 319
  - Structure and Semantics, 324-**325-327**, 365, 369, 374-375, 377
  - subvarieties of varieties of groups and monoids, 304, 321
  - subvarieties of varieties of  $G$ -sets,  $R$ -modules, 305, 326, 328
  - subvarieties of varieties of rings, 305
  - trivial and nontrivial  $-$ , 375
  - varieties generated by sets of algebras, **291**, 299, 300-307
  - “When do we consider two varieties the same?”, 306, 378
  - “Which categories are equivalent to varieties?”, 326
  - vector spaces, 237, *see also* tensor products
    - bases of  $-$ , 119, 148, 176
    - category of  $-$ , 206
    - duality of  $-$ , 150, 151, 180, 182, 202, 206, 375, 377, 381
    - linearly compact and other topological  $-$ , 150, 151, 240, 375
  - word problems, *see* normal forms
  - Yoneda's Lemma, Yoneda embedding, **217-219**, 220, 222, 255, 299, 316, 326
    - and (co)algebra objects in a category, 333, 334, 336, 339
  - Zermelo-Fraenkel set theory, ZF, ZFC, *see* foundations ... : axioms ...
  - zero, *see* operations:  $-$ -ary *and* semigroups: ...
  - Zorn's Lemma, 118-**119-121**
    - equivalence of  $-$  with “weaker” statement, 134