

## Symbol Index

Symbols of standard and uncontroversial usage are generally not included here.

As in the word index, boldface page-numbers indicate pages where definitions are given. If a symbol is defined in one place and used again without explanation more than a page or so away, I show the page(s) where it is defined, and often some of the pages where it is used or where the entity it symbolizes is discussed. But I do not attempt, as in the word index, to show all significant occurrences of each subject. For this, the word index, with its headings and subheadings, is more useful.

Order of entries: Under each letter of the alphabet, the lower-case letter is followed by the upper-case letter, then Greek and miscellaneous related symbols, in a somewhat arbitrary order. (For a particularly complicated example, the order I have set up under  $p$  is  $p, P, \pi, \Pi, \mathbb{P}, \varphi, \Phi, \psi, \Psi$ , though not all of these symbols actually occur.) Symbols that are not even approximately alphabetical are alphabetized by assigning them spellings; e.g.,  $\wedge$  and  $\vee$  are alphabetized as meet and join; the symbol  $=$ , and related symbols such as  $\cong$ , are alphabetized, in an arbitrary order, under equal; and  $\mapsto$  and  $\downarrow$  are similarly alphabetized under arrow. Fortunately, you do not have to know all the details, though some symbols will require more search than others.

Font-differences, and ‘‘punctuation’’ such as brackets, do not affect ordering unless everything else is equal. Operator-symbols are often shown in combination with letters with which they are commonly used, e.g.,  $\langle X | Y \rangle$  is alphabetized under  $XY$ .

Of the many categories that are given names in Chapter 6 and subsequently, we do not record cases where the meaning is obvious, like **Group**, nor cases discussed only in passing, like **GermAnal** (germs of analytic functions, which can be found under ‘‘functions’’ in the word index), but only category-names used in more than one place, for which some aspect of the definition (e.g., the associativity assumption in **Ring**<sup>1</sup>) or the abbreviation (as with **Ab**) is not obvious.

$\forall$	for all (universal quantifier), <b>11</b> , 152.
$\text{ari}_{\Omega}$ , <b>ari</b>	arity function ( <i>see also</i> $\Omega$ ), 17, <b>274</b> .
$\mapsto$	indicates action of a function on elements, <b>11</b> .
$\downarrow$	<i>see</i> $(S \downarrow T)$ <i>below</i> .
$ A $	underlying set (etc.) of object $A$ , <b>11</b> , <b>274</b> , <b>334</b> .
<b>Ab</b>	category of abelian groups; also, prefix for ‘‘abelian’’ in names of categories such as <b>AbMonoid</b> , <b>160</b> .
$A/\sim$ , $A/(s_i = t_i)_{i \in I}$	quotient-set or factor-algebra of $A$ , <b>23</b> , <b>37</b> , <b>62</b> .
$\text{Ar}(\mathbf{C})$	set of morphisms (‘arrows’) of the category $\mathbf{C}$ , <b>157</b> .
$A^*$	set determined by $A$ under a Galois connection, <b>148-153</b> .
$\text{Aut}(X)$	group of automorphisms of $X$ , 45, 154, 175-176.
$\aleph_0$ , $\aleph_{\alpha}$	least (resp. $\alpha$ th) infinite cardinal, 27, <b>116</b> , 117.

- Binar**, **Binar**<sup>e</sup> variety of sets with a single binary operation, respectively, with binary operation having neutral element, **365**, **371**, 380.
- $\beta: (A, B) \rightarrow C$  bilinear map (temporary notation), **56-60**.
- cl general symbol for a closure operator, **140**.
- Cat** category of all (small) categories, **177**.
- C<sup>D</sup>** category whose objects are all functors  $D \rightarrow C$ , **201**, 207-210.
- Clone**, **Clone**<sup>( $\gamma$ )</sup> category of all covariant ( $<\gamma$ -)clonal categories, **317-318-327**, 382.
- Cl(V)**,  $Cl_n(V)$  clonal theory of variety **V**, and its objects, **318**.
- CommRing**<sup>1</sup> the variety of all commutative associative rings with 1 (referred to simply as “commutative rings”), **160**, 227, 330-332, 338, 341.
- Cong  $f$  subalg. of  $S \times S$  whose underlying set is congruence induced by  $f$ , **64**.
- C<sup>pt</sup>** category of pointed objects of **C**, **192**, 199, **366**.
- C(X, Y)** set or (in Chapter 9) algebra of morphisms  $X \rightarrow Y$  in the category **C**, **157**, **334**, **337**.
- $\mathbb{C}$  the complex numbers.
- deg( $x$ ) (in §9.6) degree of element  $x$  in a comonoid object of **Monoid**, **348-350**.
- $\Delta$  diagonal functor  $C \rightarrow C^D$ , **226**.
- $\cong$  isomorphism (of algebras, categories, etc.).
- $\approx$  equivalence of categories, **206**.
- End( $X$ ) monoid of endomorphisms of  $X$ , 45, 154.
- $\langle X | Y \rangle$ ,  $\langle X | Y \rangle_V$  object (of **V**) presented by generators  $X$ , relations  $Y$ , **40**, **286**, **295**.
- E(X)** lattice of equivalence relations on  $X$ , **138-140**, 216.
- $\eta$ ,  $\varepsilon$  unit and counit of an adjunction, **223**.
- $\exists$  there exists (existential quantifier), **11**, 152.
- $f|X$  the restriction of the function  $f$  to the set  $X$ , 31, **100**.
- $F_\Omega$ ,  $F_V$  free-algebra functors of categories of algebras (*see mainly free in word index*), **285**, **295**, 301.
- $G^{ab}$  abelianization of  $G$ ; *see group: abelianization in word index*, **45**.
- $G_{cat}$  *see S<sub>cat</sub> below*.
- $GL(n, K)$  general linear group (group of all invertible  $n \times n$  matrices), 45, 46, 271, 345, 369.
- $G_{md}$  “underlying” monoid of the group  $G$ , **65-66**, 161, 175.
- $h_Y$ ,  $h^Y$  covariant and contravariant hom functors,  $C(Y, -)$  and  $C(-, Y)$ , **179**, **182**, 184.

- $\mathbf{H}(C)$  set of all homomorphic images of algebras in the set  $C$ , **301**, 372.
- $\text{Hom}(X, Y)$  in introductory chapters, set of homomorphisms from  $X$  to  $Y$ ; mostly superseded by  $\mathbf{C}(X, Y)$  in later chapters.
- HtpTop** category of topological spaces, with homotopy classes of maps for morphisms, **164**, 175, 178, 183.
- HtpTop**<sup>(pt)</sup> as above, but defined using pointed topological spaces, **339**, 381.
- $\text{id}_X$  identity morphism of the object  $X$ , **158**.
- $\text{Id}_C$  identity functor of the category  $C$ , **175**, 203, 206, 209-210.
- $\subseteq, \subset, \supseteq, \supset$  inclusion relations, **90**.
- $\vee$  “join” (in a (semi)lattice); “or” (disjunction of propositions), **11**, **125**, **126**, 153.
- $K\langle t \rangle, \mathbb{Z}\langle x_1, \dots, x_n \rangle$  free  $K$ -algebra or ring (“noncommutative polynomial ring”), 68-69.
- $K_f$  congruence determined by the map  $f$ , **62**, 64.
- $KS, \mathbb{Z}S$  monoid algebra or ring, **70**.
- $K[t], \mathbb{Z}[x_1, \dots, x_n]$  polynomial algebra or ring, 28, 67-68.
- $\leq, <, \geq, \preceq, \prec$ , etc. symbols for partial orderings or preorderings, **88**-104.
- $\limsup$  limit superior, **19**, 97.
- $\varprojlim, \varinjlim$  limits and colimits (including inverse and direct limits); *see mainly word index*, **234-242-267**.
- $\lambda, \mu, \rho$  (in §9.6) indices for writing coproducts of copies of  $R$ , e.g.,  $R^\lambda \amalg R^\mu \amalg R^\rho$ , and their elements, e.g.,  $x^\lambda$ , **347-353**, 357, 367.
- $\wedge$  “meet” (in a (semi)lattice); “and” (conjunction of propositions), **11**, 23, 153.
- $M_3$  ternary majority vote operation, **18**, 376.
- $M_5, N_5$  5-element “forbidden sublattices” in characterizations of distributivity and modularity, **130**, 136.
- Monoid** category of all monoids, **160**.
- $\mu, \mu_G$ , etc. composition operation of a group, monoid, or category (when there is need to show it explicitly), **11**, **157**.
- $\mathbf{N}$  natural numbers as subcategory of **Set**, **317**.
- $\mathbb{N}$  natural numbers as monoid, ordered set, etc., **317**.
- $1, 1_L$  multiplicative neutral element of ring; greatest element in a lattice or partially ordered set, 66, **132**, 366.
- $( )^{\text{op}}$  opposite (of a partially ordered set, semigroup, monoid, ring, or category), **89**, **175**, **180-181**, 186.

- $\text{Ob}(\mathbf{C})$  object-set of the category  $\mathbf{C}$ , **157**.  
 $\omega, \omega_\alpha$  least infinite ordinal (= {natural numbers}), respectively least ordinal of cardinality  $\aleph_\alpha$ , **110-118, 122**.  
 $\Omega$  algebra type (family of operation-symbols with specified arities), **17, 19, 274-299**.  
 $\Omega\text{-Alg}$  variety of all algebras of type  $\Omega$ , **276-280, 285-287, 293-294**.  
 $\mathbf{P}(C)$  in §8.6, set of all products of algebras in the set  $C$ ; otherwise, see  $\mathbf{P}(S)$  below, **301, 372**.  
 $\text{POSet}, \text{POSet}_<$  categories of partially ordered sets and isotone, respectively strict isotone maps, **161**.  
 $\mathbf{P}(S)$  power set (set of all subsets) of  $S$ , **76-78, 89, 91, 106, 122, 127, 216**.  
 $\pi_1(X, x_0)$  fundamental group of the pointed topological space  $(X, x_0)$ , **154, 156, 163, 179, 207, 339, 381**.  
 $\prod_{i \in I} X_i, \coprod_{i \in I} X_i$  product, coproduct of a family of objects  $X_i$ , **193-194**.  
 ${}_R B_S$  notation emphasizing that  $B$  is an  $(R, S)$ -bimodule, **360**.  
 $\text{RelSet}$  category with sets for objects, relations for morphisms, **164, 173, 176, 183, 188**.  
 $\text{Rep}(\mathbf{C}, \mathbf{V})$  category of all representable functors from  $\mathbf{C}$  to  $\mathbf{V}$ , **339-383**.  
 $\text{Ring}^1$  the variety of all associative rings with 1 (referred to simply as ‘‘rings’’), **160, 358**.  
 $R^\lambda, x^\rho$ , etc. see ‘‘ $\lambda, \mu, \rho$ ’’, **347**.  
 $R\text{-Mod}, \text{Mod-}R$  categories of left, respectively right  $R$ -modules, **160, 358**.  
 $R\text{-Mod-}S$  variety of all  $(R, S)$ -bimodules, **360, 380**.  
 $|R|^\sigma$  in §9.6, set of elements in coproduct monoid  $R^\lambda \coprod R^\mu \coprod R^\rho$  associated with index-string  $\sigma$ , **348**.  
 $s$  in §9.3, co-operation corresponding to an operation  $s$ ; in §9.8,  $s_d =$  co-scalar-multiplication by  $d$ , **337**.  
 $s_f, s_A$  function evaluating a term  $s$  at a tuple  $f$ , respectively on the whole algebra  $A$ , **16-17-25, 30-32, 43, 297**.  
 $\text{symb}_T$  map  $X \rightarrow T$  taking each element of  $X$  to the symbol representing it, **15-17, 23, 30**.  
 $(S \downarrow T), (S \downarrow C), (C \downarrow T)$  ‘‘comma categories’’, **199**.  
 $S_{\text{cat}}$  category constructed from the monoid, group, or partially ordered set  $S$ , **161, 162, 163, 165, 178, 181**.

- $\mathbf{S}(C)$  set of all subalgebras of algebras in the set  $C$ , **301**, 372.  
**Semigroup** category of all semigroups, **160**.  
 $\mathbf{Set}_{(\mathbb{U})}$ ,  $\mathbf{Group}_{(\mathbb{U})}$ , etc. explicit notation for categories of  $\mathbb{U}$ -small objects, **170**.  
 $S^{\text{gp}}$  universal enveloping group of the monoid  $S$  (obtained by adjoining inverses to all elements), **65-66**.  
 $\text{SL}(n, K)$  special linear group (group of  $n \times n$  matrices over  $K$  with determinant 1), 34, 330-332, 341, 343, 369.  
 $S_n$  symmetric group on  $n$  elements, 27.  
 $\otimes$  tensor product, **57-60**, 155, **361**, **363**.  
**2** diagram category with picture  $\cdot \rightarrow \cdot$ , **162**.  
 $T_{\text{red}}$  (in Chapter 2) set of reduced group-theoretic terms, **30**.  
 $T, T_X, \cdot, ^{-1}, e, T_{X, \Omega}$  (in Chapters 1-3) the set of all terms in a set  $X$ , and given operation-symbols, 14-**15-25**, 30-34, 38, 102.  
 $U_{\Omega}, U_{\mathbf{V}}$  underlying-set functors on categories of algebras (*see mainly* functors: forgetful *in word index*), **285**, **295**.  
 $\mathbb{U}$  a universe; *see* foundations ... : universes *in word index*, **169**.  
 $\sqcup$  disjoint union (coproduct) of sets, **48**, 78.  
 $\mathbf{Var}(C)$  variety generated by a set  $C$  of algebras, **291**, 301-304.  
 $\mathbf{V}(J)$  variety of algebras defined by the set  $J$  of identities, **290**.  
 $\mathbf{V} \circ \mathbf{W}$  variety equivalent to category of  $\mathbf{W}$ -algebra objects of  $\mathbf{V}$ , **378-382**.  
 $[x]$  frequently, equivalence class of  $x$  under an equivalence relation, 23, 95.  
 $[x, y]$  variously, commutator in a group or ring, Lie brackets, interval in a partially ordered set, **33**, **44**, **92**, 225.  
 $x^y$  conjugate of  $x$  by  $y$ , i.e.,  $y^{-1}xy$ , **33**.  
 $X \cap Y, \bigcap X_i$  intersection (of  $X$  and  $Y$ ; of the sets  $X_i$ ), 19, 76.  
 $X \cup Y, \bigcup X_i$  union (of  $X$  and  $Y$ ; of the sets  $X_i$ ), 19, 76.  
 $X^I$  the set of all maps  $I \rightarrow X$ , **13**.  
 $0, 0_L$  additive neutral element; least element in a lattice or p.o. set, 66, **132**, 366.  
 $\mathbb{Z}$  the integers, 12.  
 $\mathbb{Z}_n$  the group or ring  $\mathbb{Z}/n\mathbb{Z}$ , 42, 59, 229-231, 232-233, 296.  
 $\mathbb{Z}S, \mathbb{Z}[t], \mathbb{Z}\langle x_1, \dots, x_n \rangle$  *see*  $KS, K[t], K\langle x_1, \dots, x_n \rangle$  *above*.  
 $\hat{\mathbb{Z}}_{(p)}$  the  $p$ -adic integers (*see* integers: ... *in word index*), **230-232**.