

CLASSICAL AND QUANTUM INERTIA: A MATTER OF PRINCIPLES

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“But I, Simplicio, who have made the test can assure you that ...”
Galileo Galilei,¹

A simple, general discussion of the problem of inertia is provided both in classical physics and in the quantum world. After briefly reviewing the classical principles of equivalence (weak (WEP), Einstein (EEP), strong (SEP)), I pass to a presentation of several equivalence statements in nonrelativistic quantum mechanics and for quantum field vacuum states. It is suggested that a reasonable type of preferred quantum field vacua may be considered those possessing stationary spectra of their vacuum fluctuations with respect to accelerated classical trajectories.

1. INTRODUCTION

Either empirical evidence or pure thought scientific belief (i.e., supported by some mathematics) can produce powerful physical principles for fundamental theories whenever an appropriate interpretation is provided. This is the case of the remarkable *universality* of the *classical free fall* (Galileo’s or the Pisa free fall; first actual experiments in June 1710 at St. Paul’s in London by Newton) discovered at the very beginning of modern science, and much later, but still before the true advent of quantum mechanics, interpreted by Einstein in terms of a universal coupling of all forms of matter to a common metric tensor, an idea which was the key point in constructing general relativity. Indeed, since classical free fall motions (for the quantum case, see below), although accelerating ones, do not depend on the test mass ($t_f = \sqrt{2h/g}$), one may think of relating them to fundamental predynamical (geometrical) properties of the universe.

On the other hand, by his second law, Newton imprinted on us the idea that there is a profound relationship between material inertia and all sorts of mechanical forces. Newton classified forces into those of contact and those of action-at-a-distance. In the first case, the agent producing the force is in direct contact with the test particle (zero range forces), whereas in the latter the agent is able to exert its effect instantaneously over huge distances (infinite range forces) without any apparent transport by means of a *medium*. Newton’s second law applies more to the contact forces and/or in situations in which we can think of a close agent of forces, but of course the law is considered as general. The second category of forces are the long range forces, which usually are tackled within electromagnetism and gravitation. The discovery of a limit for the velocity of signals (i.e., the velocity of light in

vacuum, c) showed that Newton's equation may well be substituted by differential wave equations, including terms due to the limiting velocity (i.e., operators of the form $-c^2\nabla^2$). In this way, forces can be transmitted at infinite speed only if the system is connected through a 'mass' term to an infinitely rigid substrate.²

Newton was always careful with the concept of mass, as he introduced it in at least three of his basic formulas, which in fact referred to the same concept of force. Since the accelerations look different in different physical contexts, Newton distinguished between inertial, passive gravitational and active gravitational masses. A well-known review of Bondi,³ on these topics where the negative mass concept is introduced is good reading. Later, the negative gravitational mass has been restricted to antimatter by Morrison and Gold.⁴

If one focuses more on the concept of inertia there occurs another difficulty, as we cannot be sure that this apparently genuine feature of the test particle is determined by the local conditions or by some sort of global interaction with the whole rest of the universe, a famous alternative known as Mach's cosmological principle (for reviews and connection with the notion of isotropic singularity, see Ref.[5]). As is well known, already in 1710 Bishop Berkeley objected to Newton's absolute space, insisting on the idea that it is not meaningful from the experimental point of view, thus being a forerunner of Einstein, who acted in the same way regarding the mechanical ether. In the spirit of the same idea that only systems in relative motion can be detected, Mach attributed inertial forces to acceleration relative to the "heaven of fixed stars". In more practical terms, Mach's principle of inertia (MPI) can be formulated as follows,⁶

MPI: more or less standard

The inertia of particles and bodies on earth and in the solar system is due to their acceleration relative to all matter outside the solar system.

The idea of a cosmological scale of inertia was tackled in interesting works by Sciamia,⁷ still within the framework of general relativity, and culminated in the Brans-Dicke scalar-tensor theory of gravitation.⁸

The main purpose of this paper is to bring together in one place works of many people both on the classical inertia and the non-relativistic and relativistic quantum features of this fundamental physical concept. Several formulations of the principles of equivalence are reviewed and possible connections and hints on further progress in this broad research area are suggested.

2. CLASSICAL EQUIVALENCES: WEP, EEP, SEP

Due to their very general/philosophical content the equivalence assertions are subject to many contradictory opinions. Will,⁹ has written excellent reviews presenting the various formulations of the classical principles of equivalence, as well as their tests. In Will's works one can find some of the most general statements of the classical inertia principles, which are quoted in the following. The assertions of

Galilei and Newton are known at the present time as the weak equivalence principle (WEP) and reads

WEP: Will

if an uncharged test body is placed at an initial event in space-time and given an initial velocity there, then its subsequent trajectory will be independent of its internal structure and composition.

By uncharged test body one means an electrically neutral body that has negligible self-gravitational energy and moreover is small enough in size in order to neglect the coupling to the inhomogeneities of the external fields. By means of modern Eötvös-type experiments (Roll, Krotkov and Dicke,¹⁰ Braginsky and Panov,¹¹ Adelberg,¹²) the WEP is clearly correct at a fractional precision better than 10^{-11} .

What is known as the Einstein equivalence principle (EEP) is the following statement

EEP: Will

the outcome of any local non-gravitational test experiment is independent of where and when in the universe it is performed.

By a local non-gravitational test experiment one should mean any experiment that is performed in a freely falling laboratory which is small and shielded with respect to the inhomogeneities in the external fields and for which self-gravitational effects are negligible. If EEP is valid, then gravitation must be a curved space-time phenomenon, and therefore one can think of metric theories of gravity. This is the reason why EEP is so important. The EEP as stated by Will is perhaps an excessive general phrase. Following the work of Kreinovich and Zapatrin,¹³ I quote the standard EEP (i.e., almost as given by Einstein)

EEP: standard

What ever measurements we perform inside some spacetime region we cannot distinguish between the case when there is a homogeneous gravitational field and the case when all bodies in this region have constant acceleration with respect to some inertial frame. And since any field can be considered homogeneous in a small enough region, the principle can be applied to a neighborhood of any point.

V.A. Fock,¹⁴ pointed out that this formulation is not exact enough, because in the presence of gravity the spacetime curvature tensor is nonzero, while in a uniformly accelerated frame the curvature tensor is zero.

The standard EEP shows plainly the correctness of Ohanian's opinion,¹⁵: "Einstein's theory of general relativity was conceived in an attempt at formulating a relativity of acceleration".

The most general equivalence formulation appears to be the strong equivalence principle (SEP) dealing with situations in which one considers in addition to the metric field other types of dynamical fields and/or prior-geometric fields. The hy-

pothesis is that all these fields yield local gravitational physics which may have both location and velocity-dependent effects. SEP states that

SEP: Will

the outcome of any local test experiment is independent of the velocity of the freely falling apparatus and of where and when in the universe it is performed.

The distinction between SEP and EEP is the inclusion of bodies with self-gravitational interactions (planets, stars, black-holes). Actually, SEP means the equality of the so-called passive gravitational mass and inertial mass. If SEP is valid, there must be one and only one gravitational field in the universe given by a unique metric. Laboratory experiments test only WEP in the form of composition dependent interactions of Yukawa form. Tests of SEP, on the other hand, are only possible in astrophysical environments, first of all in the Solar System, since one needs bodies with a significant contribution to their inertia from their gravitational binding energy. A useful parameter η measuring the deviations from SEP has been introduced by Nordtvedt,¹⁶ in 1968. In other words, SEP means to determine if gravitational binding energies are falling with the same acceleration. The predilect test of SEP is lunar laser ranging (i.e., the change in round-trip radar time between Earth and Moon when the radar path passes close to the Sun). The data show that the fractional difference in the falling accelerations toward the Sun between the (iron-dominated) Earth and the (silica-dominated) Moon is $(2.7 \pm 6.2) \times 10^{-13}$.¹⁷ This is still a weak binding case since the gravitational binding energy reduces the mass of the Earth by only 5.1 parts in 10^{10} . More recently, Earth-Mars ranging was proposed,¹⁸ as well as strong field regimes.¹⁹

As a matter of fact, in all forms of SEP the weak point is the notion of locality. This problem has been tackled in the important paper of Bertotti and Grishchuk.²⁰ Usually, the locality concept required by equivalence is to say that the effects of curvature on the local metric are negligible. According to Bertotti and Grishchuk, in an appropriate inertial frame and in the slow-motion approximation a *local gravitational system* can be defined whenever the measurement errors are greater than the corresponding effects of tidal forces. The formulation of SEP belonging to these authors is the following

SEP: Bertotti and Grishchuk

We say that SEP is fulfilled if, when the size r of the system is sufficiently small, its dynamical behaviour, to a given accuracy, is universal and not affected by the external world.

Moreover, these authors discuss the problem of the universality of *gravitational clocks*, commenting on the three cases previously considered in the works of Will,²¹ namely the *rotating relativistic star*, the *slowly rotating black hole* and the *binary system*. In Newtonian gravity the gravitational (Kepler) clocks can be considered *universal*, if their size is small enough. This comes out when one takes into account

the effects of a third body on the two-body system. Will has found in all the three cases the common changes in the frequency due to special relativity and the gravitational shift formula. The problem is to estimate the changes in the laws of gravitational two-body systems when the relativistic corrections are included in order to see to what approximation they can be considered universal. On the other hand, there is considerable technological interest in the development of highly stable spaceborne clocks that may lead to the detection of the gravitomagnetic field of the earth according to the recent proposal of Mashhoon and collaborators.²²

Coming back to WEP, one should mention that various authors put it at the ground of detailed theoretical constructions leading to nice mathematical consequences. One of the best known procedures is the Ehlers-Pirani-Schild scheme.²³ These authors argue that WEP has two important features from the theoretical point of view. For a space-time manifold with a pure gravitational field, one can say that (a) the possible motions of all freely falling test particles are the same, and (b) at any point p in space-time, there exists a neighborhood $U(p)$ of p and a four-dimensional coordinate system, such that the trajectories of *every* freely falling test particle through p satisfies $d^2x^\mu/d\lambda^2 = 0$ at p for a suitable parameter λ along the trajectory. This is just a local form of the law of inertia, and such a coordinate system is said to be locally inertial at p . The latter condition is a property shared only by gravitational fields, which being related to the connection coefficients, can be coordinate transformed away. Using this special property for the massive and massless cases, it was shown by Ehlers, Pirani and Schild, that there exists an affine connection ω , independent of the test particles used, such that the trajectories of freely falling test particles are affinely parametrized geodesics with respect to it.

3. INERTIA PRINCIPLES IN THE QUANTUM WORLD

Two fundamental concepts of quantum theories are the intrinsic vacuum noise known as zero point energy and the quantum state (although one may prefer the path-integral formalism). The classical concept of trajectory occurring in the classical formulations of the inertia principles can be considered a sort of zero-order approximation at the best, and the space-time picture is only one of the many possible representations. Moreover, time enters the quantum formalism merely as a parameter and it is hard if not impossible to think of a time operator as happens for other common observables. Many different concepts of quantum time and/or clocks are actively pursued. I recall here only the *optimal quantum clock* concept of Bužek, Derka and Massar,²⁴ based on trapped ions, the *tunneling times*,²⁵ and the *flavor-oscillation clock*.²⁶ Probabilistic arguments are practically unavoidable when discussing scales at and below the molecular ones, and therefore the quantum equivalence statements are expected to be substantially different from their classical counterparts. Moreover, considering quantum mechanics as a sort of wave theory, the mass parameter will manifest itself at the experimental level mainly through de Broglie, Compton, and Planck (wave)lengths, i.e., in (non-relativistic) matter

interferometry,^{27,28} particle production (one can look at some bremsstrahlung literature), and ‘gravity-wave’ interferometry,²⁹ experiments.

3.1 Nonrelativistic quantum inertia

For the nonrelativistic quantum mechanics, one can find interesting aspects of the equivalence principles in various theoretical (see e.g., Ref.[30]) and experimental approaches, especially those related to neutron,²⁷ and atom,²⁸ beam interferometry.

The first quantum experiments in the EP context have been of Galilei type (Pisa gedanken experiment) and have been performed with neutrons,³¹ according to the philosophy “let’s see how they fall !”, although the free fall of atoms in molecular beams was easily observable since late thirties, but was used for other purposes, such as to measure the Bohr magneton by compensating gravity through magnetic fields.³² The WEP has been confirmed for neutrons to within $3 \cdot 10^{-3}$ accuracy, by measuring the fall height of a neutron, initially moving horizontally at a known velocity. At present, there are hopes that experiments with ultracold neutrons,³³ can lead to an accuracy as high as 10^{-6} . Many other proposals have been made over the years, such as antimatter in free fall,³⁴ cooled atoms in optical molasses,³⁵ and opto-gravitational cavities,³⁶ and recently free-falling mesoscopic Schrödinger cat states,³⁷ and, possibly, atomic Bose condensates.

Recently, Ahluwalia,³⁸ revived an argument provided by Kenyon,³⁹ on the possible observability of constant gravitational potentials by gravitationally induced CP violation this time in the context of earth and/or solar-bound *quantum-mechanical free falls*. Using the simplest example of a linear superposition of two different mass eigenstates, which in neutrino physics led him and Burgard to the concept of flavor-oscillation clocks,²⁶ Ahluwalia argues that the redshift-inducing phases of such freely-falling clocks depend directly on the extremely small constant gravitational potential of the local cluster of galaxies, the so-called Great Attractor. It will be interesting to investigate Ahluwalia’s suggestion using the wave packet representation,⁴⁰ for both meson and neutrino oscillations. Furthermore, it is known that Kenyon’s paper was criticized by Nieto and Goldman in their classic 1991 *Physics Report*, according to the canonical opinion that no independent experimental means are available to measure absolute gravitational potentials. However, as pointed out by Ahluwalia,⁴¹ the weak field limits of classical gravity and *any* theory of quantum gravity have different behavior with respect to the gravitational potential. Thus, the possibility of such a Aharonov-Bohm type situation in quantum free-fall remains open. Also, according to Ahluwalia, for the case of quantum freely falling frames there is the possibility of violation of the *local position invariance*,⁴² which together with the *local Lorentz invariance*,⁴³ stands at the ground of EEP.

In the context of matter wave interferometry, Lämmerzahl,²⁸ proposed a generalized quantum WEP (QWEP), which reads as follows

QWEP: Lämmerzahl

For all given initial states the input independent result of a physical experiment is independent of the characteristic parameters (like mass, charge) of the quantum

system.

Lämmerzahl carefully added a few more comments clarifying the notions he was using in the above statement, and proceeded to show that some important classes of quantum quantities, like the gravity-induced phase shifts of atom beams and neutron interference experiments and the time evolution of expectation values and uncertainties support his formulation.

3.2 Relativistic field inertia

There are deep insights in the problem of quantum field inertia that have been gained in the last two decades as a consequence of Hawking effect and Unruh effect, and consequently the ‘imprints’ of gravitation in the relativistic quantum physics have been substantially clarified.⁴⁴ The method of quantum detectors (accelerated elementary particles) proved to be very useful for the understanding of the quantum field inertial features. A new way of thinking of quantum fluctuations has emerged and new pictures of the vacuum states have been provided, of which the landmark one is the heat bath interpretation of the Minkowski vacuum state from the point of view of a uniformly accelerating non-inertial quantum detector. This interpretation is mostly attributed to Unruh because of his 1976 seminal paper, although the corresponding mathematical formula has been obtained more or less at the same time by several people.⁴⁵ One can think of *zero-point fluctuations*, *gravitation* and *inertia* as the only three universal phenomena of nature. This idea has been popularized by Smolin,⁴⁶ some time ago. However, one may also think of inertia as related to a peculiar sort of collective degrees of freedom known as vacuum expectation values (vev’s) of Higgs fields. As we know, these vev’s do not follow from the fundamentals of quantum theory. On the other hand, one can find papers claiming that inertia can be assigned to a Lorentz type force generated by electromagnetic zero-point fields,⁴⁷ It is also quite well known the *Rindler condensate* concept of Gerlach.⁴⁸ The point is that there exist completely coherent zero-point condensates (like the Rindler-Gerlach one) entirely mimicking the Planck spectrum, without any renormalization (Casimir effect).

According to Unruh,⁴⁹ simple model particles of uniform, one-dimensional proper acceleration a in Minkowski vacuum are immersed in a scalar quantum field ‘heat’ bath of temperature

$$T_a = \frac{\hbar}{2\pi ck} \cdot a ,$$

where \hbar is Planck’s constant, c is the light speed in vacuum, and k is Boltzmann’s constant. For first order corrections to this formula see works by Reznik.⁵⁰ The Unruh temperature is proportional to the lineal uniform acceleration, and the scale of such noninertial quantum field ‘heat’ effects is fixed by the numerical values of universal constants to the very low value of 4×10^{-23} in cgs units). In other words, the huge acceleration of 2.5×10^{22} cm/s² can produce a black body spectrum of only 1 K. In the case of Schwarzschild black holes, using the surface gravity $\kappa = c^4/4GM$ instead of a , one gets the formula for their Hawking temperature, T_κ . In a more physical picture, the Unruh quantum field heat reservoir is filled with

the so-called Rindler photons (Rindler quasi-particles), and therefore the quantum transitions are to be described as absorptions or emissions of the Rindler reservoir photons. The Unruh picture can be used for interpreting Hawking radiation in Minkowski space.⁵¹ In order to do that, one has to consider the generalization(s) of the equivalence principle to quantum field processes. A number of authors have discussed this important issue with various degrees of detail and meaning and with some debate.⁵²

Nikishov and Ritus,⁵³ raised the following objection to the heat bath concept. Since absorption and emission processes take place in finite space time regions, the application of the local principle of equivalence requires a constant acceleration over those regions. However, the space-time extension of the quantum processes are in general of the order of inverse acceleration. In Minkowski space it is not possible to create homogeneous and uniform gravitational fields having accelerations of the order of a in spacetime domains of the order of the inverse of a .

Pinto-Neto and Svaiter,⁵⁴ summarized the detailed discussions of Grishchuk, Zel'dovich, and Rozhanskii, and of Ginzburg and Frolov, concerning the formulations of quantum field equivalence principles from the point of view of the response functions of quantum detectors, in particular the Unruh-DeWitt (UDW) two-level monopole detector. Recall that in the asymptotic limit the response function is the integral of the quantum noise power spectrum. Or, since the derivative of the response function is the quantum transition rate, the latter is just the measure of the vacuum power spectrum along the chosen trajectory (worldline) and in the chosen initial (vacuum) state. This is only in the asymptotic limit and there are cases requiring calculations in finite time intervals.⁵⁵ Denoting by $R_{M,I}$, $R_{R,A}$, and $R_{M,A}$ the detection rates with the first subscript corresponding to the vacuum (either Minkowski or Rindler) and the second subscript corresponding to either inertial or accelerating worldline, one can find for the UDW detector in a scalar vacuum that $R_{M,I} = R_{R,A}$ expressing the dissipationless character of the vacuum fluctuations in this case, and a thermal factor for $R_{M,A}$ leading to the Unruh heat bath concept. In the case of a uniform gravitational field, the candidates for the vacuum state are the Hartle-Hawking (HH) and the Boulware (B) vacua. The HH vacuum is defined by choosing incoming modes to be positive frequency modes with respect to the null coordinate on the future horizon and outgoing modes as positive frequency ones with respect to the null coordinate on the past horizon, whereas the B vacuum has the positive frequency modes with respect to the Killing vector which makes the exterior region static. For a uniform gravitational field the HH vacuum can be thought of as the counterpart of the Minkowski vacuum, while the B vacuum is the equivalent of the Rindler vacuum. Then, the quantum field equivalence principle (QFEP) can be formulated in one of the following ways

Quantum detector-QFEP: $HH - M$ equivalence

i) The detection rate of a free-falling UDW detector in the HH vacuum is the same as that of an inertial UDW detector in the M vacuum.

ii) A UDW detector at rest in the HH vacuum has the same DR as a uniformly accelerated detector in the M vacuum.

Quantum detector-QFEP: $B - R$ equivalence

iii) A UDW detector at rest in the B vacuum has the same detection rate as a uniformly accelerated detector in the R vacuum.

iv) A free-falling UDW detector in the B vacuum has the same detection rate as an inertial detector in the R vacuum.

The above formulations seem reasonable enough, but their generalization to more realistic cases must be carefully considered in the future. Let us record one more formulation due to Kolbenstvedt,⁵²

Quantum detector-QFEP: Kolbenstvedt

A detector in a gravitational field and an accelerated detector will behave in the same manner if they feel equal forces and perceive radiation baths of identical temperature.

In principle, since the Planck spectrum is Lorentz invariant (and even conformal invariant) its inclusion in equivalence statements looks quite natural. The linear connection between temperature and one-dimensional, uniform, proper acceleration, which is also valid in some important gravitational contexts, is indeed a fundamental relationship, because it allows for an absolute meaning of quantum field effects in such an *ideal* noninertial frame, as soon as one recognize thermodynamic temperature as the only *absolute*, i.e., fully *universal* energy type physical concept. In general, the scalar quantum field vacua are not stationary stochastic processes (stationary vacuum noises) for all types of classical trajectories. Nevertheless, the lineal acceleration is *not* the only case with that property as was shown by Letaw,⁵⁶ who extended Unruh's considerations obtaining six types of worldlines with stationary vacuum excitation spectrum (SVES-1 to SVES-6, see below), as solutions of some generalized Frenet equations and under the condition of constant curvature invariants of the worldline (curvature, torsion and hypertorsion, i.e., κ , τ and ν , respectively). The six stationary cases are the following

1. $\kappa = \tau = \nu = 0$, (inertial worldlines; trivial cubic SVES-1).
2. $\kappa \neq 0$, $\tau = \nu = 0$, (hyperbolic worldlines; SVES-2 is Planckian allowing the interpretation of $\kappa/2\pi$ as 'thermodynamic' temperature).
3. $|\kappa| < |\tau|$, $\nu = 0$, $\rho^2 = \tau^2 - \kappa^2$, (helical worldlines; SVES-3 is an analytic function corresponding to case 4 below only in the limit $\kappa \gg \rho$).
4. $\kappa = \tau$, $\nu = 0$, (the spatially projected worldlines are semicubical parabolas containing a cusp where the direction of motion is reversed; SVES-4 is analytic in the dimensionless energy variable involving κ , but is not Planckian).
5. $|\kappa| > |\tau|$, $\nu = 0$, $\sigma^2 = \kappa^2 - \tau^2$, (the spatially projected worldlines are catenaries; SVES-5 cannot be found analytically in general, but for $\tau/\sigma \rightarrow 0$ tends to become Planckian (SVES-2), whereas for $\tau/\sigma \rightarrow \infty$ tends toward SVES-4).
6. $\nu \neq 0$, (rotating worldlines uniformly accelerated normal to their plane of

rotation; SVES-6 forms a two-parameter set of curves).

As one can see only the hyperbolic worldlines allow for a Planckian SVES and actually for a one-to-one mapping between the curvature invariant κ and the ‘thermodynamic’ temperature. Thus, one can infer that in some cases it is possible to determine the classical worldline on which a quantum particle is moving from measurements of the vacuum noise spectrum. There is much interest in considering the radiation patterns at accelerators in this perspective,⁵⁷ and it is in this sense that a sufficiently general and acceptable statement on the *universal* nature of the kinematical parameters occurring in a few important quantum field model problems can be formulated as follows

There exist accelerating classical trajectories (worldlines) on which moving ideal (two-level) quantum systems can detect the scalar vacuum environment as a stationary quantum field vacuum noise with a spectrum directly related to the curvature invariants of the worldline, thus allowing for a radiometric meaning of those invariants.

Another important byproduct is the possibility to choose a class of preferred vacua of the quantum world,⁵⁸ as *all* those having stationary vacuum noises with respect to the classical (geometric) worldlines of constant curvature invariants because in this case one may find some necessary attributes of universality in the more general quantum field radiometric sense,⁵⁹ including as a particular case the Planckian thermal spectrum. Of course, much work remains to be done towards a more “experimental” picture of highly academic calculations in quantum field theory, which are to be considered as useful only as a guide for more definite and therefore more complex situations.⁵⁷ A careful look to the literature shows that there are already steps in this direction. For example, Nagatsuka and Takagi,⁶⁰ studied radiation from a quasi-uniformly accelerated charge; Cresser,⁶¹ considered a model electron detector similar to the DeWitt monopole detector allowing him to develop a theory of electron detection and photon-photoelectron correlations in two-photon ionization processes; Klyshko,⁶² discussed the possible connection between photodetection, squeezed states and accelerated detectors; Frolov and Ginzburg,⁶³ pointed out that the radiation associated to uniformly accelerated detectors moving in vacuum is similar to that occurring in the region of anomalous Doppler effect, which take place when a quantum detector is moving at a constant superlight velocity in a medium. Marzlin and Audretsch,⁶⁴ considered constantly accelerated multi-level atoms and concluded that the magnitude of the Unruh effect is not modified. However, one should notice that all the aforementioned quantum field vacua look extremely ideal from the experimental standpoint. Indeed, it is known that only strong external fields can make the electrodynamic vacuum to react and show its physical properties, becoming similar to a magnetized and polarized medium, and only by such means one can learn about the structure of QED vacuum. Important, recent results on the relationships between Schwinger mechanism and Unruh effect have been reported in recent works.⁶⁵

At the axiomatic level, Hessling,⁶⁶ published new results on the algebraic quan-

tum field equivalence principle (AQFEP). Hessling's formulation is too technical to be reproduced here. The difficulties are related to the rigorous formulation of local position invariance, a requisite of equivalence, for the singular short-distance behavior of quantum fields, and to the generalization to interacting field theories. Various general statements of locality,⁶⁷ for linear quantum fields are important steps toward proper formulations of AQFEP. These are nice but technical results coming out mainly from clear mathematical exposition involving the Kubo-Martin-Schwinger (KMS) states of Hadamard type. Hessling's AQFEP formulation is based on the notion of quantum states *constant up to first order* in an arbitrary space-time point, and means that for these states a certain scaling limit should exist, and moreover a null-derivative condition with respect to a local inertial system around that arbitrary point is to be fulfilled for all n-point functions. In a certain sense this is similar to the properties of Gaussian noise. For example, the vacuum state of the Klein-Gordon field in Minkowski space with a suitable scaling function fulfills Hessling's AQFEP. Hessling showed, using as a toy model the asymptotically free ϕ^3 theory in six-dimensional Minkowski space, that the derivative condition of his QEP is not satisfied by this interacting quantum field theory, which perturbatively is similar to QCD. This failing is due to the running coupling constant which does not go smoothly to zero in the short-distance limit. The complexity of the Yang-Mills vacuum (YMV) is noteworthy.⁶⁸ Interestingly, Reuter and Wetterich,⁶⁹ claim that the true YMV is characterized by a nonvanishing gluon condensate. If so, one may think of a gluon-vev inertial contribution of the YMV. Also, the ground state of quantum gravity,⁷⁰ although a highly speculative topic might allow considerations from the inertia standpoint.

Finally, it is worth mentioning that the *time-thermodynamics* relation in general covariant theories and the connection with Unruh's temperature and Hawking radiation is an active area of research due to the remarkable correspondence between causality and the modular Tomita-Takesaki theory.⁷¹ It would be interesting to formulate in this context some sort of AQFEP statement beyond that of Hessling.

3.3 Brief miscellany

This subsection is a browsing through further literature.

(i) A discussion involving EEP in the Schwarzschild geometry has been recently made by Moreau, Neutze, and Ross.⁷² They found a coordinate transformation separating the line element into a pure acceleration, diagonal part and an off-diagonal, pure curvature contribution allowing for a good understanding of the equivalence issue in that case.

(ii) Punsly,⁷³ deals with the problem of equivalence as related to black hole evaporation adopting the premises that at each point in spacetime all the inertial observers can accurately postulate the relativistic quantum field theories of flat spacetime on open sets with dimensions much less than the radii of curvature of spacetime. Thus, by pure local considerations all the local observers can formulate number representations of the field through local particle creation and annihilation

operators. Each local freely falling observer transports with him his own definition of the vacuum state. The proposal of Punsly is to introduce a global vacuum state defined throughout the spacetime outside of the horizon by “integrating” the local vacua along a space-filling family of freely falling trajectories. Such a proposal satisfies the principle of equivalence.

(iii) Kleinert,⁷⁴ speaks about a new QEP which determines short-time action and measure of fluctuating orbits in spaces with curvature and torsion that he gets from a simple mapping procedure by which classical orbits and path integrals for the motion of a point particle in flat space can be transformed directly into those in curved space with torsion. Alvarez and Mann,⁷⁵ evaluated the constraints on non-metric violations of the EEP over a wide sector of the electroweak standard model of particle physics. On the other hand, Kauffmann,⁷⁶ tackled a gravity-induced birefringence of space due to nonmetric coupling between gravity and electromagnetism. He got an upper bound from time delay data of the pulsar PSR 1937+21.

(iv) Anandan,⁷⁷ introduces the concept of quantum physical geometry by applying the Ehlers-Pirani-Schild scheme to freely falling quantum wavepackets.

(v) Jaekel and Reynaud,⁷⁸ discuss for the case of a Fabry-Perot cavity the radiation pressure due to quantum fluctuations, which induces mechanical effects on scatterers. They calculate the correction to the total mass of the cavity resulting from the Casimir force between the two mirrors, and show that energy stored in the vacuum fluctuations contributes to inertia in conformity with the law of inertia of energy. It comes out that inertial masses exhibit quantum fluctuations with a characteristic mass noise spectrum. For a recent review of Casimir effect(s), see Ref.[79].

(vi) There is a conjecture due to Grøn and Eriksen,⁸⁰ that Einstein’s field equations do not permit the existence of empty space-times with a uniform gravitational field, i.e., a field in which the proper acceleration of a free particle instantaneously at rest is the same everywhere in the field. Apparently, as in electromagnetism, the closest one can come to a uniform gravitational field in empty four-dimensional space-time is the parallel gravitational field outside a massive plane of infinite extension.

(vii) Carlini and Greensite,⁸¹ show that the classical field equations of general relativity can be expressed as a single geodesic equation, describing the free fall of a point particle in superspace, and applied the result to several minisuperspace cosmological models.

(viii) I also mention a work of Baumann and collaborators,⁸² who reported highly interesting experiments regarding the free fall of immiscible vortex rings in liquids, which might have an impact on WEP, if WEP oriented.

(ix) In a series of papers of Faraggi and Matone,⁸³ a sort of mathematical equivalence postulate is introduced stating that all physical systems can be connected by a coordinate transformation to the free system with vanishing energy, uniquely leading to the quantum analogue of the Hamilton-Jacobi equation, which is a third-order non-linear differential equation. By this means a trajectory representation of

the quantum mechanics is derived, depending on the Planck length.

(x) When the vacuum noises are not stationary, one can nevertheless perform their tomographical processing,⁸⁴ requiring joint time and frequency information. Alternatively, since in the quantum detector method the vacuum autocorrelation functions are the essential physical quantities, and since according to fluctuation-dissipation theorem(s) (FDT) they are related to the linear (equilibrium) response functions to an initial condition/vacuum, more FDT type work, especially their generalization to the out of equilibrium case,⁸⁵ will be useful in this framework. In fact, there is some recent progress due to Hu and Matacz,⁸⁶ in making more definite use of FDT for vacuum fluctuations. Very recently, Gour and Sriramkumar,⁸⁷ questioned if small particles exhibit Brownian motion in the quantum vacuum and concluded that even though the answer is in principle positive the effect is extremely small and thus very difficult to detect experimentally. For the well-known method of influence functionals and generalizations, see Ref.[88].

4. CONCLUSION

As I have presented here in some detail, considerations of equivalence type in quantum field theories may well guide the abstract research towards the highly required feature of universality (going up to the act of measurement itself, see Ref.[89]) which is one of the ultimate purposes of the meaningful research.

The equivalence principles are related to a number of fundamental problems that may be considered as ever-open-issues, like those of physical *mass*,⁹⁰ *locality*,⁹¹ and more rigorous definitions of *reference frames*,⁹² either in classical physics or in the quantum approach. Indeed, referring to the latter issue, since the equivalence principles are connected to the type of geometrical structure of spacetime(s), more geometrical-axiomatic formulations,⁹³ of such fundamental statements are required for example in the context of fractal geometry,⁹⁴ and even beyond geometry,⁹⁵ in order to learn for example under what conditions one can get a unique metric.

To this end, one can say that as any matter of interpretation at a very general level, the principles of equivalence are open to many opinions and discussions. They have been the beginning of modern physics, and probably they will ever frustrate us in one way or another.

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