

# Continuous Quantum Hidden Subgroup Algorithms

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## ABSTRACT

In this paper we show how to construct two continuous variable and one continuous functional quantum hidden subgroup (QHS) algorithms. These are respectively quantum algorithms on the additive group of reals  $\mathbb{R}$ , the additive group  $\mathbb{R}/\mathbb{Z}$  of the reals  $\mathbb{R} \bmod 1$ , i.e., the circle, and the additive group PATHS of  $L^2$  paths  $x : [0, 1] \rightarrow \mathbb{R}^n$  in real  $n$ -space  $\mathbb{R}^n$ . Also included is a curious discrete QHS algorithm which is dual to Shor's algorithm.

## Contents

1	Introduction	1
2	Definition of the hidden subgroup problem (HSP) and hidden subgroup algorithms (HSAs)	2
3	A QHS Algorithm $\text{Alg}_{\mathbb{R}}$ on the reals $\mathbb{R}$	3
4	A QHS algorithm $\text{Alg}_{\mathbb{R}/\mathbb{Z}}$ on the circle $\mathbb{R}/\mathbb{Z}$	4
5	A curious algorithmic dual of Shor's algorithm	6
6	A QHS Algorithm $\text{Alg}_{\text{Paths}}$ on the space Paths	6
7	More speculations and questions	8
8	Acknowledgements	8

## 1. INTRODUCTION

In this paper we show how to construct two continuous variable and one continuous functional quantum hidden subgroup (QHS) algorithms. These are quantum algorithms  $\text{Alg}_{\mathbb{R}}$ ,  $\text{Alg}_{\mathbb{R}/\mathbb{Z}}$ , and  $\text{Alg}_{\text{Paths}}$ , respectively on the additive group of reals  $\mathbb{R}$ , the additive group  $\mathbb{R}/\mathbb{Z}$  of the reals  $\mathbb{R} \bmod 1$ , i.e., the circle, and the additive group PATHS of  $L^2$  paths  $x : [0, 1] \rightarrow \mathbb{R}^n$  in real  $n$ -space  $\mathbb{R}^n$ . With the methods found in a recent paper,<sup>12</sup> it is a straight forward exercise to extend  $\text{Alg}_{\mathbb{R}}$  and  $\text{Alg}_{\mathbb{R}/\mathbb{Z}}$  to a wandering algorithms respectively on the additive group of real  $n$ -tuples  $\mathbb{R}^n$ , and the additive group of the  $n$ -dimensional torus  $T^n = \bigoplus_1^n \mathbb{R}/\mathbb{Z}$ .

The chief advantage of the QHS algorithm  $\text{Alg}_{\mathbb{R}/\mathbb{Z}}$  over  $\text{Alg}_{\mathbb{R}}$  is that the ambient space  $\mathbb{R}/\mathbb{Z}$  is compact. As a result,  $\text{Alg}_{\mathbb{R}/\mathbb{Z}}$  can easily be approximated by a sequence  $\text{Alg}_{\mathbb{Z}_Q}$  of QHS algorithms over suitably chosen finite groups. (Each of these algorithms  $\text{Alg}_{\mathbb{Z}_Q}$  is a natural dual to Shor's algorithm.) The last QHS algorithm  $\text{Alg}_{\text{Paths}}$  is a quantum functional integral algorithm which is highly speculative. The algorithm, in the spirit of Feynman, is based on functional integrals whose existence is difficult to determine, let alone approximate. However, in the light of Witten's functional integral approach<sup>25</sup> to the knot invariants, this algorithm has the advantage of suggesting a possible approach toward developing a QHS algorithm for the the Jones polynomial.

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## 2. DEFINITION OF THE HIDDEN SUBGROUP PROBLEM (HSP) AND HIDDEN SUBGROUP ALGORITHMS (HSAS)

We now proceed by defining what is meant by a hidden subgroup problem (HSP) and a corresponding hidden subgroup algorithm. There are other perspectives to be found in the open literature.<sup>11 10 18</sup>

**DEFINITION 2.1.** *A map  $\varphi : A \rightarrow S$  from a group  $A$  into a set  $S$  is said to have **hidden subgroup structure** if there exists a subgroup  $K_\varphi$  of  $A$ , called a **hidden subgroup**, and an injection  $\iota_\varphi : A/K_\varphi \rightarrow S$ , called a **hidden injection**, such that the diagram*

$$\begin{array}{ccc} A & \xrightarrow{\varphi} & S \\ \nu \searrow & & \nearrow \iota_\varphi \\ & A/K_\varphi & \end{array}$$

*is commutative, where  $A/K_\varphi$  denotes the collection of right cosets of  $K_\varphi$  in  $A$ , and where  $\nu : A \rightarrow A/K_\varphi$  is the natural map of  $A$  onto  $A/K_\varphi$ . We refer to the group  $A$  as the **ambient group** and to the set  $S$  as the **target set**. If  $K_\varphi$  is a normal subgroup of  $A$ , then  $H_\varphi = A/K_\varphi$  is a group, called the **hidden quotient group**, and  $\nu : A \rightarrow A/K_\varphi$  is an epimorphism, called the **hidden epimorphism**.*

The hidden subgroup problem can be expressed as follows:

**Hidden Subgroup Problem (HSP).** *Given a map with hidden subgroup structure*

$$\varphi : A \rightarrow S ,$$

*determine a hidden subgroup  $K_\varphi$  of  $A$ . An algorithm solving this problem is called a **hidden subgroup algorithm (HSA)**.*

The corresponding quantum form of this HSP is:

**Hidden Subgroup Problem: Quantum Version.** *Let*

$$\varphi : A \rightarrow S$$

*be a map with hidden subgroup structure. Construct a quantum implementation of the map  $\varphi$  as follows:*

*Let  $\mathcal{H}_A$  and  $\mathcal{H}_S$  be Hilbert spaces (or rigged Hilbert spaces) defined respectively by the orthonormal bases*

$$\{ |a\rangle \mid a \in A \} \text{ and } \{ |s\rangle \mid s \in S \} ,$$

*and let  $s_0 = \varphi(0)$ , where  $0$  denotes the identity of the ambient group  $A$ . Finally, let  $U_\varphi$  be a unitary transformation such that*

$$\begin{aligned} U_\varphi : \mathcal{H}_A \otimes \mathcal{H}_S &\longrightarrow \mathcal{H}_A \otimes \mathcal{H}_S \\ |a\rangle |s_0\rangle &\longmapsto |a\rangle |\varphi(a)\rangle \end{aligned} ,$$

### 3. A QHS ALGORITHM $\text{ALG}_{\mathbb{R}}$ ON THE REALS $\mathbb{R}$

Let

$$\varphi : \mathbb{R} \longrightarrow \mathbb{C}$$

be a periodic admissible function of minimum period  $P$  from the reals  $\mathbb{R}$  to the complex numbers  $\mathbb{C}$ . We will now create a continuous variable Shor algorithm to find integer periods. This algorithm can be extended to rational and irrational periods.<sup>15</sup>

We construct two quantum registers

$$|\text{LEFT REGISTER}\rangle \text{ and } |\text{RIGHT REGISTER}\rangle$$

called left- and right-registers respectively, and ‘living’ respectively in the rigged Hilbert spaces  $\mathcal{H}_{\mathbb{R}}$  and  $\mathcal{H}_{\mathbb{C}}$ . The left register was constructed to hold arguments of the function  $\varphi$ , the right to hold the corresponding function values.

We assume we are given the unitary transformation

$$U_{\varphi} : \mathcal{H}_{\mathbb{R}} \otimes \mathcal{H}_{\mathbb{C}} \longrightarrow \mathcal{H}_{\mathbb{R}} \otimes \mathcal{H}_{\mathbb{C}}$$

defined by

$$U_{\varphi} : |x\rangle |y\rangle \longmapsto |x\rangle |y + \varphi(x)\rangle$$

Finally, we choose a large positive integer  $Q$ , so large that  $Q \geq 2P^2$ .

The quantum part of our algorithm consists of **Step 0** through **Step 4** as described below:

**Step 0** Initialize

$$|\psi_0\rangle = |0\rangle |0\rangle$$

**Step 1** Apply the inverse Fourier transform to the left register, i.e. apply  $\mathcal{F}^{-1} \otimes 1$  to obtain

$$|\psi_1\rangle = \int_{-\infty}^{\infty} dx e^{2\pi i x \cdot 0} |x\rangle |0\rangle = \int_{-\infty}^{\infty} dx |x\rangle |0\rangle$$

**Step 2** Apply  $U_{\varphi} : |x\rangle |u\rangle \longmapsto |x\rangle |u + \varphi(x)\rangle$  to obtain

$$|\psi_2\rangle = \int_{-\infty}^{\infty} dx |x\rangle |\varphi(x)\rangle$$

**Step 3** Apply the Fourier transform to the left register, i.e. apply  $\mathcal{F} \otimes 1$  to obtain

$$\begin{aligned} |\psi_3\rangle &= \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx e^{-2\pi ixy} |y\rangle |\varphi(x)\rangle = \int_{-\infty}^{\infty} dy |y\rangle \delta_P(y) \int_0^{P-} dx e^{-2\pi ixy} |\varphi(x)\rangle \\ &= \sum_{n=-\infty}^{\infty} \left| \frac{n}{P} \right\rangle \left( \frac{1}{|P|} \int_0^{P-} dx e^{-2\pi ix \frac{n}{P}} |\varphi(x)\rangle \right) = \sum_{n=-\infty}^{\infty} \left| \frac{n}{P} \right\rangle \left| \Omega \left( \frac{n}{P} \right) \right\rangle \end{aligned}$$

where

$$\left| \Omega \left( \frac{n}{P} \right) \right\rangle = \frac{1}{|P|} \int_0^{P-} dx e^{-2\pi ix \frac{n}{P}} |\varphi(x)\rangle ,$$

and where  $\int_0^{P-} dx e^{-2\pi ix \frac{n}{P}} |\varphi(x)\rangle$  denotes the formal integral  $\int_0^P dx e^{-2\pi ix \frac{n}{P}} [1 - \delta(x - P)] |\varphi(x)\rangle$ .

**Step 4** Measure the left register with respect to the observable\*

$$\mathcal{O} = \int_{-\infty}^{\infty} dy \frac{[Qy]}{Q} |y\rangle \langle y|$$

to produce a random eigenvalue

$$\frac{m}{Q}$$

for which there exists a rational of the form  $\frac{n}{P}$  such that  $\frac{m}{Q} \leq \frac{n}{P} < \frac{m+1}{Q}$ .

**Step 5** If  $Q \geq 2P^2$ , then  $\frac{n}{P}$  is unique and is a convergent of the continued fraction expansion of  $\frac{m}{Q}$ . Thus  $\frac{n}{P}$  can be computed from the standard continued fraction recursion.

#### 4. A QHS ALGORITHM $\text{ALG}_{\mathbb{R}/\mathbb{Z}}$ ON THE CIRCLE $\mathbb{R}/\mathbb{Z}$

Given a periodic admissible function with rational period  $P$  from the circle  $\mathbb{R}/\mathbb{Z}$  to the complex plane  $\mathbb{C}$

$$\varphi : \mathbb{R}/\mathbb{Z} \longrightarrow \mathbb{C},$$

we will now create a QHS algorithm to find its period . This algorithm can be extended to irrational periods.<sup>16</sup>

**PROPOSITION 4.1.** *If  $\varphi : \mathbb{R}/\mathbb{Z} \longrightarrow \mathbb{C}$  has a rational period, then its minimum non-trivial period  $P$  is a reciprocal integer  $P = \frac{1}{a} \bmod 1$ .*

We assume we are given the unitary transformation

$$U_\varphi : \mathcal{H}_{\mathbb{R}/\mathbb{Z}} \otimes \mathcal{H}_{\mathbb{C}} \longrightarrow \mathcal{H}_{\mathbb{R}/\mathbb{Z}} \otimes \mathcal{H}_{\mathbb{C}}$$

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\* $[Qy]$  denotes the greatest integer  $\leq Qy$ .

defined by

$$U_\varphi : |x\rangle |y\rangle \mapsto |x\rangle |y + \varphi(x) \bmod 1\rangle$$

The quantum part of our algorithm consists of **Step 0** through **Step 4** as described below:

**Step 0** Initialize

$$|\psi_0\rangle = |0\rangle |0\rangle \in \mathcal{H}_{\mathbb{Z}} \otimes \mathcal{H}_{\mathbb{C}}$$

**Step 1** Apply the inverse Fourier transform to the left register, i.e. apply  $\mathcal{F}^{-1} \otimes 1$  to obtain

$$|\psi_1\rangle = \oint dx e^{2\pi i x \cdot 0} |x\rangle |0\rangle = \oint dx |x\rangle |0\rangle \in \mathcal{H}_{\mathbb{R}/\mathbb{Z}} \otimes \mathcal{H}_{\mathbb{C}}$$

**Step 2** Apply  $U_\varphi : |x\rangle |u\rangle \mapsto |x\rangle |u + \varphi(x) \bmod 1\rangle$  to obtain

$$|\psi_2\rangle = \oint dx |x\rangle |\varphi(x)\rangle$$

**Step 3** Apply the Fourier transform to the left register, i.e. apply  $\mathcal{F} \otimes 1$  to obtain

$$|\psi_3\rangle = \sum_{n \in \mathbb{Z}} \oint dx e^{-2\pi i n x} |n\rangle |\varphi(x)\rangle = \sum_{n \in \mathbb{Z}} |n\rangle \oint dx e^{-2\pi i n x} |\varphi(x)\rangle \in \mathcal{H}_{\mathbb{Z}} \otimes \mathcal{H}_{\mathbb{C}}$$

This can be shown to reduce to

$$|\psi_3\rangle = \sum_{\ell \in \mathbb{Z}} |\ell a\rangle |\Omega(\ell a)\rangle$$

where

$$|\Omega(\ell a)\rangle = \int_0^{1/a} dx e^{-2\pi i n x} |\varphi(x)\rangle .$$

**Step 4** Measure  $|\psi_3\rangle = \sum_{\ell \in \mathbb{Z}} |\ell a\rangle |\Omega(\ell a)\rangle$  with respect to the observable

$$\mathcal{O} = \sum_{n \in \mathbb{Z}} n |n\rangle \langle n|$$

to produce a random eigenvalue  $\ell a$ .

**Step 5** If the above steps are run twice to produce eigenvalues  $\ell_1 a$  and  $\ell_2 a$ , then the probability that the integers  $\ell_1$  and  $\ell_2$  are relatively prime is  $\zeta(2)^{-1} \approx 0.6079$ , where  $\zeta(2)$  denote the Riemann zeta function at  $k = 2$ . Hence, with high probability, the Euclidean algorithm will produce the desired answer  $a$ .

**Remark.** Please note that, unlike Shor's algorithm which uses the classical continued fraction algorithm in its last step to determine the period, this algorithm uses in its last step only the much faster classical Euclidean algorithm to find the period  $\frac{1}{a}$ .

## 5. A CURIOUS ALGORITHMIC DUAL OF SHOR'S ALGORITHM

Let us now construct a class of approximating QHS algorithms over finite groups which asymptotically approach in the limit the QHS algorithm  $\text{ALG}_{\mathbb{R}/\mathbb{Z}}$ . To do so, for each positive integer  $Q$ , we approximate the infinite groups  $\mathbb{Z}$  and  $\mathbb{R}/\mathbb{Z}$  respectively by the finite groups

$$\left\{ \begin{array}{l} \mathbb{Z} \approx \mathbb{Z}_Q = \{k \in \mathbb{Z} \bmod Q : 0 \leq k < Q\} \\ \mathbb{R}/\mathbb{Z} \approx \mathbb{Z}_Q = \left\{ \frac{r}{Q} : r = 0, 1, \dots, Q-1 \right\} \end{array} \right.$$

and we approximate the map  $\varphi : \mathbb{Z} \rightarrow \mathbb{C}$  by the obvious map  $\tilde{\varphi} : \mathbb{Z}_Q \rightarrow \mathbb{C}$ . The resulting algorithm  $\text{ALG}_{\mathbb{Z}_Q}$  "lives" in the Hilbert space  $\mathcal{H}_{\mathbb{Z}_Q} \otimes \mathcal{H}_{\mathbb{C}}$  and uses the approximating observable

$$\mathcal{O}_Q = \sum_{n=0}^{Q-1} n |n\rangle \langle n|$$

The result for each  $Q$  is a QHS algorithm  $\text{ALG}_{\mathbb{Z}_Q}$  which is the algorithmic dual of Shor's algorithm. Because of the remark found at the end of the previous section, it appears to run much faster.

## 6. A QHS ALGORITHM $\text{ALG}_{\text{PATHS}}$ ON THE SPACE PATHS

The reader should be cautioned that the following algorithm is highly speculative. This algorithm, in the spirit of Feynman, is based on functional integrals whose existence is difficult to determine, let alone approximate.

Let  $\text{PATHS}$  be the space of all paths  $x : [0, 1] \rightarrow \mathbb{R}^n$  in real  $n$ -space  $\mathbb{R}^n$  which are  $L^2$  with respect to the inner product

$$x \cdot y = \int_0^1 ds x(s)y(s)$$

We make  $\text{PATHS}$  into a vector space over the reals  $\mathbb{R}$  by defining scalar addition and vector addition respectively as

$$\left\{ \begin{array}{l} (\lambda x)(s) = \lambda x(s) \\ (x + y)(s) = x(s) + y(s) \end{array} \right.$$

We wish to solve the following problem:

**Problem.** *Given a functional  $\varphi : \text{Paths} \rightarrow \mathbb{R}^n$  and a hidden subspace  $V$  of Paths such that*

$$\varphi(x + v) = \varphi(x) \quad \forall v \in V,$$

*create a QHS algorithm that finds the hidden subspace  $V$ .*

Let  $\mathcal{H}_{\text{PATHS}}$  be the rigged Hilbert space with orthonormal basis

$$\{|x\rangle : x \in \text{PATHS}\}$$

where we have defined the bracket product as

$$\langle x|y\rangle = \delta(x - y)$$

Keeping in mind that the space PATHS can be written as the disjoint union

$$\text{PATHS} = \bigcup_{v \in V} (v + V^\perp) ,$$

where  $V^\perp$  denotes the orthogonal complement of  $V$  in PATHS, we proceed with the following QHS algorithm:

**Step 0** Initialize

$$|\psi_0\rangle = |0\rangle |0\rangle \in \mathcal{H}_{\text{PATHS}} \otimes \mathcal{H}_{\mathbb{R}^n}$$

**Step 1** Apply the inverse Fourier transform to the left register, i.e. apply  $\mathcal{F}^{-1} \otimes 1$  to obtain

$$|\psi_1\rangle = \int_{\text{PATHS}} \mathcal{D}x e^{2\pi i x \cdot 0} |x\rangle |0\rangle = \int_{\text{PATHS}} \mathcal{D}x |x\rangle |0\rangle$$

**Step 2** Apply  $U_\varphi : |x\rangle |u\rangle \mapsto |x\rangle |u + \varphi(x)\rangle$  to obtain

$$|\psi_2\rangle = \int_{\text{PATHS}} \mathcal{D}x |x\rangle |\varphi(x)\rangle$$

**Step 3** Apply the Fourier transform to the left register, i.e. apply  $\mathcal{F} \otimes 1$  to obtain

$$|\psi_3\rangle = \int_{\text{PATHS}} \mathcal{D}y \int_{\text{PATHS}} \mathcal{D}x e^{-2\pi i x \cdot y} |y\rangle |\varphi(x)\rangle = \int_{\text{PATHS}} \mathcal{D}y |y\rangle \int_{\text{PATHS}} \mathcal{D}x e^{-2\pi i x \cdot y} |\varphi(x)\rangle$$

Using the decomposition  $\text{PATHS} = \bigcup_{v \in V} (v + V^\perp)$ , we can formally show that this reduces to

$$|\psi_3\rangle = \int_{V^\perp} \mathcal{D}u |y\rangle |\Omega(u)\rangle ,$$

where

$$|\Omega(u)\rangle = \int_{V^\perp} \mathcal{D}x e^{-2\pi i x \cdot u} |\varphi(x)\rangle .$$

**Step 4** Measure  $|\psi_3\rangle = \int_{V^\perp} \mathcal{D}u |y\rangle |\Omega(u)\rangle$  with respect to the observable

$$\mathcal{O} = \int_{\text{PATHS}} \mathcal{D}w w |w\rangle \langle w|$$

to produce a random element of  $V^\perp$ .

## 7. MORE SPECULATIONS AND QUESTIONS

Can the above highly speculative algorithm be modified in such a way to create a QHS algorithm for the Jones polynomial? In other words, can it be modified by replacing the space PATHS with the space  $\mathcal{A}$  of gauge connections, and by making suitable modifications of the functional integral

$$\widehat{\psi}(K) = \int_{\mathcal{A}} \mathcal{D}A \psi(A) \mathcal{W}_K(A)$$

where  $\mathcal{W}_K(A)$  denotes the Wilson loop

$$\mathcal{W}_K(A) = \text{tr} \left( P \exp \left( \oint_K A \right) \right)$$

The functional integral  $\widehat{\psi}(K)$  transforms the function  $\psi(A)$  of gauge connections to a function of closed curves in three dimensional space. Witten<sup>25</sup> showed that, if  $\psi(A)$  is chosen correctly (an exponentiated integral of the Chern-Simons form), then, up to framing corrections,  $\widehat{\psi}(K)$  is a knot and link invariant. This invariant reproduces the original Jones polynomial for appropriate choice of the gauge group, and many other invariants for other such choices. By continuing our exploration of quantum algorithms as in the last section, we hope to give a quantum algorithm for these topological invariants, thereby forging a new connection between quantum computing and topological quantum field theory.

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