

Decoherence, re-coherence, and the black hole information paradox

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Abstract. We analyze a system consisting of an oscillator coupled to a field. With the field traced out as an environment, the oscillator loses coherence on a very short *decoherence timescale*; but, on a much longer *relaxation timescale*, predictably evolves into a unique, pure (ground) state. This example of *re-coherence* has interesting implications both for the interpretation of quantum theory and for the loss of information during black hole evaporation. We examine these implications by investigating the intermediate and final states of the quantum field, treated as an open system coupled to an unobserved oscillator.

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1 Introduction

1.1 Overview of decoherence

Recent years have witnessed a significant increase of interest in the process of *decoherence*[1, 2, 3, 4] — the loss of quantum coherence suffered by a quantum system in contact with an *environment*. An *environment* consists of degrees of freedom which are coupled with, but not regarded as an integral part of the system. External variables are an obvious example, but even internal degrees of freedom may constitute an environment, if they cannot be followed by the observer. Many quantum systems are therefore subject to decoherence, and the phenomenon is thus widespread and important.

For example, it has been demonstrated that collective observables of macroscopic quantum systems will lose quantum coherence very quickly by this means. This loss of coherence will proceed at very different rates, depending on the initial state of the system. Indeed, in the simple models of quantum apparatus proposed to describe the process of measurement, one can select an interaction Hamiltonian which commutes with the observable of the recording apparatus[1]. The varying susceptibility of initial states to decoherence then allows one to model apparent collapse of the wave packet. The results can be taken to imply that different outcomes of a measurement are all present, but — in the language of Everett[5] — belong to different branches of the universal wave vector. Their simultaneous detection is impossible: decoherence leads to *environment-induced superselection rules*[1], which effectively exclude a majority of states from the Hilbert space of the open system. In the context of the Many Worlds Interpretation (MWI) of quantum mechanics, environment-induced superselection supplies a preferred basis, which selects the “branches” into which the universal state vector is “splitting”. Decoherence can thus be thought of as a “missing link” between the quantum universe and classical reality, in that it provides the criterion for selection of preferred observables (such as position) while supplying an effective definition for classicality, as well as the rationale for the apparent “collapse of the wave packet”.

The process of decoherence has also been studied in the somewhat less idealized, but still exactly solvable, model of an oscillator system coupled to a quantum field representing the environment. The evolution of this system is known as *quantum Brownian motion*[6, 7, 8], and it also exhibits

environment-induced superselection[9]. One can describe superselection in the case of quantum brownian motion by appealing to the *predictability sieve*[10] — a formal implementation of the idea that the preferred quantum states will be stable (*i.e.*, will minimize entropy production) in spite of the coupling to the environment. For example, the predictability sieve selects *coherent states* as the preferred states of an underdamped harmonic oscillator[11]. Moreover, because decoherence occurs on timescales which are typically much smaller than a system’s dynamical timescales (or even timescales associated with monitoring by an observer), one can convincingly argue that similar “nice” states will be singled out by environment-induced superselection in more realistic (and more complicated) situations.

Much of the discussion of the implications of the decoherence process is based on the tacit assumption that “decoherence is forever”; a concern is often voiced that any sign of re-coherence would be trouble[12]. In this paper we explore the problem of re-coherence, by exhibiting a model — quantum Brownian motion with a zero-temperature environment — in which decoherence happens quickly, but then gets “undone” slowly. Since decoherence and information are intimately related in quantum theory, it will turn out that this system can serve as an instructive toy model for the information problem in black hole evaporation.

1.2 Outline of the calculation

A brief and simplified preview of our calculation is in order. We will start our oscillator in a “Schrödinger Cat” state, a superposition of two well-separated coherent states. The initial state of the oscillator-field system will therefore be¹

$$|\Psi_i\rangle = (c_+|\psi_+\rangle + c_-|\psi_-\rangle) |0\rangle_{field} . \quad (1)$$

(More complicated pure initial conditions can also be considered.) Interaction with the environment will (approximately, and for a few decoherence timescales) lead to

$$|\Psi_i\rangle = (c_+|\psi_+\rangle + c_-|\psi_-\rangle) |0\rangle_{field} \rightarrow c_+|\psi_+(t)\rangle|\Phi_+(t)\rangle_{field} + c_-|\psi_-(t)\rangle|\Phi_-(t)\rangle_{field} = |\Psi(t)\rangle , \quad (2)$$

¹In fact, we will consider states that are only approximately direct products of field and oscillator states; but the simplified outline presented in this Section still describes the essential physics involved.

where

$$\langle \Phi_+(t) | \Phi_-(t) \rangle \ll 1 . \quad (3)$$

Hence the density matrix of the oscillator will be given by

$$\hat{\rho} = |c_+|^2 |\psi_+(t)\rangle\langle\psi_+(t)| + |c_-|^2 |\psi_-(t)\rangle\langle\psi_-(t)| + \mathcal{O}(\langle\Phi_+(t)|\Phi_-(t)\rangle) . \quad (4)$$

Thus, at this stage one might feel justified in “declaring victory”: the decoherence has happened, as the form of the density matrix of the system demonstrates.

However, our system is a *damped* harmonic oscillator. In contact with the vacuum of a quantum field, it will slowly (on the relaxation timescale) approach the unique ground state *regardless of the initial state*. Hence, after a sufficiently long time, $|\Psi(t)\rangle$ will approach

$$\begin{aligned} |\Psi(\infty)\rangle &\equiv c_+|0\rangle|\Phi_+\rangle + c_-|0\rangle|\Phi_-\rangle \\ &= |0\rangle(c_+|\Phi_+\rangle + c_-|\Phi_-\rangle) , \end{aligned} \quad (5)$$

where $|0\rangle$ is — in our example — the ground state of the harmonic oscillator. Thus, decoherence seems to have “gone away”: it did not prevent the state of the oscillator from re-cohering into a unique, pure state. Moreover, as a consequence of this *recoherence* the environment (field) has been put into a very awkward, *pure* “Schrödinger Cat” state of its own!

Before this process has been completed, each of the two systems involved (the oscillator or the field, with the other traced out as unobserved) was in a mixed state, by virtue of the correlations between them. However, in the end these correlations have all disappeared. Or, to put it more precisely, the oscillator-field correlations have been “used up” to force the field into a highly non-trivial state. This state has the property (as one can anticipate intuitively, and as we shall prove in more detail below) that, in spite of its undeniable purity, it appears to be mixed when explored by approximately local measurements (limited in time and space to less than the duration of the recoherence episode). Thus, the information which appeared to be “lost” to observers who could access only one of the two systems (namely, the field) eventually re-emerges, but in a very obscure and hard-to-exhibit form.

1.3 Analogy with black hole evaporation

This sequence of events — possible rapid loss of information, and its eventual re-emergence after a long time, but in a form difficult to decipher — is

analogous to another interesting and fundamental process which has received a lot of attention in recent years: black hole evaporation[13, 14, 15, 16]. There, gravitational collapse rapidly increases the entropy of the Universe by the (usually large) difference between the entropy of the collapsing material and the final entropy of the black hole[17], which is given (in bits) by the area of its horizon measured in units of square Planck length.

The collapse seems to increase the entropy of the Universe, because the inside of the black hole horizon is inaccessible to external observers. Furthermore, black hole evaporation puts the field outside the horizon into a mixed state which, when analyzed layer by layer, appears to contain approximately black body radiation, with entropy at least as large as the entropy shed by the black hole. There appears to be no information in the emitted radiation. So, when at the end of the process the black hole is gone (as seems likely), the entropy of the Universe is larger by at least the entropy increase which occurred during the collapse. This process can be analyzed in some detail in the case of the Witten black hole in $1 + 1$ dimensional spacetime[15, 16]. Results in this case are usually cited (in spite of the facts that the analogy with the $3 + 1$ dimensional case is only partial, and that the calculation cannot really be carried out for the time when the black hole remnant ultimately disappears) as evidence for the *black hole information paradox*[18]. This paradox is that the fundamental equations of gravitation and field theory seem to imply an irreversible increase of entropy in the process of collapse, despite the fact that they are themselves reversible. Reversibility cannot, of course, be established in this case simply by appealing to the dynamics, since Einstein's theory necessarily predicts a singularity inside the horizon, where the known laws of physics, including general relativity, are thought to fail. It is easy to imagine that the reversibility of the the whole process is an early victim in this failure[19].

The decoherence *cum* re-coherence process described and analyzed in this paper supports an alternative view. One can imagine that the information lost beyond the black hole horizon eventually re-emerges, but not in any obvious form such that it could be detected by looking at "natural" observables (anything reasonably local, or at least confined to finite shells of the radiation emitted by the black hole). Rather, the information is re-emitted in a horribly "scrambled up" manner, where the state left behind after the black hole evaporates can still be pure (or at least no more mixed than the pre-collapse state), but this purity can only be revealed by measurement of some uncom-

promisingly *global* observable, which coherently and simultaneously samples *all* of the emitted quanta.

Motivated by this analogy, we shall exhibit such global observables, which can be computed exactly in our system by virtue of its linearity. It should of course be emphasized that this linearity that allows our model to be exactly solvable also makes it a rather distant analogue for the black hole evaporation process. It is precisely the inherent nonlinearity of general relativity which is responsible for the central singularity, the event horizon, the “no hair” theorems, and therefore for the unique value of the black hole entropy. Nevertheless, the complexity of the pure “global states” of the field generated in our simple example suggests how the information can be preserved but remain “hidden”, and thus, suggests a possible resolution of the black hole information paradox.

2 The calculation

2.1 The model

Our model consists of a simple harmonic oscillator Q coupled to a massless scalar field $\phi(x)$ in one dimension. To ensure that the energy is bounded below, we choose the following Lagrangian [7], with an ultraviolet cut-off:

$$L = \frac{M_0}{2}(\dot{Q}^2 - \Omega_0^2 Q^2) + \frac{1}{2} \int_{-\infty}^{\infty} dx (\partial_t \phi)^2 - (\partial_x \phi)^2 - gQ \int_{-\infty}^{\infty} dx F(x) \partial_t \phi(x) . \quad (6)$$

The coupling constant g has dimensions such that we can define from it the frequency $\gamma_0 \equiv \frac{g^2}{4M_0}$. Once renormalized, this frequency will correspond to a relaxation timescale. $F(x)$ is a smearing function which implements the cut-off on the field-oscillator interaction. For our later convenience, we choose the particular form

$$F = \frac{1}{\pi} \int_0^{\infty} dk \frac{\Gamma_0}{\sqrt{\Gamma_0^2 + k^2}} \cos kx = \frac{\Gamma_0}{\pi} K_0(\Gamma_0 x) . \quad (7)$$

$K_0(\Gamma_0 x)$ is the modified Bessel function of order zero, which is concentrated within $x < \Gamma_0^{-1}$, and is a delta-function representation in the limit $\Gamma_0 \rightarrow \infty$.

We quantize our model by defining the Hamiltonian operator

$$\begin{aligned} \hat{H} = & \frac{1}{2} \int_{-\infty}^{\infty} dx \left([\hat{\pi}(x) + gF(x)\hat{Q}]^2 + [\partial_x \hat{\phi}(x)]^2 \right) \\ & + \frac{1}{2} \left(M_0^{-1} \hat{P}^2 + M_0 \Omega_0^2 \hat{Q}^2 \right) . \end{aligned} \quad (8)$$

Here we set $\hbar = c = 1$, and introduce the convention that operators have circumflex accents, while c-numbers do not. \hat{P} and $\hat{\pi}(x)$ are the canonical momenta.

In fact, this model is unitarily equivalent to several other much-studied systems, including even the free massless field. In writing (8), therefore, we are really choosing how the Hilbert space is to be divided into field and oscillator degrees of freedom. Our criteria for doing this in the way that leads to (8) are that we demand that an identifiable oscillator exists, that it be coupled locally (save for some UV smearing) to the field at the origin, and that the expected energies for direct product states of field and oscillator are finite. (The latter stipulation is needed in order for weak coupling to imply small entanglement between oscillator and field at low energies; it amounts to a demand for an ultraviolet cut-off.)

To see this unitary equivalence, and to proceed in our calculation, we diagonalize (8) by defining the following normal modes, for $\omega > 0$:

$$\begin{aligned} \hat{\phi}(x) &= \frac{1}{\sqrt{\pi}} \int_0^{\infty} d\omega \hat{A}_\omega u_\omega(x) + \hat{B}_\omega \sin \omega x \\ \hat{\pi}(x) &= \frac{1}{\sqrt{\pi}} \int_0^{\infty} d\omega \hat{\Pi}_\omega^A v_\omega(x) + \hat{\Pi}_\omega^B \sin \omega x \\ \hat{Q} &= \frac{1}{\sqrt{\pi}} \int_0^{\infty} d\omega \hat{\Pi}_\omega^A q(\omega) \\ \hat{P} &= \frac{1}{\sqrt{\pi}} \int_0^{\infty} d\omega \hat{A}_\omega p(\omega) . \end{aligned} \quad (9)$$

The mode functions u_ω and v_ω , and the co-efficients $q(\omega)$ and $p(\omega)$, are found by solving ordinary differential equations derived by iterating the Heisenberg equations of motion. One obtains

$$\begin{aligned}
u_\omega(x) &= C(\omega) \left(\left[1 - \frac{\omega^2}{\Omega_0^2} + \frac{2\gamma_0\Gamma_0\omega^2}{\Omega_0^2(\Gamma_0^2 + \omega^2)} \right] \cos \omega x \right. \\
&\quad \left. + \frac{4\gamma_0\Gamma_0^2\omega^2}{\pi\Omega_0^2\sqrt{\Gamma_0^2 + \omega^2}} \int_0^\infty \frac{dk}{\sqrt{\Gamma_0^2 + k^2}} \frac{\cos kx}{k^2 - \omega^2} \right) \\
v_\omega(x) &= u_\omega(x) + \frac{4\gamma_0\Gamma_0C(\omega)}{\Omega_0^2\sqrt{\Gamma_0^2 + \omega^2}} F(x) \\
q(\omega) &= -gC(\omega) \frac{\Gamma_0}{M_0\Omega_0^2\sqrt{\Gamma_0^2 + \omega^2}} \\
p(\omega) &= -M_0\omega^2q(\omega). \tag{10}
\end{aligned}$$

We will soon see that $u_\omega(x)$ can be given in a much more transparent form.

The important normalization co-efficient $C(\omega)$ is defined so that

$$\begin{aligned}
C^2(\omega) &= \left[\left(1 - \frac{\omega^2}{\Omega_0^2} + \frac{2\gamma_0\Gamma_0\omega^2}{\Omega_0^2(\Gamma_0^2 + \omega^2)} \right)^2 + \left(\frac{2\gamma_0\Gamma_0^2\omega}{\omega^2(\Gamma_0^2 + \omega^2)} \right)^2 \right]^{-1} \\
&\equiv \frac{\Omega_0^4(\Gamma_0^2 + \omega^2)}{[\omega^2 + \Gamma^2][\omega^2 - (\Omega + i\gamma)^2][\omega^2 - (\Omega - i\gamma)^2]}. \tag{11}
\end{aligned}$$

The convenience of the cut-off scheme defined in (7) lies in the fact that $C^2(\omega)$ has only six simple poles, occurring in pairs of equal magnitude. This will make it easy to perform analytically several integrals that appear in the time evolutions discussed in later subsections. In Eq.(6) the new quantities Γ, Ω, γ are modified versions of the frequencies $\Gamma_0, \Omega_0, \gamma_0$, renormalized by the interaction. The renormalized frequencies may be expressed in terms of the bare ones only through a cumbersome (although analytically solvable) cubic equation. It turns out, however, that if we fix the renormalized parameters, which are the physically relevant ones, then the corresponding bare parameters may be expressed relatively simply:

$$\begin{aligned}
\Gamma_0 &= \Gamma + 2\gamma \\
\Omega_0^2 &= \frac{\Gamma}{\Gamma + 2\gamma}(\Omega^2 + \gamma^2) \\
\gamma_0 &= \gamma \left(\frac{\Gamma}{\Gamma + 2\gamma} + \frac{\Omega^2 + \gamma^2}{(\Gamma + 2\gamma)^2} \right). \tag{12}
\end{aligned}$$

From (12) one can see that if $\Gamma \gg \Omega \gg \gamma$, then the same inequalities will hold for the bare quantities as well, and the proportional differences between the two sets of frequencies will all be of order $(\frac{\gamma}{\Omega})^2$, $(\frac{\Omega}{\Gamma})^2$, or $\frac{\gamma}{\Gamma}$. *The results in this subsection 2.1 are all exact, and do not assume any particular relationships between the three frequencies.* In the remainder of this paper, however, we will be interested in the case of extreme underdamping, with high cut-off frequency. We will therefore set

$$\begin{aligned}\gamma &= \epsilon\Omega \\ \Omega &= \epsilon\Gamma ,\end{aligned}\tag{13}$$

where ϵ is small. We will also consider $\frac{1}{\pi}\epsilon(1+\ln \epsilon)$ to be negligible. Only under this stronger assumption will the oscillator's final state be approximately pure[7].

The mode functions u_ω and v_ω are both even functions in x . (Since the oscillator couples only to the even modes of the field, the odd mode functions are merely the usual sines. In our scenario, the odd modes will simply remain in their ground states forever, and so we will not refer to them explicitly hereafter.) For ω far from Ω_0 , u_ω and v_ω are essentially cosines; but they are distorted near the origin for ω close to Ω_0 , as one might expect. They possess several properties analogous to the orthonormality of cosines:

$$\begin{aligned}\int_{-\infty}^{\infty} dx v_\omega(x)u_{\omega'}(x) &= \pi\delta(\omega - \omega') + q(\omega)p(\omega') \\ \int_0^{\infty} d\omega v_\omega(x)u_\omega(y) &= \frac{\pi}{2}[\delta(x - y) + \delta(x + y)] \\ \int_0^{\infty} d\omega q(\omega)u_\omega(x) &= \int_0^{\infty} d\omega p(\omega)v_\omega(x) = 0 \\ \int_0^{\infty} d\omega p(\omega)q(\omega) &= -\pi .\end{aligned}\tag{14}$$

These relations may all be verified by contour integration. The computation is made easier if we use (12) to re-write (10) as

$$\begin{aligned}
u_\omega(x) &= \int_{-\infty}^{\infty} dx' F(x-x') \tilde{u}_\omega(x') \\
\tilde{u}_\omega(x) &= \frac{C(\omega)}{\Omega_0^2 \Gamma_0 \sqrt{\Gamma_0^2 + \omega^2}} \left([(\Omega_0^2 - \omega^2)(\Gamma_0^2 + \omega^2) + 2\gamma_0 \Gamma_0 \omega^2] \cos \omega x \right. \\
&\quad \left. - 2\gamma_0 \Gamma_0^2 \omega \sin \omega |x| \right) \\
&= -\frac{C(\omega)}{\Omega_0^2 \Gamma_0 \sqrt{\Gamma_0^2 + \omega^2}} \\
&\quad \times \Re \left([\omega - (\Omega + i\gamma)][\omega + (\omega - i\gamma)][\omega - i\Gamma][\omega + i(\Gamma + 2\gamma)] e^{i\omega|x|} \right) .
\end{aligned} \tag{15}$$

Using (14) we can invert the transformation (9), to obtain

$$\begin{aligned}
\hat{A}_\omega &= \frac{1}{\sqrt{\pi}} \left[\int_{-\infty}^{\infty} dx \hat{\phi}(x) v_\omega(x) - q(\omega) \hat{P} \right] \\
\hat{\Pi}_\omega^A &= \frac{1}{\sqrt{\pi}} \left[\int_{-\infty}^{\infty} dx \hat{\pi}(x) u_\omega(x) - p(\omega) \hat{Q} \right] .
\end{aligned} \tag{16}$$

It is then straightforward to verify from the standard commutation relations of $\hat{\phi}$ and $\hat{\pi}$, and \hat{Q} and \hat{P} , that (16) implies the canonical relations

$$\begin{aligned}
[\hat{A}_\omega, \hat{A}_{\omega'}] &= 0 \\
[\hat{\Pi}_\omega^A, \hat{\Pi}_{\omega'}^A] &= 0 \\
[\hat{A}_\omega, \hat{\Pi}_{\omega'}^A] &= i\delta(\omega - \omega') .
\end{aligned} \tag{17}$$

Furthermore, we have

$$\hat{H} = \frac{1}{2} \int_0^\infty d\omega \left((\hat{\Pi}_\omega^A)^2 + \omega^2 \hat{A}_\omega^2 \right) + \hat{H}_{odd} , \tag{18}$$

where \hat{H}_{odd} contains the unimportant odd mode operators B_ω and Π_ω^B . This demonstrates that (8) is indeed equivalent to a free massless field. It is of course obvious that any quadratic model is equivalent to some spectrum of decoupled oscillators; but the fact that the single oscillator simply disappears like a drop in the continuous bucket of field modes, and does not alter the spectral density at all, is not so trivial.

2.2 Initial state

Previous investigations of decoherence have typically considered direct product initial states, of the form

$$|\Psi_i\rangle = |0\rangle_{field} |\psi_a\rangle_{oscillator} , \quad (19)$$

where $|0\rangle_{field}$ is the vacuum state of the free field, and $|\psi_a\rangle$ is a bimodal oscillator state, such as

$$|\psi_a\rangle = c_+ e^{-ia\hat{P}} |0\rangle_{osc} + c_- e^{ia\hat{P}} |0\rangle_{osc} , \quad (20)$$

$|0\rangle_{osc}$ being the free oscillator ground state. For calculational convenience, we will instead use the initial state

$$|\Psi_i\rangle = c_+ e^{-ia\hat{P}} |0\rangle + c_- e^{ia\hat{P}} |0\rangle , \quad (21)$$

where $|0\rangle$ is the true, interacting ground state of the field-oscillator system.

Since our objective in this paper is to study cases in which the initial oscillator entropy is effectively zero, we must show that the field-oscillator entanglement in the initial state (21) leads only to negligible initial entropy when the field is traced out. Since we have diagonalized the full Hamiltonian, the wave functional of the interacting ground state $|0\rangle$ in the Π_ω^A variables is simply a product of Gaussians:

$$\langle \Pi^A | 0 \rangle = Z^{-1} e^{-\frac{1}{2} \int_0^\infty \frac{d\omega}{\omega} (\Pi_\omega^A)^2} . \quad (22)$$

Z is a normalization constant; a state $\langle \Pi_\omega^A | 0 \rangle$ is an eigenstate of the $\hat{\Pi}_\omega^A$ operators defined in (16)².

We can readily obtain from (22) the corresponding reduced density matrix for the oscillator,

$$\rho_0(Q, Q') = \int \mathcal{D}\pi \langle Q, \pi | 0 \rangle \langle 0 | Q', \pi \rangle , \quad (23)$$

where we write $\mathcal{D}\pi$ for the measure to indicate that we want the limit where the integral is continuously infinite dimensional. Since according to (16) the

²Both entities are of course only well-defined if we consider the continuous and infinite spectrum of oscillators to be the limit of a sequence of systems of finite numbers of oscillators.

operators $\hat{\Pi}_\omega^A$ are linear combinations of the operators $\hat{\pi}(x)$ and \hat{Q} , the states $|\pi\rangle|Q\rangle$ are just different labellings of the states $|\Pi_\omega^A\rangle$. We therefore already have in equation (22) the integrand of (23), and we need only now express the path-like integral over π in terms of the Π_ω^A variables.

We can deduce a straightforward way to do this from Equation (16). We first change variables in (23) by substituting

$$\Pi_\omega^A[\pi(x), Q] \rightarrow \bar{\Pi}_\omega^A[\pi(x)] - \frac{p(\omega)}{\sqrt{\pi}}Q, \quad (24)$$

in (22), where

$$\bar{\Pi}_\omega^A \equiv \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dx u_\omega(x) \pi(x). \quad (25)$$

Equation (14) then implies

$$\int_0^\infty d\omega q(\omega) \bar{\Pi}_\omega = 0. \quad (26)$$

To integrate over $\pi(x)$, then, we will integrate over $\bar{\Pi}_\omega$, with a delta-function inserted to enforce (26) and thus remove the Q sector from the integral:

$$\begin{aligned} \rho_0(Q, Q') &= \int d\lambda \int \mathcal{D}\bar{\Pi} e^{i\frac{\lambda}{\sqrt{\pi}} \int_0^\infty d\omega q(\omega) \bar{\Pi}_\omega} e^{-\frac{1}{2} \int_0^\infty \frac{d\omega}{\omega} \left[(\bar{\Pi}_\omega^A - \frac{p(\omega)}{\sqrt{\pi}}Q)^2 + (\bar{\Pi}_\omega^A - \frac{p(\omega)}{\sqrt{\pi}}Q')^2 \right]} \\ &= N e^{-\frac{1}{4\pi} \int_0^\infty \frac{d\omega}{\omega} p^2(\omega) (Q-Q')^2} e^{-\frac{\pi}{4} [\int_0^\infty d\omega \omega q^2(\omega)]^{-1} (Q+Q')^2} \\ &\equiv N e^{-\frac{1}{2} M \Omega (Q^2 + Q'^2)} e^{-\Delta M \Omega (Q-Q')^2}. \end{aligned} \quad (27)$$

Here we introduce the renormalized mass M and ground state entanglement parameter Δ , which with our choice of frequency ratios (13) are given by

$$\begin{aligned} M &= M_0 \times \left(1 + \frac{2}{\pi} \frac{\gamma}{\Omega} + \mathcal{O}(\epsilon^2) \right) \\ \Delta &= -\frac{1}{\pi} \epsilon (1 + \ln \epsilon) + \mathcal{O}(\epsilon^2). \end{aligned} \quad (28)$$

Since we have assumed (in order to have a pure state at late times) that Δ is negligibly small, we now observe that (27) is effectively equal, for all Q, Q' , to the density matrix of the (pure) ground state of an oscillator with mass M and natural frequency Ω . Since the translation operators $e^{\pm ia\hat{P}}$ in (21) only shift Q and Q' in (27) by $\pm a$, it is also true that the reduced density matrix

formed from (21) differs negligibly from the one derived from (19). We have therefore shown that our oscillator has negligible entropy, when the field is traced out of our chosen initial state. In fact, our choice of (21) instead of (19) will have no significant effect on our results (because we assume such weak coupling).

For the remainder of this paper, we will set $M\Omega = 1$, and assume that, in the choice of units this implies, a^2 is large (of order ϵ^{-1}). This will mean that, even though the oscillator is very weakly coupled to the field, and has a dissipation timescale much longer than the dynamical timescale, the two Gaussians that are superposed in the initial state are far enough apart from each other that significant decoherence will occur on a very short timescale.

2.3 Oscillator evolution

By using the transition matrix in the momentum representation for a harmonic oscillator of frequency ω , we easily obtain the wave functional of the final state into which the initial state (21) evolves at time t :

$$\begin{aligned} \langle \Pi^A, \Pi^B | \Psi_t \rangle &= N^{\frac{1}{2}} e^{-\frac{1}{2} \int_0^\infty \frac{d\omega}{\omega} \left[(\Pi_\omega^A)^2 + ia^2 \frac{p^2(\omega)}{\pi} \sin \omega t e^{-i\omega t} \right]} \\ &\quad \times \left(c_+ e^{i\frac{a}{\sqrt{\pi}}} \int_0^\infty \frac{d\omega}{\omega} p(\omega) e^{-i\omega t} + c_- e^{-i\frac{a}{\sqrt{\pi}}} \int_0^\infty \frac{d\omega}{\omega} p(\omega) e^{-i\omega t} \right) \end{aligned} \quad (29)$$

We then use a technique like that employed in deriving (27) above to obtain the reduced density matrix.

There will be four components:

$$\rho(Q, Q'; t) = |c_+|^2 \rho_{++} + |c_-|^2 \rho_{--} + c_+ c_-^* \rho_{+-} + c_- c_+^* \rho_{-+} . \quad (30)$$

We find that the ‘diagonal’ components are given by

$$\begin{aligned} \rho_{\pm\pm}(Q, Q'; t) &= N e^{-\frac{1}{2} ([Q \mp ar(t)]^2 + [Q' \mp ar(t)]^2)} \\ &\quad \times e^{\mp ias(t)(Q-Q')} \\ &\quad \times e^{-\Delta [Q-Q']^2} ; \end{aligned} \quad (31)$$

while the cross-terms are

$$\begin{aligned} \rho_{\pm\mp}(Q, Q'; t) &= N e^{-\frac{1}{2} ([Q \mp ay(t)]^2 + [Q' \pm ay(t)]^2)} \\ &\quad \times e^{\mp iaz(t)(Q+Q')} \\ &\quad \times e^{-\Delta [Q-Q' \mp 2y(t)]^2} \\ &\quad \times e^{-a^2 ((1+4\Delta)[1-y^2(t)] - z^2(t))} . \end{aligned} \quad (32)$$

Since Δ is negligible, we can ignore the last line of (31) and the second-last line of (32). This leaves each of the four terms in $\rho(Q, Q'; t)$ as a separated function of the form

$$\rho_{\pm\pm'} = \psi_{\pm\pm'}(Q)\psi_{\pm\pm'}^*(Q'). \quad (33)$$

The functions $r(t)$ and $s(t)$ are simple enough:

$$\begin{aligned} r(t) &= e^{-\gamma t}[\cos \omega t - \epsilon \sin \omega t] + \mathcal{O}(\epsilon^2) \\ s(t) &= e^{-\gamma t}\left[\left(1 - \frac{2}{\pi}\epsilon\right) \sin \Omega t + 2\epsilon \cos \Omega t\right] - 2\epsilon e^{-\Gamma t} + \mathcal{O}(\epsilon^2). \end{aligned} \quad (34)$$

It is clear from (31) and (34) that the diagonal terms describe Gaussian wave packets performing weakly damped oscillations.

The functions $y(t)$ and $z(t)$, on the other hand, include some exponential-integral terms:

$$\begin{aligned} y(t) &= e^{-\gamma t}\left[\left(1 - \frac{2}{\pi}\epsilon - 4\Delta\right) \cos \Omega t - 2\epsilon \sin \Omega t\right] - \frac{2}{\pi}\epsilon + \mathcal{O}(\epsilon^2) \\ &\quad - \frac{4}{\pi}\epsilon \Re\left(\left(1 + i\frac{\Omega t}{2}\right)e^{i\Omega t}[\text{Ei}(-i\Omega t) + i\pi]\right) \\ &\quad + \frac{2}{\pi}\epsilon\left(e^{\Gamma t}\text{Ei}(-\Gamma t) + e^{-\Gamma t}\text{Ei}(\Gamma t)\right) \\ z(t) &= e^{-\gamma t}[\sin \Omega t + \epsilon \cos \Omega t] + \mathcal{O}(\epsilon^2) \\ &\quad - \frac{2}{\pi}\epsilon \Im\left(\left(1 + i\Omega t\right)e^{i\Omega t}[\text{Ei}(-i\Omega t) + i\pi]\right). \end{aligned} \quad (35)$$

(The exponential-integral functions of imaginary argument appear together with $i\pi$ because we actually need a different branch of the Ei function than the standard one.)

The cross-terms are very rapidly suppressed, because the exponent of the last term in (32),

$$D(t) \equiv a^2(1 + 4\Delta)[1 - y^2(t)] - a^2 z^2(t), \quad (36)$$

grows on the cut-off timescale (see Figure 1), and a^2 is large. We emphasize that this decoherence occurs even in the extremely underdamped limit where $\Delta \rightarrow 0$, and even when the initial state is not an exact direct product.

The decohering factor $D(t)$ grows rapidly because at times much less than a dynamical time Ω^{-1} , $y(t)$ is approximately given by

$$y(t) \sim 1 + \frac{2}{\pi}\epsilon\left(e^{\Gamma t}\text{Ei}(-\Gamma t) + e^{-\Gamma t}\text{Ei}(\Gamma t) - 2C_{Euler} - 2\ln(\Gamma t)\right) + \mathcal{O}(\epsilon^2). \quad (37)$$

This function drops from unity on the cut-off timescale Γ^{-1} . We can therefore see that $y(t)$ differs from $r(t)$, and $z(t)$ differs from $s(t)$, on the same timescale as $D(t)$ suppresses the crossterms. Hence there is a very short, very early time interval during which the four wave functions $\psi_{\pm\pm'}$ appearing in $\rho(Q, Q'; t)$ are all significant and distinct. Orthogonality, however, as opposed to mere distinctness, is what will be important for determining the eigenvalues and hence the entropy of the reduced density matrix. By the time $y(t)$ has diverged from $r(t)$ enough that $\psi_{\pm\mp}$ are effectively orthogonal to $\psi_{\pm\pm}$, $D(t)$ is already large, and $\psi_{\pm\mp}$ may be ignored. We will therefore be able to neglect the distinction between $\psi_{\pm\pm}$ and $\psi_{\pm\mp}$, and approximate the density matrix by the simpler, two-state form

$$\begin{aligned} \rho(Q, Q'; t) \doteq & |c_+|^2 \psi_+(Q) \psi_+^*(Q') + |c_-|^2 \psi_-(Q) \psi_-^*(Q') \\ & + e^{-D(t)} c_+ c_-^* \psi_+(Q) \psi_-^*(Q') \\ & + e^{-D(t)} c_- c_+^* \psi_-(Q) \psi_+^*(Q') , \end{aligned} \quad (38)$$

where

$$\psi_{\pm}(Q) = \psi_{\pm\pm}(Q) \equiv N^{\frac{1}{2}} e^{-\frac{1}{2}[Q \mp r(t)]^2 \mp i a s(t) Q} . \quad (39)$$

2.4 Entropy evolution

We can explicitly diagonalize $\hat{\rho}(t)$ in the approximation that (38) is valid by assuming that the eigenvector wave functions are of the form

$$\phi_{\lambda}(Q) = \sum_{\pm} A_{\pm}(\lambda) \psi_{\pm}(Q) . \quad (40)$$

We then solve for the co-efficients A_{\pm} by requiring them to be elements of the eigenvectors of the matrix

$$\mathcal{M} \equiv \begin{pmatrix} |c_+|^2 + c_+ c_-^* e^{-D(t)+a^2(r^2+s^2)} & |c_+|^2 e^{-D(t)} + c_+ c_-^* e^{-a^2(r^2+s^2)} \\ |c_-|^2 e^{-D(t)} + c_- c_+^* e^{-a^2(r^2+s^2)} & |c_-|^2 + c_- c_+^* e^{-D(t)+a^2(r^2+s^2)} \end{pmatrix} . \quad (41)$$

The quadratic characteristic equation for \mathcal{M} yields the two eigenvalues

$$\lambda_{\pm} = \frac{1}{2} \left[1 \pm \sqrt{1 - 4|c_+|^2 |c_-|^2 (1 - e^{-2a^2 e^{-2\gamma t}} - e^{-2D(t)})} \right] + \mathcal{O}(\epsilon^2) , \quad (42)$$

when we take advantage of the facts that $|c_+|^2 + |c_-|^2 = 1$, $e^{-a^2(r^2+s^2)} \simeq e^{-a^2 e^{-2\gamma t}}$, and $e^{-D(t)} e^{-a^2 e^{-2\gamma t}} \simeq 0$.

We now specialize to the case $c_+ = c_- = \frac{1}{\sqrt{2}}$, where we have

$$\lambda_{\pm} \simeq \frac{1}{2} \left(1 \pm [e^{-a^2 e^{-2\gamma t}} + e^{-D(t)}] \right), \quad (43)$$

since $e^{-a^2 e^{-2\gamma t}} e^{-D(t)}$ is extremely small for any t .

The entropy $S(t)$ for this case is plotted in Figure 2. It initially rises on the decoherence timescale from its initial negligible value to $\ln 2$, where it persists for many dynamical times, before declining on the dissipative timescale. The re-establishment of the purity of the oscillator state is clearly due to the fact that after a time on the order of $\gamma^{-1} \ln a$ the two shifted gaussians have lost so much amplitude that they begin to overlap and become indistinguishable. It then becomes less and less true that the oscillator is in a mixture of two orthogonal states. In the limit of complete relaxation, the ground state is reached, and this is of course a pure state.

We can assess the accuracy of the approximation (38) by finding the eigenvalues of $\hat{\rho}(t)$ using (33) instead. In this case we would have a fourth rank matrix in the analogue of (41), and we would find four eigenvalues. Only two of these would be non-negligible, however, and they would turn out to differ insignificantly from (42). The eigenvectors we would find by this more accurate technique would differ somewhat, at very early times, from the two ϕ_{λ} implied by (40) and (41). At these early times, the more exact eigenvectors would also include some non-vanishing amplitudes for the states represented by $\psi_{\pm\mp}(Q)$, with their dependencies on $y(t)$ and $z(t)$ instead of $r(t)$ and $s(t)$.

2.5 Field evolution

We now wish to consider the massless scalar field as the observed system, and to trace out the harmonic oscillator as unobserved. Since the initial state of the total system is pure, the non-zero eigenvalues of the reduced density matrix of the field are the same as those of the reduced density matrix of the oscillator. The entropy of the field is therefore the same as the oscillator entropy discussed in the preceding subsection. The problem that still remains, and which did not arise for the oscillator, is that of assessing where in spacetime the information associated with this entropy may be said to reside.

There are of course many possible definitions of the term ‘‘information’’, but for the purposes of this paper we shall consider that the information

problem will be solved by identifying the field state as a simple mixture of states that can be created from the vacuum by external sources. The external sources will be functions in spacetime, and the required information will be considered to reside in the regions where these sources have support.

We will present the reduced density matrix $\hat{R}(t)$ for the field in the basis of field operator eigenstates, and so we will need to transform the field-oscillator state $\langle \Pi_\omega^A, \Pi_\omega^B | \Psi_t \rangle$ of Equation (29) from the $\Pi_\omega^A, \Pi_\omega^B$ representation into the A_ω, B_ω representation. We then invoke one of the eigenvalue relations corresponding to Equation (16)

$$A_\omega = \frac{1}{\sqrt{\pi}} \left[\int_{-\infty}^{\infty} dx \phi(x) v_\omega(x) - q(\omega) P \right], \quad (44)$$

and trace over P to obtain the reduced density matrix

$$R[\phi, \phi'; t] = \int dP \langle A_\omega[\phi, P] | \Psi_t \rangle \langle \Psi_t | A_\omega[\phi', P] \rangle. \quad (45)$$

The reduced density matrix for the field will again be the sum of four contributions,

$$R[\phi, \phi'; t] = |c_+|^2 R_{++} + |c_-|^2 R_{--} + c_+ c_-^* R_{+-} + c_- c_+^* R_{-+}, \quad (46)$$

where

$$\begin{aligned} R_{\pm\pm} &= \Psi_\pm[\phi; t] \Psi_\pm^*[\phi'; t] e^{\left[\int_{-\infty}^{\infty} dx (\phi + \phi') L(x) \right]^2} \\ R_{\pm\mp} &= \Psi_\pm[\phi; t] \Psi_\mp^*[\phi'; t] e^{\left[\int_{-\infty}^{\infty} dx (\phi + \phi') L(x) \right]^2} \\ &\quad \times e^{-a^2(r^2 + s^2)} e^{\pm a[ir(t) - s(t)] \int_{-\infty}^{\infty} dx (\phi + \phi') L(x)}. \end{aligned} \quad (47)$$

Here we have defined

$$\begin{aligned} L(x) &\equiv \frac{1}{\pi} \int_0^\infty d\omega \omega q(\omega) v_\omega(x) \\ \Psi_\pm[\phi(x); t] &\equiv \tilde{Z}^{-\frac{1}{2}} e^{-\frac{1}{2} \int_0^\infty d\omega \frac{\omega}{\pi} \left(\frac{a p(\omega)}{\omega} \sin \omega t + a q(\omega) z(t) - \int_{-\infty}^\infty dx \phi(x) v_\omega(x) \right)^2} \\ &\quad \times e^{\pm i a \int_{-\infty}^\infty dx \phi(x) K(x, t)}. \end{aligned} \quad (48)$$

The function $K(x, t)$ will be defined below.

As in (38), we can simplify (47) by approximating it as

$$\begin{aligned} R_{\pm\pm} &\simeq \Psi_{\pm}[\phi; t] \Psi_{\pm}^*[\phi'; t] \\ R_{\pm\mp} &\simeq \Psi_{\pm}[\phi; t] \Psi_{\mp}^*[\phi'; t] \times e^{-a^2 e^{-2\gamma t}} . \end{aligned} \quad (49)$$

This approximation can evidently lead to large errors in evaluating expectation values of $\hat{\phi}(x)$ for x close to the origin, where $L(x)$ is non-negligible; but as with the analogous approximation in (38), it retains the actual behavior of the entropy very well. Equation (49) and the inner product

$$\langle \Psi_{-}(t) | \Psi_{+}(t) \rangle = e^{-D(t)} \quad (50)$$

imply that the eigenvalues of the density matrix (46) are again given by (42), as should be the case.

In (49), there are only two wave functionals Ψ_{\pm} that characterize the state of the field. The states they represent may be created by an external source, linearly coupled to both $\hat{\phi}$ and $\hat{\pi}$:

$$|\Psi_{\pm}(t)\rangle = e^{\pm ia \int_{-\infty}^{\infty} dx [J(x,t)\hat{\pi}(x) + K(x,t)\hat{\phi}(x)]} |\Psi_0\rangle , \quad (51)$$

where the time-independent state $|\Psi_0\rangle$ is simply $|\Psi_{\pm}\rangle$ with a set equal to zero. (This state is not precisely the vacuum state of the field, but it resembles it closely except near $x = 0$, where it has been polarized by the unobserved oscillator.)

J and K are the external sources which describe the spacetime location of the information:

$$\begin{aligned} J(x, t) &= \frac{g}{2} \int_{-t}^t dx' F(x - x') r(t - |x'|) \\ K(x, t) &= \frac{g}{2} (2r(t)F(x) - F(x + t) - F(x - t) \\ &\quad - (1 + \frac{2}{\pi}\epsilon) \int_{-t}^t dx' F(x - x') s(t - |x'|)) . \end{aligned} \quad (52)$$

From (52) it is clear that in the late time limit, all the information has propagated away from the origin. In this late time limit $e^{-a^2 e^{-2\gamma t}} \rightarrow 1$, and the state of the field, with the oscillator traced out, becomes pure.

Note that the behavior of J and K is not particularly sensitive to the small amounts of initial field-oscillator entanglement that are at issue in

choosing (21) instead of a direct product state. The operator exponent in (51) is simply the projection, onto the field sector of the total Hilbert space, of the time-evolved \hat{P} operator:

$$\hat{P}(t) = r(t)\hat{P} - M\Omega s(t)\hat{Q} + \int_{-\infty}^{\infty} dx [J(x,t)\hat{\pi}(x) + K(x,t)\hat{\phi}(x)]. \quad (53)$$

If the information initially in the oscillator is characterized by \hat{P} , then the information propagation into the field will be described (up to ambiguities near the origin) by (52). This clearly shows that the information which was characterized by the initial value of P of the oscillator will become non-local in the late time limit.

3 Discussion

The evolution of the oscillator-field system shows that the entanglement entropy is roughly constant until the oscillator approaches its ground state. Moreover we have shown that the information is hidden in a very non-local way.

Both $J(x,t)$ and $K(x,t)$ are important in characterizing the state of the field. $J(x,t)$ behaves very much like classical radiation from a source, and implies that some of the information initially in the oscillator propagates into the field in the same way that one might naively expect. In contrast, $K(x,t)$ possesses sharp spikes, whose width is on the cut-off scale, which propagate away or decay in place. The propagating spikes are evidently the couriers for the rapid shedding of information that is associated with decoherence. The spike in $K(x,t)$ which remains at $x = 0$ decays only on the dissipative timescale. This could be interpreted as describing information which remains in the ‘quantum hair’ of the oscillator, and is only slowly radiated away. It should be pointed out, however, that our approximation in (49) breaks down near the origin.

Furthermore, assigning the degrees of freedom near the origin to the field or to the unobserved oscillator is to a large degree arbitrary, since unitary transformations that alter such assignments can leave the rest of the model essentially unchanged. It could well be that the information pool described by the non-propagating term in $K(x,t)$ is best considered as belonging to a ‘dressed’ version of the oscillator. This type of ambiguity seems likely to

be a common feature of problems in which information loss and ultraviolet regulators are closely related, since one does not expect a uniquely specified ultraviolet cut-off to exist. In fact, the best procedure in such cases might be to search for an optimal kind of UV regulation based on the location of information.

We can arrange to present the state of the field (with the oscillator treated as unobserved) in the form of a Wigner function. We can do this by noting that (i) the field state is effectively a mixture of pure states $|\Psi_{\pm}(t)\rangle$, and (ii) at any fixed time t , the states $|\Psi_{\pm}(t)\rangle$ belong to a subspace of the field's Hilbert space which may be mapped onto the Hilbert space of a single harmonic oscillator. The second assertion is justified by the inner product Equation (50), which is the correct inner product for two coherent states with annihilation operator eigenvalues $\alpha_{\pm} = \pm\sqrt{D(t)/2}$.³ Therefore, by applying to (46) and (49) the mapping

$$|\Psi_{\pm}(t)\rangle \rightarrow |\pm\sqrt{D(t)/2}\rangle_{osc}, \quad (54)$$

the state of the field may be described at any given time by a mixture of single-oscillator coherent states $|\alpha_{\pm}\rangle_{osc}$.

Wigner functions for such mixed states are easily calculated. The function for the field at $t = 0$ is single gaussian corresponding to the ground state. The Wigner functions which represent the state of the field at several later instants are plotted in Figures 3.1 – 3.3. Figure 3.1 shows the state of the field very soon after $t = 0$, as it is just getting excited into a mixture of two coherent states. The two overlapping gaussian peaks that can be discerned in 3.1 will separate on the decoherence timescale. (No rapid growth in energy is associated with this sudden separation: it is only the inner products of the field states, and not their energies, that fit the analogy with oscillator coherent states.) Figure 3.2 describes the field during the long intermediate epoch of its evolution: a mixture of two well-separated gaussians. Finally, when the oscillator ends up in its ground state, the field regains its purity. The rapid oscillations near the origin in Figure 3.3 are symptoms of quantum coherence.

³By repeating the analysis of Section 2 starting with more general initial states, it is straightforward to define a class of field states which represent, in the sense we are considering, the full complex plane of coherent states, with complex eigenvalues α .

These figures clearly show the loss and eventual restoration of quantum coherence in the field. In order to determine the degree of purity of a harmonic oscillator, however, one must have some means of measuring at least the gross features of its Wigner function. In our case, the observables which must thus be measured (in whatever combination) are the non-local operators $\int_{-\infty}^{\infty} dx [J(x, t)\hat{\pi}(x) + K(x, t)\hat{\phi}(x)]$ and its canonical conjugate. Since these operators are non-local on the dissipation scale, it would seem to be very difficult to actually observe the asymptotic purity of the state of the field.

This system can be thought as a specific example of Page's[20] alternative outcome of black hole evaporation. He suggests that information might get out of black holes through radiation, and has shown that the information might not show up in an analysis perturbative in M_{Planck}/M . Our system seems to behave in this way, as in our case the information is not recovered until the very end of the decay of the oscillator. If we were to start the oscillator in a *mixture* of the two gaussian states, instead of in the superposition we have discussed, we would not notice any difference by examining the field until the oscillator had relaxed to very near its ground state. How long this takes to occur, in our model, is dependent on the initial state.

We should also point out that we do not find a strict relation between the energy of the oscillator and the rate of information exchange (one of the folkloric statements used as an argument for the loss of coherence in black hole radiation). Thus it might well be possible that black hole evolution does preserve quantum coherence, but that it is very hard, if not impossible, to recover all the initial information in a set of local observations of the final state.

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Figure captions.

Figure 1. Behavior of $D(t)$ of equation (36), with the parameter a set equal to 10. $D(t)$ is the term responsible for the suppression of the interference term of the two-gaussian initial state; it increases rapidly (on the decoherence timescale) and remains large.

Figure 2. Evolution of the entropy of the oscillator for three initial states of different values of the initial separation parameter a . The entropy increases rapidly during the decoherence phase and remain essentially constant until the oscillator enters the final stages of its decay towards the ground state. Since this occurs only when the oscillator has lost all but a fixed amount of its initial energy, the length of time before recoherence depends on the initial state.

Figure 3. Evolution of the the degree of freedom of the field which is excited by the oscillator (*i.e.*, the mode spanning linear combinations of the time-dependent, non-locally excited states $|\Psi_{\pm}(t)\rangle$). It starts in its ground state, is excited into a mixed state by the oscillator, and approaches a pure excited state as the oscillator decays to its own ground state. The sharp oscillations in 3.3 are witnesses to the purity of the field state at late times.

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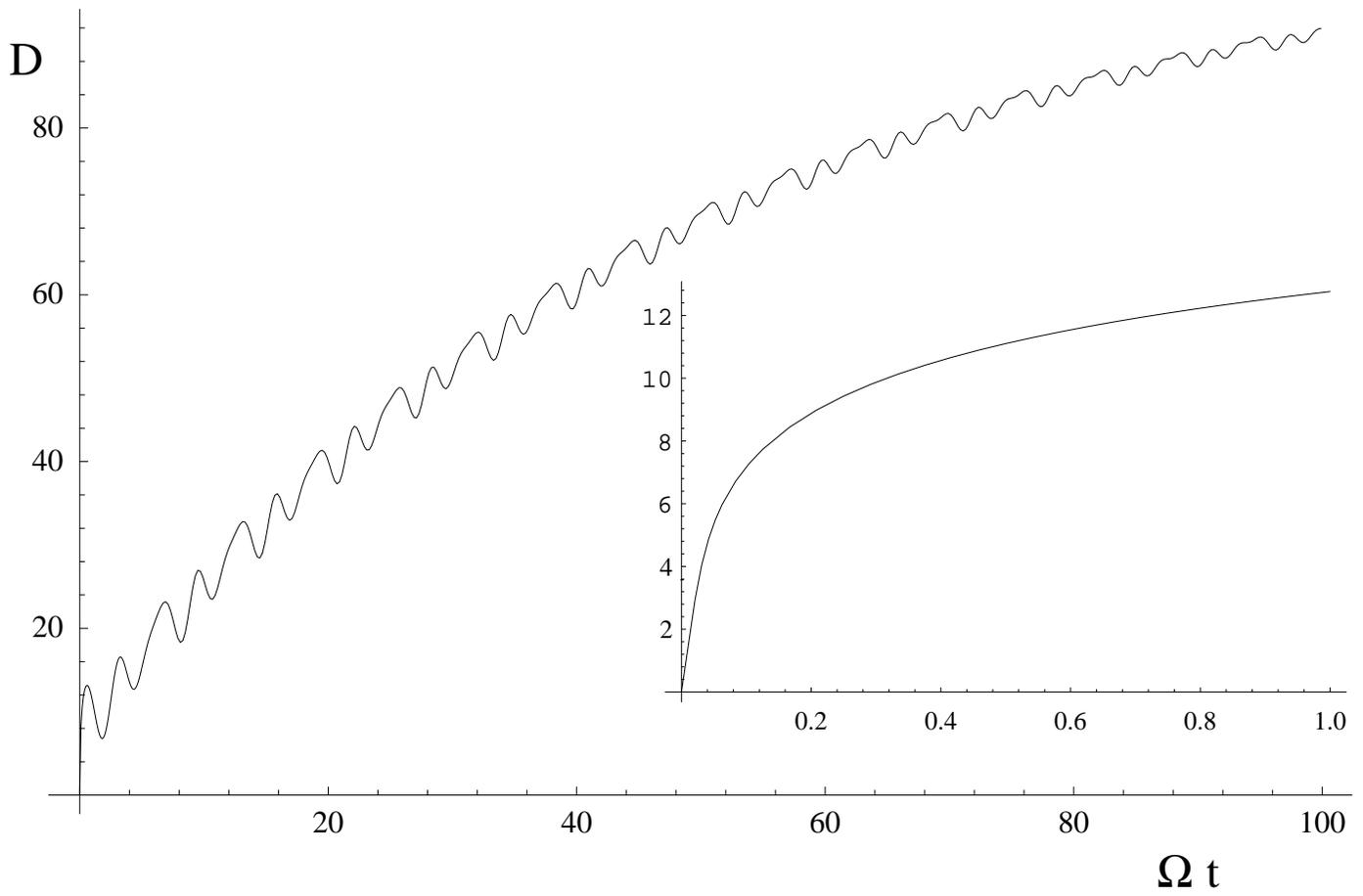


Figure 1

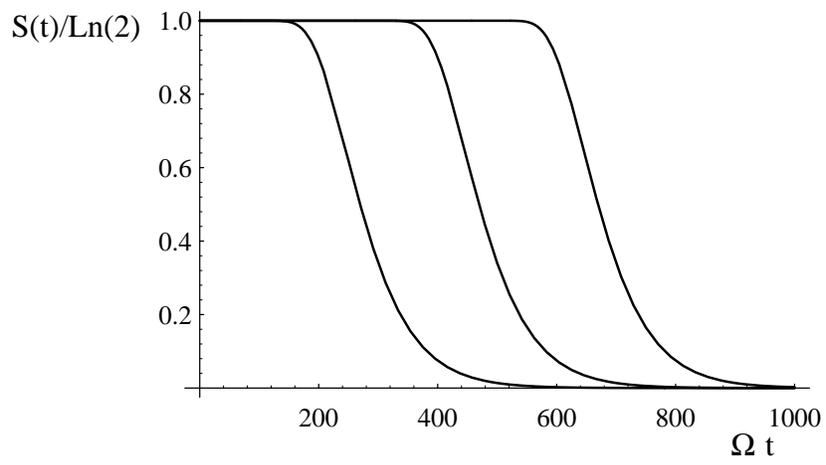


Figure 2