

Foundational Methods in Computer Science 2013 Abstracts

Mount Allison University, May 31st-June 3rd, 2013

Tutorials and invited lectures

Joachim Kock (Barcelona): *Polynomial functors over groupoids: from program semantics to quantum field theory – and back*

The theory of polynomial functors has origins in program semantics: on one hand polynomial functors serve as data type constructors, often called containers, and on the other hand their initial algebras are well-founded trees, accounting for inductive data types. It is useful to upgrade the theory to groupoid coefficients instead of set coefficients: this allows to capture also data types with symmetries, such as multisets, and gives a more flexible notion of well-founded tree, covering in particular non-planar trees. Polynomial functors over groupoids cover also combinatorial species as well as Baez-Dolan stuff types, and allow for “type-theoretic” methods in combinatorics.

More specifically, Feynman graphs, as used in perturbative quantum field theory, can be interpreted as well-founded trees. The fixpoint equations that generate them are the so-called combinatorial Dyson-Schwinger equations, a combinatorial skeleton of the quantum equations of motion.

They are of great interest in quantum field theory, and categorical methods can help understanding them. Conversely, methods used in combinatorics and QFT, such as Hopf algebras and renormalisation schemes, should have applications in computer science. Manin has recently suggested a renormalisation approach to the theory of computation, and to the halting problem in particular. Polynomial functors should be a convenient framework to understand these connections.

Some of these connections are a bit speculative at the moment, and I am not an expert in either program semantics or quantum field theory. The goal of the lectures is first of all to explain the parts that are on reasonably firm footing, and second to try to involve theoretical computer scientists to help with the developments. No prior knowledge of physics is assumed, the main prerequisite being familiarity with category theory.

Ernie Manes (Massachusetts): *Atoms in orbital extensive categories*

If the vertices of a regular icosahedron can be colored black or white, how many such objects are there if opposite vertices have different color? This is an undergraduate-level problem. I believe the following further problem has not been examined: for how many of these can the icosahedron be rotated into the maximum possible number of differently-appearing positions?

We view this question as belonging to the study of combinatorics in certain extensive categories (for basic background, see Carboni, Lack and Walters, Introduction to extensive and distributive categories, JPAA 84, 1993, 145-158). A characterization of these categories emphasizes a category of conjugacy classes of subgroups of a finite group. An “alchemy theorem” turns all atoms into G .

Bring your calculator. There will be lots of homework!

Eugenio Moggi (Genoa): *Categories for Collection Types*

Collection types have been proposed as a suitable way to capture database query languages in a type-theoretic setting.

In 1998 Manes introduced the notion of collection monad on the category of sets as a suitable semantics for collection types. The canonical example of collection monad is the finite powerset monad.

In order to account for the algorithmic aspects of database languages, the category of sets is unsuitable, and should be replaced with other categories, whose arrows are functions computable by low complexity algorithms.

We propose a class of categories, that are more general than locales, and meet the following desiderata:

- in them one can define an analogue of the finite powerset monad and recast the notion of collection monad;
- concrete examples include standard set-theoretic models, and also models based on low complexity numeric functions (like linear space computable functions);
- in them one can interpret a significant fragment of Martin-Lof extensional type theory.

Michael Shulman (IAS): *Homotopical Computation*

Martin-Löf type theory can function both as a foundation for mathematics and as a powerful dependently typed programming language. Recently it has emerged that it also admits a natural interpretation in homotopy theory. This raises the possibility of “programming” directly with homotopy-theoretic objects and concepts, such as higher-dimensional spaces and isomorphism-invariance. Making precise sense of this is an ongoing research topic, but so far the results have been very promising. Specifically, existing implementations only allow homotopical computation “up to homotopy”, but this already suffices for proofs of many homotopy-theoretic theorems. I will describe the basics of homotopical computation and how we use them in proofs of this sort, and give an overview of some current open problems.

Contributed talks

Robin Cockett (Calgary): *The linear maps of a differential storage category DO form a tensor differential category*

Differential storage categories were designed to answer the question “What does the cokerisli category of a differential look like?” Embarrassingly we were unable to prove that they actually did answer the question! The talk is about the proof that differential storage categories do answer the question. Important in coming up with a proof was the realization that there are different sorts of (tensor) differentials and one had to be careful to choose the right one!

Subashis Chakraborty (Calgary): *MPL, the message passing language*

Message passing is a key element of concurrent programming. In “The Logic of Message Passing” (2007), Cockett and Pastro introduced a proof theory that included message passing as a primitive.

I am currently working on a programming language, MPL, which is based on this logic, where programs and communication between programs are typechecked. In this talk, we will describe the syntax and reduction semantics of MPL programs. We will also see how the types guide the communication, and we will describe how to typecheck MPL programs.

In MPL, we also add datatypes to the message passing aspects of the logic. This has the effect of adding communication protocol specification to the language, and ensuring that the program will implement this protocol at compile time. We will also see examples of how these protocols work in MPL.

Jeff Egger (Dalhousie): TBA

Jonathan Gallagher (Calgary): *Partial Term Rewriting*

Combinatory logic is an important example of an orthogonal term rewriting system. In combinatory logic, the partiality of a function arises as the nontermination of the rewriting relation. Partial combinatory logic was introduced to make this partiality explicit.

In this talk we will introduce the notion of the partialization of an orthogonal term rewriting system; this yields, when applied to combinatory logic, its partial counterpart. We will then show that the rewriting system on a partialized orthogonal rewriting system enjoys many of the nice properties of the original system, most importantly, confluence.

Brett Giles (Calgary): *Exact synthesis of quantum circuits*

An important question in the field of compilers for quantum programming languages is how best to approximate an arbitrary quantum transform in terms of a given set of quantum operators. This is referred to as “synthesis”. One can further break this down into the questions of “exact synthesis”, where the original operator is expressed exactly, and “approximate synthesis” where the operator is expressed up to some error ϵ . Much of the work on synthesis targets the set of operators “Clifford+T”.

A recent result by Kliuchinkov, Maslov, and Mosca answered the question of exact synthesis for single qubit operators - any unitary operation whose matrix is expressible in $Z[\frac{1}{\sqrt{2}}, i]$ is a product of Clifford+T operators. In that same paper, they conjectured this would also hold for n -qubit operators.

Peter Selinger and I showed this conjecture to be correct, and I will describe the proof in this talk. The proof uses elements of abstract and linear algebra (at a level normally found in 1st or 2nd year courses) and induction on the size of the matrix. If time permits, I will also discuss an alternative method of exact synthesis of single-qubit operators due to Matsumoto and Amano.

Pieter Hofstra (Ottawa): *A new category of games*

Categories of games have been studied extensively in the context of game semantics, where strategies are used to model proofs and programs. However, the games used in that paradigm are not nearly as general as those used in traditional game theory; for example, they don't include games with more than two players, as a key ingredient in game semantics is the duality between player and opponent. In this talk, I will present a new category of games which is not motivated by logical considerations but by the aim to give a conceptual interpretation of some of the classical results in traditional game theory. (It should be noted that morphisms of games

are conspicuously absent in the traditional theory.) The category contains games with arbitrary numbers of players, simultaneous or sequential, as well as multi-round games. We focus on describing various base-change operations in this category, a naturally arising notion of equivalence of games as well as an interpretation of some solution concepts.

Rory Lucyshyn-Wright (York): *Riesz-Schwartz extensive quantities and vector integration in closed categories*

We develop aspects of functional analysis in an abstract axiomatic setting, through monoidal and enriched category theory. We work in a given closed category, whose objects we call *spaces*, and we study R -module objects therein (or algebras of a commutative monad), which we call *linear spaces*. Building on ideas of Lawvere and Kock, we study functionals on the space of scalar-valued maps, including compactly-supported Radon measures and Schwartz distributions. We develop an abstract theory of vector-valued integration with respect to these scalar functionals and their relatives. We study three axiomatic approaches to vector integration, including an abstract Pettis-type integral, showing that all are encompassed by an axiomatization via monads and that all coincide in suitable contexts. We study the relation of this vector integration to relative notions of completeness in linear spaces. One such notion of completeness, defined via enriched orthogonality, determines a symmetric monoidal closed reflective subcategory consisting of exactly those separated linear spaces that support the vector integral. We prove Fubini-type theorems for the vector integral. Further, we develop several supporting topics in category theory, including enriched orthogonality and factorization systems, enriched associated idempotent monads and adjoint factorization, symmetric monoidal adjunctions and commutative monads, and enriched commutative algebraic theories.

Susan Niefield (Union): *Projective and Totally Continuous Modules over a Quantale*

A complete lattice is projective in the category of suplattices if and only if it is totally continuous. In this talk, we consider projectives in the category $Q\text{-Mod}$ of modules over a commutative quantale Q . Our interest in projective modules stems from the fact that a quantale morphism $Q \rightarrow A$ is coexponentiable in the category $Q\text{-Alg}$ of Q -algebras if and only if the induced Q -module is projective. We know that the categories $L\text{-Mod}$ and $L\text{-Alg}$ are equivalent to the categories of internal suplattices and quantales, respectively, in the topos $\mathbf{Sh}(L)$ of sheaves on a locale L . Thus, an L -module is projective if and only if it is internally totally continuous, and so an L -algebra is coexponentiable if and only if the induced internal suplattice is totally continuous in $\mathbf{Sh}(L)$.

Dorette Pronk (Dalhousie): *Bicategorical Fibration Structures and Stacks*

The familiar construction of categories of fractions, due to Gabriel and Zisman, allows one to invert a class W of arrows in a category in a universal way. Similarly, bicategories of fractions allow one to invert a collection of arrows in a bicategory. In this case the arrows are inverted in the sense that they are made into equivalences. As with categories of fractions, bicategories of fractions suffer from the defect that they need not be locally small even when the bicategory in which W lives is locally small. Similarly, in the case where W is a class of arrows in a 2-category, the bicategory of fractions will not in general be a 2-category.

In this talk I will discuss two notions — *systems of fibrant objects* and *fibration systems*— which will allow us to associate to a bicategory B a homotopy bicategory $\text{Ho}(B)$ in such a way

that $\text{Ho}(B)$ is the universal way to invert weak equivalences in B . This construction resolves both of the difficulties with bicategories of fractions mentioned above. As an example of such structures we describe a fibration system on the 2-category of prestacks on a site and prove that the resulting homotopy bicategory is the 2-category of stacks. If there is time I will also discuss algebraic, differentiable and topological stacks.

This is joint work with Michael Warren.

Robert Seely (McGill): *Cartesian storage categories*

Cartesian differential categories were introduced to provide a direct axiomatization of the differential structure of the coKleisli category of a (tensor) differential category. In particular, it was proved that the coKleisli category of any (tensor) differential (storage) category is always a Cartesian differential category. However, Cartesian differential categories arise in many different and independent ways so that the converse is clearly not true. However, it is reasonable to ask whether every Cartesian differential category admits a full and faithful embedding into the coKleisli category of some (tensor) differential category.

In the past year or so, we have made some progress towards achieving such embedding theorems. This has involved developing a general theory of “storage” categories in order to have a suitably abstract characterization of the target of these representation theorems. This talk is an introduction to the talk by Robin Cockett, which will describe the most recent work; my purpose is to “remind” listeners of the basic framework of Cartesian storage categories, and so will repeat some of the content of my talk last year.

Joint work with R. Blute and J.R.B. Cockett.

Peter Selinger (Dalhousie): *Efficient Clifford+T approximation of unitary operators*

Let U be the monoidal category of finite dimensional Hilbert spaces and unitary maps. The Clifford+T groupoid is the smallest monoidal subcategory of U generated by the Hadamard gate, the controlled-not gate, the T-gate, and scalars. This groupoid is dense in U , and the approximation of arbitrary members of U by Clifford+T operators (up to given ϵ) is an important problem in quantum computation. Previously, the standard solution to this problem was the Solovay-Kitaev algorithm, a geometric algorithm that achieved gate sequences of length $K \log^c(1/\epsilon)$, where $c > 3$. I will present an efficient number theoretic algorithm that achieves length $K + 4\log(1/\epsilon)$ for single-qubit z-rotations, and $K + 12\log(1/\epsilon)$ for arbitrary single-qubit operators.

Alanod Sibih (Dalhousie): *Orbifold atlas groupoids*

We study orbifolds and strong maps of orbifolds. We begin by introducing a representation for orbifolds that consists of internal categories in the category of topological spaces. These categories represent orbifolds and atlas maps, but do not work well for general strong maps. We generalize the notion of category of fractions to internal categories in the category of topological spaces and find its universal property. We apply this to the atlas category to obtain an atlas groupoid. We can then use this to give a description of strong maps of orbifolds and the equivalence relation on them in terms of atlas groupoids. If there is enough time we will define paths in orbifolds as strong maps and give a description of the equivalence classes on such paths in terms of charts and chart embeddings.

Polina Vinogradova (Ottawa): *Formalizing Abstract Computability: Turing Categories in Coq*

The presentation will begin with a description of an existing abstraction of the notion of computability using categorical tools, called a Turing category, as described by Cockett and Hofstra. The concept of a Turing category presents a generalization of certain key features of computability - the partiality and recursive enumerability of computable maps, by means of objects and arrows, while abandoning much of the structure found in r.e. sets. Next, our approach to formalization (using the Coq proof assistant) of the definitions and results arising with the Turing category abstraction will be outlined. This formalization effort yields a new perspective on computability (in that it is using an r.e. language to describe a computational setting greater than that of the traditional r.e computation) as well as a new tool for effectively reasoning about it.

Richard Wood (Dalhousie): *The waves of a total category and total distributivity*

Street and Walters defined a locally small category \mathcal{K} to be *total(ly cocomplete)* if its Yoneda functor $Y : \mathcal{K} \rightarrow \widehat{\mathcal{K}}$ has a left adjoint, X . We say that \mathcal{K} is *totally distributive* if X has a left adjoint, W . It transpires that every total category admits a functor $W : \mathcal{K} \rightarrow \widehat{\mathcal{K}}$ for which the associated hom-like functor, $\widetilde{\mathcal{K}}(-, -) : \mathcal{K}^{op} \times \mathcal{K} \rightarrow \mathbf{set}$, has interesting properties.

Joint work with Francisco Marmolejo and Bob Rosebrugh.