

Full Day Tutorial on Quantum Models of Cognition and Decision

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causal reasoning, decision making, conceptual combinations, memory recognition, and associative memory. This tutorial is needed to introduce and train cognitive scientists on this promising new theoretical approach to cognitive science.

General Purpose

This *full day* tutorial is an exposition of a rapidly growing new alternative approach to building computational models of cognition and decision based on quantum theory. The cognitive revolution that occurred in the 1960's was based on classical computational logic, and the connectionist/neural network movements of the 1970's were based on classical dynamical systems. These classical assumptions remain at the heart of both cognitive architecture and neural network theories, and they are so commonly and widely applied that we take them for granted and presume them to be obviously true. What are these critical but hidden assumptions upon which all traditional theories rely? Quantum theory provides a fundamentally different approach to logic, reasoning, probabilistic inference, and dynamical systems. For example, quantum logic does not follow the distributive axiom of Boolean logic; quantum probabilities do not obey the disjunctive axiom of Kolmogorov probability; quantum reasoning does not obey the principle of monotonic reasoning. It turns out that humans do not obey these restrictions either, which is why we consider a quantum approach. This tutorial will provide an exposition of the basic assumptions of classic versus quantum information processing theories. These basic assumptions will be examined, side by side, in a parallel and elementary manner. The logic and mathematical foundation of classic and quantum theory will be laid out in a simple and elementary manner that uncovers the mysteries of both theories. Our main point will be to show that quantum theory provides a unified and powerful explanation for a wide variety of paradoxes found in human cognition and decision ranging across findings from attitudes, inference,

Presenters

Jerome Busemeyer is a professor of Cognitive Science at Indiana University. He was editor of the *Journal of Mathematical Psychology* and he is now *Associate Editor of Psychological Review*. His research interests include decision making and dynamic modeling. Peter Bruza is a professor of information science and he is a pioneer in the field of quantum interaction (QI). He also serves on the editorial boards of *Information Retrieval*, *Journal of Applied Logic*, *The Logic Journal of the IGPL*. Jerome and Peter are co-authors of a new book "*Quantum models of cognition and decision*" Cambridge University Press, 2012. Taiki Takahashi is a professor at Haikkado University working in the field of neuroeconomics, but also with expertise in quantum decision theory, and he has published articles on this topic in *Physical Letters A*. Jennifer Trueblood is a PhD student at Indiana University with several publications on the topic of quantum cognition including one in *Cognitive Science*.

Previous Tutorials and Symposia

This tutorial was presented for a full day at the Cognitive Science meetings in Nashville, 2007, Washington DC 2008, and Amsterdam, 2009, which included around 30 people each time. The ratings obtained from participants after the tutorial were all very good. Also this tutorial follows a symposium on quantum cognition presented at the Cognitive Science meeting 2011, and these papers will appear as a special issue in *Topics in Cognitive Science*. A similar tutorial was presented at the 3rd and 4th Annual Meetings on Quantum Interaction held

at Saarbruecken, Germany, 2009, and Aberdeen Scotland, 2010 with about 40 participants.

Participant Background

This tutorial will introduce participants to an entirely new area and no previous experience or background with quantum theory will be assumed. *No background in physics is required.* In fact, except for a few simple examples to motivate the idea, little or no reference to physics will be made during main part of the tutorial. What is required is an elementary background in classic logic and probability. During the tutorial, we will review basic concepts of linear algebra needed for quantum theory. (e.g., vectors, projectors, unitary transformations).

Material to be Covered

1. The first topic will examine the major differences between classic versus quantum theories of probability. The concept of superposition is introduced and distinguished from classic probability mixtures. The important issue of measurement in classical and quantum systems will be compared and examined. The key to this section will be several dramatic empirical examples illustrating empirical violations of the classic laws of probability (e.g., conjunction, disjunction, total probability) and the parsimonious explanation of all these violations by quantum theory. (1 hr).
2. Next we examine the differences between classical and quantum dynamical systems. The basic idea of a Markov processes will be introduced and compared with quantum processes. (Cognitive architectures and many neural networks can be represented as Markov processes). A parallel development of Markov and quantum processes will be shown. The concept of a state will be distinguished for Markov and quantum systems. The effects of measurement on the state of the system are compared for Markov and quantum systems. A key feature of this section is to show when and how quantum processes depart from Markov processes. (1 hr)
3. The third part will present the details of Matlab and R programs used to compute the choice probability and response time predictions of a dynamic quantum model that has been developed to explain three ongoing research programs in cognitive and decision making: violations of the “sure thing principle” of rational decision theory, violations of dynamic consistency in decisions, and interference of categorization on decisions. (45 min)
4. The fourth part will introduce quantum computing and information processing ideas. The concepts of a bit and a qubit will be contrasted. The concept of a conjunction of properties used in classic information processing theory will be related to the concept of a tensor product space used in quantum theory. The controlled U-gate will be introduced and compared with a production rule. The linear transformation of states used by quantum theories will be related to the distributed representation and content addressable properties of connectionist/neural networks.

The concept of fuzzy representation and probabilistic representation will be discussed and compared for fuzzy set, Bayesian, and quantum theories. The idea of an entangled state will be described. Bell’s inequality will be introduced, and violations found in conceptual combinations are reviewed. The dramatic implications of violations of this inequality for classical theories will be discussed. (45 min)

5. This part will present the details of Matlab programs used to perform quantum computing for some complex information processing tasks. This includes pattern recognition and planning event dependent action sequences under uncertainty. Basic tools of quantum computing will be used including Kronecker products to perform U-gate operations, and partial traces for measurement of components of a complex system. (45 min).
6. This part will detail how quantum theory is being used to model the human mental lexicon. In particular, quantum entanglement will be described as a means of modeling cognitive phenomena in non-reductionist way, e.g., conceptual combinations. A key feature of this section is to introduce formal tools and experimental methods which can determine whether cognitive phenomena can be validly modeled in a decompositional way. (1.0 hr)
7. Review and future directions (30 min).

References

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