

# How and why the wave function collapses after a measurement

Massimo Fioroni<sup>1</sup> and Giorgio Immirzi<sup>1,2</sup>

<sup>1</sup> Dipartimento di Fisica, Università di Perugia

<sup>2</sup> Istituto Nazionale di Fisica Nucleare, sezione di Perugia

via A. Pascoli, 06100 Perugia, Italy

*email: fioroni@perugia.infn.it , immirzi@perugia.infn.it*

November 15, 1994

## ABSTRACT

We explain the collapse of the wavefunction with the notion that, in a measurement, the system observed nucleates a first order phase transition in the measuring device. The possible final states differ by the values of macroscopic observables, and their relative phase is therefore unobservable. The process is irreversible, but needs no separate postulate.

The purpose of this paper is to restate and expand the explanation of the collapse of the wavefunction suggested by various authors [1, 2, 3, 4]<sup>1</sup>, perhaps most clearly by K. Hepp [6]. Briefly, this is the view that Von Neumann’s “process 1” [7], which occurs in a system when a measurement is performed, is a short hand description of what happens when the system observed triggers a first order phase transition in the measuring apparatus. The possible final states of the system+detector become macroscopically different and therefore “disjoint”, i.e. there is no way to measure their relative phases, hence they are best described by a density matrix. This being the nature of the measuring process, it is clearly irreversible, but does not require a separate postulate.

We have found that this point of view, expressed in a variety of ways, is informally accepted by many working physicists, and even taken for granted by experimentalists (e.g. [8]). Unfortunately, L.D. Landau held the opposite opinion [9], and conjectured that on the contrary, the law of increase of entropy might follow from the postulate that the wave function collapses, i.e. that process 1 occurs when a measurement is performed.

Let us briefly recall what the problem is [10], in the idealized description given by J. Von Neumann [7]. To measure the value of an observable  $O$ , with discrete spectrum (i.e.  $\hat{O}\psi_n = o_n\psi_n$ , or  $\hat{O} = \sum o_n\hat{P}_n$ ) on a system S with wavefunction  $\psi$ , we need to couple it to a classical measuring apparatus M, initially in a state  $\Psi_0$ , and to arrange the interaction between the two in such a way that for each  $n$ ,  $\psi_n\Psi_0$  quickly evolves to some  $\psi_n\Psi_n$ ; then inspecting the state of M we can figure out what the state of S was. Von Neumann proves that a unitary operator realizing this transition exists, and therefore, if S is initially in a superposition of eigenstates of  $O$ , we would expect:

$$\sum c_n\psi_n\Psi_0 \longrightarrow \sum c_n\psi_n\Psi_n \quad (1)$$

On the contrary, we find that the wave function “collapses”, so that after the mea-

---

<sup>1</sup>see [1, 5] for an adequate list of references, much longer than this paper.

surement the compound system has to be described by the density matrix

$$\hat{\rho} = \sum |c_n|^2 \psi_n \Psi_n \otimes \psi_n^\dagger \Psi_n^\dagger \quad (2)$$

How can this irreversible process be compatible with the unitary evolution one expects from the Schrödinger equation? <sup>2</sup>

Reflecting on concrete examples of measuring devices, one comes to the conclusion that M must be “classical” in the sense that it can be, and is described by thermodynamic variables, i.e. macroscopic order parameters defined by averages over a large number of microscopic observables. Initially M is in a metastable state, so that its free energy is at some local minimum. The interaction with the microscopic system S triggers a first order phase transition for M, to a state of lower free energy which is a true minimum. This process is typically irreversible: of course the microscopic dynamics is governed by the Schrödinger equation, but the evolution of the order parameter(s) is irreversible, something we have known since the work of L. Boltzmann<sup>3</sup>.

If the final states correspond to different phases of M, that are macroscopically different, i.e. differ by the value of a macroscopic quantity, they cannot be meaningfully superimposed, because no observation can reveal their relative phases, and *must* be treated like mixtures, as in eq. 2: this basic superselection rule, which disposes of Schrödinger’s cat paradox, is the point emphasized and elucidated by K. Hepp [6]. This fact is familiar when the different phases correspond to different directions of spontaneous symmetry breaking [12], but holds much more generally as a natural consequence of the modern formulation of quantum statistical mechanics [13].

In this language, one takes as primary objects the  $C^*$  algebra  $\mathcal{A}$  of the local *observables*, and the *states*, positive linear functionals which associate to every observable

---

<sup>2</sup>note that in either case the final state of the system will be described by a density matrix if we ignore the result of the measurement, i.e. trace over the variables of M. This is all one needs e.g. in Feynman’s approach.

<sup>3</sup>a beautiful discussion of Boltzmann’s work on the origin of irreversibility has been given by J.L. Lebowitz [11], whose point of view on the collapse of the wave function is, incidentally, completely consistent with ours.

an expectation value. For every state we may find a representation of  $\mathcal{A}$  as an algebra of operators on a Hilbert space. In the thermodynamic limit, e.g. when the number of constituents  $N \rightarrow \infty$ , these representations may be equivalent or not, with equivalence classes labeled by the values of the macroscopic observables, obtained as limits from local ones. Non equivalence means that no observable has matrix elements between the state vectors of the two representations, which are therefore “disjoint”.

For example [6], if we label the states of a spin- $\frac{1}{2}$  by the unit 3-d vectors  $\mathbf{e} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$  such that  $(\boldsymbol{\sigma} \cdot \mathbf{e}) u_{\mathbf{e}} = u_{\mathbf{e}}$ , we find:

$$u_{\mathbf{e}} = \begin{pmatrix} \cos \frac{\theta}{2} e^{-i\phi/2} \\ \sin \frac{\theta}{2} e^{+i\phi/2} \end{pmatrix} ; \quad |u_{\mathbf{e}'}^\dagger \cdot u_{\mathbf{e}}|^2 = \frac{1 + \mathbf{e}' \cdot \mathbf{e}}{2} \leq \exp(-\frac{1}{4}|\mathbf{e}' - \mathbf{e}|^2) \quad (3)$$

(by  $1 + x < e^x$ ). Then for the states of a system made of  $N$  spin- $\frac{1}{2}$

$$|\langle \mathbf{e}'_1 \dots \mathbf{e}'_N | \mathbf{e}_1 \dots \mathbf{e}_N \rangle|^2 \leq \exp(-\frac{1}{4} \sum |\mathbf{e}'_k - \mathbf{e}_k|^2) \leq \exp(-\frac{1}{4} N |\frac{1}{N} \sum \mathbf{e}'_k - \frac{1}{N} \sum \mathbf{e}_k|^2) \quad (4)$$

The same bound will drive to zero the matrix elements of any operator involving a finite number of spins; therefore, in the thermodynamic limit  $N \rightarrow \infty$ , no physical measurement can measure the relative phase between states which differ by the mean value of the spin vector  $\frac{1}{N} \sum \mathbf{e}_k$ , because no operator can change all the spins simultaneously.

There are other reasons that make the idea that quantum measurements happen through first order phase transitions in the measuring device a rather attractive one. Very loosely speaking, one may wonder in general how can a “small” system affect a “large” one. But typically metastability is precipitated by a microscopic nucleation mechanism, which only requires microscopic energies, although the energy released in the transition is certainly macroscopic. In this respect simple explicit models (e.g. [14, 6, 4]) which simple or no internal dynamics of M, are inevitably inadequate [15, 16, 5]. On the other hand nucleation theory has a long history, from Van der Waals, to Becker and Döring, to a vast modern literature. A well studied theoretical

model [17] is the two dimensional Ising model with

$$-\beta H = b \sum_{\langle ij \rangle} \sigma_i \sigma_j + h \sum_i \sigma_i \quad (5)$$

taken below the Curie point  $b = 0.44..$  with, say, positive magnetization  $\langle \sigma \rangle$ , and a small magnetic field in the opposite direction. A finite system like this is not at equilibrium, but actually quite stable, unless a “droplet” of spins is inverted. As an example, we run a Montecarlo simulation on a 200x200 lattice, thermalizing it with 3000 runs at  $h = 0$ ,  $b = 0.6$ , then going slowly to  $h = -0.05$ . No inversion of  $\langle \sigma \rangle$  takes place for 9000 runs, unless a square of at least 12x12 spins is inverted, whereupon  $\langle \sigma \rangle$  changes sign within a few hundred runs. Of course this is a classical model, but one does not expect a quantum model to behave very differently.

It may be that the explanation we give makes the wave function collapse rather trivial; and indeed, it is difficult to justify the interest and the immense literature the matter has attracted over the years, to say nothing about the extravagant philosophical implications (partly listed in [4]). Let us emphasize that in our opinion the wave function collapse does not play an important role in the theory or the practice of quantum mechanics, and has nothing directly to do with the profound and important epistemological questions that arise because of the probabilistic nature of quantum mechanics, nor with the possibility of reconstructing a consistent history of a phenomenon from a sequence of observations. An early paper by N.F. Mott [18] is particularly illuminating on this last subject: the straight tracks produced in a Wilson chamber by an  $\alpha$  particle emitted in S-wave follow from the application of the Schrödinger equation to the combined system  $\alpha$ -particle+atoms, without resorting to Von Neumann’s process 1.

On the other hand, it may be that the notion of “disjointness” we have put at the centre of our analysis has a broader meaning than the one given here. For example, it appears that the detection of a particle by a grain of emulsion is a more subtle and complex phenomenon than commonly realized. A more extravagant example

could be a measuring device which worked permuting the sequence of bases of a DNA molecule. In such cases, the possible final states of the measuring device cannot be said to be different phases of a macroscopic system, although it is doubtful that they can be meaningfully superimposed. A similar question has been discussed in [19] in a simpler context, while A.J. Leggett has proposed a quantitative measure of "disjunctivity" [20]. On balance, we cannot consider settled the question of wave function collapse.

We would like to thank M. Cini and M. Pascazio for some very helpful discussion.

## References

- [1] J. A. Wheeler and W. H. Zurek, eds, Quantum Theory and Measurement, (Princeton University Press, Princeton, N.J., 1983).
- [2] G. Ludwig, Die Grundlagen der Quantenmechanik (Springer, Berlin, 1954).
- [3] A. Daneri, A. Loinger and G.M. Prosperi, Nucl. Phys. **33** (1962) 297. Reprinted in [1].
- [4] M. Cini, Il Nuovo Cim. **73B** (1983) 27.
- [5] M. Namiki, S. Pascazio, Phys. Rep. **232** (1993) 30.
- [6] K. Hepp, Helv. Phys. Acta **45** (1972) 237.
- [7] J. Von Neumann, Mathematische Grundlagen der Quantenmechanik, (Springer, Berlin, 1932). Chapters V and VI of the english translation by R.T. Beyer are reprinted in [1].
- [8] R. Battiston, in: Problems of fundamental modern Physics, eds. Cherubini, DalPiaz and Minetti, (World Scientific, Singapore, 1990) p. 377.

- [9] L.D. Landau and E.M. Lifshitz, Statistical Physics,§8; Quantum Mechanics, §7. ( Pergamon, Oxford, 1958).
- [10] M. Namiki, S. Pascazio and C. Schiller,Phys. Lett. **A187** (1994) 17.
- [11] J.L. Lebowitz, “Boltzmann’s entropy and time’s arrow”, Physics Today, 1993-9, p.32; Physica A **194** (1993) 1.
- [12] K. Huang, Statistical Mechanics, II ed., (John Wiley, New York, 1987) p. 408.
- [13] D. Ruelle, Statistical Mechanics (Benjamin, New York, 1969).
- [14] H.S. Green, Il Nuovo Cim. **9** (1958) 880.
- [15] W.H. Furry, Boulder Lectures in Theoretical Physics, ed. W.E. Brittin (University of Colorado Press, Boulder, 1966).
- [16] J.S. Bell, Helv. Phys.Acta **48** (1975) 93.
- [17] R.H. Shonmann, Commun. Math. Phys., **161** (1994) 1.
- [18] N.F. Mott, Proc. Roy. Soc. London A126 (1929) 214, reprinted in [1].
- [19] M. Cini, M. Serva, Phys. Lett. **A167** (1992) 319.
- [20] A.J. Leggett, Supplement of the Progress of Theoretical Physics, **69** (1980) 80.