

Maths-in-Action

Theme Guide

2012-13

Read This First!

1. Register your choices for MA469 Project on *Mathstuff* by Sunday, 28 October 2012 at the latest.
2. This guide contains details of the 7 Maths-in-Action themes offered this year. For each theme you'll find:
 - A Synopsis (a short introduction to help you decide)
 - The Brief (a formal definition of your terms of reference)
 - Further Information and Sources (mainly books and URLs for Web pages)
3. Each theme will be restricted to at most SIX takers (first come, first served). A collaborating pair counts as one taker.

Table of Contents

Theme 1: Quantum Computing.....	3
Theme 2: Tomography - Imaging our Insides.....	6
Theme 3: Evolution.....	10
Theme 4: Space Travel.....	14
Theme 5: Weather Forecasting.....	17
Theme 6: The Human Cell	21
Theme 7: Virtual Reality.....	24

Theme 1: Quantum Computing

Synopsis

In December 2001 physicists at IBM's Almaden Research Centre in California made a vial of designer molecules act like a 7 quantum-bit (qbit) computer implementing a program to carry out on Shor's factorisation algorithm. The program's output, that

$$15 = 3 \times 5,$$

was hardly earth-shaking, but its implications for the future of computing were enormous. It proved that quantum phenomena, usually confined to the world of atoms and photons, have the potential to transform our lives through the processing of information.

Whereas factorizing large numbers on classical computers takes exponential time (every extra digit roughly doubles the run-time) a quantum computer can theoretically do it in polynomial time and thereby render useless the codes currently used for Internet security. It is unlikely that the approach used in the IBM experiment can be scaled up, but many other ideas for creating larger quantum computers are being explored and may one day transform computing beyond all imagination.

- In 2004 quantum entanglement in a semiconductor was reported by Albert Chang of Duke University. He next plans to build the world's first quantum gate in an electronically controlled semiconductor device, enabling the creation of a key building block in quantum computation.
- The UCLA team succeeded in flipping a single electron spin upside down in an ordinary commercial transistor chip, and detected current changes when the electron flips (*Nature*, 22 July 2004).
- World's First 28 qubit Quantum Computer Demonstrated Online at Supercomputing 2007 Conference (*Nanowerk News*) D-Wave Systems puts the world's first commercial quantum computer on display in an online demonstration. (Source: <http://www.nanowerk.com/news/newsid=3274.php>)

More recent progress can be seen on the timeline of quantum computing in Wikipedia:

http://en.wikipedia.org/wiki/Timeline_of_quantum_computing

Your Brief

The main focus of your brief should be

- the mathematics that describes the particular quantum phenomena that are exploited by (currently mainly theoretical) quantum computers, and
- the mathematics informing algorithms (like Shor's) that take advantage of the special features of quantum computers.

In order to present these ideas in context, you will need to give the reader an insight into the background of quantum mechanics and computer algorithms. You can enliven your account with excursions into the controversial history of quantum physics, the philosophical and psychological difficulties of interpreting quantum phenomena, the story of the visionaries who first saw the potential of quantum computation and believed in the possibility of creating

‘machines’ to carry it out, and recent progress in developing the practical technology to build working quantum computers.

Important: *This brief will be strictly adhered to when assessing your work. If you wish to deviate from it, you must apply to the Course Organiser (either in person or by sending an email to ma469@warwick.ac.uk).*

Further Information

To begin, you might do worse than read the still-evolving Wikipedia article:

http://en.wikipedia.org/wiki/Quantum_computing

You can also type “quantum computing” into Google or the search engine at the top of the IBM page. Many of these links are worth exploring further.

A good best source of information and links (for novices up to researchers at the cutting edge) can be found at the Web site of the *Centre for Quantum Computation* in Oxford; its URL <http://www.qubit.org>

Its ‘Introduction and Tutorials’ link is a good place to start. If you click on “world” and choose an area on the map, you get a good selection of academic links, including the Cambridge site <http://cam.qubit.org/> on the European list.

Some resources for general Quantum Computing

For obvious reasons, there is great interest in quantum computing in the academic community (it’s hot intellectual property and so research grants are forthcoming) and there have been a number of conferences and workshops.

This is such a new and rapidly-growing area, that many books are already out of date when they hit the library shelves. A search for “quantum computing” on Amazon.co.uk throws up over 100 titles, many published in the last four years. Here are a few that caught my eye:

1. *Quest for the Quantum Computer* by Julian Brown (Simon & Schuster Inc 2001) ISBN:0684870045. A lively, non-technical introduction by a regular contributor to the *New Scientist* magazine. Expect style of popular journalism.
2. *Mathematics of Quantum Computation* edited by Rane K. Brylinski and Goong Chen (CRC Press 2002), ISBN: 1584882824. A collection of individual papers whose authors include the editors themselves, Lov Grover, David Meyer, Michael Freedman, and many more. A deep understanding of the mathematics of quantum computation is present in this book’s 15 chapters. There are omissions but what is there is central, well written, and well explained: entanglement, kinematics and invariants for systems of a small number of qubits, universal quantum gates, quantum search, complexity, error-correction codes, teleportation, and quantum cryptography, the reader of this lovely book gets a good idea of what they are all about (paraphrase of Amazon review).
3. *Quantum Computation and Quantum Information* by Michael A. Nielsen and Isaac L. Chuang, (Cambridge University Press 2000) ISBN: 0521635039. A comprehensive textbook, good in parts like the curate’s egg, but with mixed reviews on Amazon.
4. *Quantum Computing* by Mika Hirvensalo (Springer-Verlag Berlin and Heidelberg 2003) ISBN: 3540407049. A book of high quality, giving a short but accurate

Theme 1: Quantum Computing

introduction to the 2 main quantum algorithms: the quantum Fourier transform and Grover's search. Suitable for advanced undergraduate with a basic maths degree background. The first 40 pages are an introduction to computation and the last 80 pages are an overview of quantum mechanics and mathematics. (Summary of Amazon review.)

5. *The Physics of Quantum Information* edited by Dirk Bouwmeester, Artur K Ekert, Anton Zeilinger, (Springer-Verlag, Berlin/Heidelberg 2001) ISBN: 3540667784. A collection of well-written tutorials by authorities in the field covering the main trends.
6. *Programming the Universe: A Quantum Computer Scientist Takes on the Cosmos.* Seth Lloyd. xii + 221 pp. Alfred A. Knopf, 2006. Sounds entertaining, modulo some caveats. Read the review by Jürgen Schmidhuber here:

<http://www.americanscientist.org/bookshelf/pub/2006/4/the-computational-universe>

Basic Quantum Theory

Here are some books from which to learn more about the background of quantum mechanics. They are written for physicists and you won't need much beyond the mathematics you learnt in your first year (Linear Algebra, Analysis, DEs):

7. *Quantum Mechanics* by P.C.W. Davies and D.S. Betts (Chapman and Hall 1994) ISBN: 0748744460. A gentle introduction to QM.
8. *The Feynman lectures on Physics, Vol III*, by R.P. Feynman, R.B. Leighton and M. Sands (Addison-Wesley 1998) ISBN: 0201021153
A great introduction to QM by one of the heroes of QT: not always easy to follow for beginners but makes fascinating reading.
9. *Visual Quantum Mechanics* by B.Thaller (Springer Verlag 2002) ISBN: 0387989293
There is a CD attached with colourful Quicktime animations that aim to help you to visualise the solutions of Schrödinger's Equation!

Interpretations of Quantum Mechanics

There are many books to choose from. Here are just a few:

10. *In Search of Schrödinger's Cat* by John Gribbin (Bantam Books, Toronto 1984) ISBN: 0553341030.
11. *Quantum Physics: Illusion or Reality?* by Alastair Rae, (Cambridge University Press 1986) ISBN: 0521260233.
12. *Nature Loves to Hide* by Shimon Malin, (Oxford University Press Inc, USA 2003) ISBN: 0195161092.

Theme 2: Tomography - Imaging our Insides

Synopsis

Tomography is the science building up two- or three-dimensional images of the internal structure of solid objects from various kinds of scattered emissions. It has many applications, for example to seismology, but its uses as a diagnostic tool in medicine are the best known and one of the most important. The many forms of medical tomography include, for instance:

- **X-ray computed tomography (X-Ray CT)** (the classical version popularly known as the CAT scan)
- **Magnetic Resonance Imaging (MRI)** (the tomographs are those big tubes like wind tunnels that doctors use look at the internal organs of their patients)
- **Ultrasound** (uses high-frequency sound-wave echos for non-invasive imaging, such as for foetal development)
- **Positron Emission Tomography (PET) and Single Photo Emission Computed Tomography (SPECT)** (they use emissions from injected radio-isotopes)
- **Electrical Imaging** (a promising new area not yet widely applied to medical diagnosis)

All these forms of medical tomography rely for their effectiveness on a close partnership between mathematics, number-crunching computers, and clever engineering. X-Ray CT, PET, and SPECT all reconstruct images from line integrals. MRI uses the discrete Fourier transform, while the mathematics for ultrasound models reassembles 2-D layers produced by an echo scanner.

A useful introduction can be found in Wikipedia under “medical imaging”

http://en.wikipedia.org/wiki/Medical_imaging .

And for a broader context see <http://en.wikipedia.org/wiki/Tomography> .

Your Brief

1. Your first task is to survey the different imaging techniques listed in the Synopsis above, to understand the physical principles they are based on, and to compare the kinds of mathematical techniques that are used in conjunction with computers to produce images.
2. Your second task is to select JUST ONE of these techniques and to study *in greater depth* the way mathematics, computing, physics, and engineering combine to make it work.

Important: *This brief will be strictly adhered to when assessing your work. If you wish to deviate from it, you must apply to the Course Organiser (either in person or by sending an email to ma469@warwick.ac.uk).*

Further Information

1. X-Ray Computed Tomography

This was the first non-invasive process capable of

- picturing internal body parts without other parts getting in the way, and
- differentiating soft tissues.

These machines first became available in the 70s and since then their speed and resolution has improved by several orders of magnitude. They can be classified according to the geometrical configuration of the X-ray source (for instance, linear, cylindrical or helical). An account of this technique can be found at

<http://books.nap.edu/books/0309053870/html/23.html#pagetop>

This URL takes you to an on-line book with chapters on each of the technologies listed above. Its final chapter contains a summary of the associated mathematical techniques.

2. Magnetic Resonance Imaging

Atoms with an unequal number of protons and neutrons in their nuclei, like hydrogen with one proton only, line up in a magnetic field. When the field is suddenly changed, they give off radio waves and produce a resonance image. Because the human body consists largely of water, whose molecules contain two hydrogen atoms for every oxygen atom, a magnetic resonance tomograph finds a great number of hydrogen atoms to excite in human tissue. In all, hydrogen accounts for more than 60 % of the atoms in our bodies.

An MRI image distinguishes different types of tissue essentially according to their water content. Fatty tissues, which have little water, appear as bright areas in an MRI. Blood vessels and other fluid-filled areas are dark. MRIs are especially useful in producing images of the brain, which contain a great deal of water!

There is a light-hearted discussion of MRI at

<http://electronics.howstuffworks.com/mri.htm>

as well as a complete on-line book on the subject at <http://www.cis.rit.edu/htbooks/mri> (see [7] below).

3. Ultrasound

Ultrasound (or ultrasonography) is a medical imaging technique that uses high frequency sound waves and their echoes to build up 2-dimensional images of internal layers, and out of these to construct 3-dimensional representations.

- The ultrasound machine transmits high-frequency (1 to 5 megahertz) sound pulses into your body using a probe.
- The sound waves travel into your body and hit a boundary between tissues (e.g. between fluid and soft tissue, soft tissue and bone).
- Some of the sound waves get reflected back to the probe, while some travel on further until they reach another boundary and get reflected.

- The reflected waves are picked up by the probe and relayed to the machine.
- The machine calculates the distance from the probe to the tissue boundaries using the speed of sound in tissue (5.005 ft/s or 1.540 m/s) and the time of the each echo's return (usually in the order of millionths of a second).
- The machine displays the distances and intensities of the echoes as a series of two-dimensional images of boundaries at a given distance.
- A computer then reconstructs a 3-dimensional image out of these plane cross-sections.

Another form of ultrasound tomography exploits the Doppler effect to represent internal *moving* objects, such as the flow of blood through an artery.

4. Positron Emission Tomography

PET produces images of the body by detecting the radiation emitted from injected radioactive substances; these are usually tagged with a radioactive atoms such as Carbon-11, Fluorine-18, Oxygen-15, or Nitrogen-13 with a short decay time.

The patient lies on a table in a tube-like machine, which detects the gamma rays given off at the site where a positron emitted from the radioactive substance collides with an electron in the tissue. The machine's cylindrical housing contains a circular gamma-ray detector array, which has a series of scintillation crystals, each connected to a photomultiplier tube. The crystals convert the gamma rays from the patient's body into photons of light, and the photomultipliers amplify the photons and convert them into electrical signals, which are processed by the computer to generate an image.

The table is then moved, and the process is repeated, resulting in a series of thin slice images of the body over the region of interest (e.g. brain, breast, liver). These thin slices can be assembled into a three-dimensional representation of the patient's body.

5. Single Photon Emission Computed Tomography

SPECT is a technique similar to PET. But the radioactive substances used in SPECT (Xenon-133, Technetium-99, Iodine-123) have longer decay times than those used in PET, and emit single instead of double gamma rays. SPECT can provide information about blood flow and the distribution of radioactive substances in the body. Its images have less sensitivity and are less detailed than PET images, but the SPECT technique are cheaper to set up and run. Again, Wikipedia has something useful to offer:

http://en.wikipedia.org/wiki/Single_photon_emission_computed_tomography

Sources

1. *Image Reconstruction from Projections: The Fundamentals of Computerized Tomography* by G.T. Herman, Academic Press, New York, 1980. A classic in the field, now out of print.
2. *Medical Images: Formation, Handling and Evaluation* by A.E. Todd-Pokropek and M.A. Viergever (Editors) Springer-Verlag Berlin and Heidelberg (1993). This conference proceedings contains a number of useful survey articles in Part I.

Theme 2: Tomography - Imaging our Insides

3. *Information Processing in Medical Imaging* by A. Kuba, M. Samal, and A. Todd-Pokropek, Springer, Heidelberg, 1999. Proceedings of another conference held in Hungary in 1999.
4. *Mathematical Methods in Image Reconstruction* by Frank Natterer + Frank Wübbeling Monographs on Mathematical Modeling and Computation 5 SIAM (2001) ISBN: 0898714729. This book describes the state of the art in the mathematical theory and numerical analysis of imaging.
5. *The Mathematics of Computerized Tomography* by Frank Natterer, SIAM Society for Industrial & Applied Mathematics (2001); ISBN: 0898714931. The book handles the serious mathematics and numerical algorithms behind tomography.
6. *Image Analysis for the Biological Sciences* by C. A. Glasbey and G. W. Horgan, published by Wiley in 1995.
7. *The Basics of MRI* by Joseph P. Hornak, an on-line book at <http://www.cis.rit.edu/htbooks/mri>
8. *Fourier Analysis and Imaging* by Ronald Bracewell (Kluwer Academic / Plenum Publishers 2003) ISBN: 0306481871.
9. *Medical Image Analysis* by A.P. Dhawan (John Wiley & Sons Inc 2003) ISBN: 0471451312.
10. *Mathematics of Medical Imaging* by Charles Epstein (Prentice Hall 2003) ISBN:0130675482.
11. *Pattern Recognition for Medical Imaging* by Anke Meyer-Baese (Academic Press 2004) ISBN: 0124932908.

Theme 3: Evolution

Synopsis

Taxonomy is a name for the science of classification. A special kind of taxonomy is *cladistics*, the systematic classification of groups of living things on the basis of their assumed common ancestry. (A *clade* is a group of organisms thought to have evolved from a common ancestor.) Until recently the ‘tree of life’ was constructed by studying the so-called morphological characteristics of fossils and extant species (how many legs, the presence of a backbone, etc.) to make qualitative judgements about where best to fit a species in. This was a very hit-and-miss affair, especially as most of the evidence is missing; most species have died out and so far only relatively few have been found in the fossil record.

Botanists, zoologists, and paleontologists once spent their professional lives classifying new species in this way. But now that we have the ability to analyse the DNA of living things, this drudgery and guesswork is largely a thing of the past. Knowing an organism’s genetic make-up raises the stakes by several orders of magnitude and turns speculation into much harder science. At the same time, dealing with mountains of genetic data calls for more sophisticated forms of analysis and comparison, and this is where mathematic comes in, ably supported by the number-crunching capabilities of modern computers. Many branches of mathematics are proving useful: for instance, combinatorics, optimization, probability and statistics, and the theory of algorithms. The subject is still in its infancy. New methods and models are being developed and studied. This project gives you an opportunity to learn about research at a very active and creative stage.

Your Brief

1. As usual, the main focus of your project should be on the mathematics (including the statistics and probability) that underlies the study of evolutionary theory, and, in particular, the study of *phylogenetics*, which is concerned with

- ancestor/descendent relationships between all groups of organisms,
- how these relationships can be discovered, and
- how they might have come about.

(the root word *phylogeny* is a diagram representing these relationships). There is a simple introduction to phylogeny at

<http://www.ucmp.berkeley.edu/alllife/threedomains.html>

and glossaries of phylogenetic terms at

<http://www.ucmp.berkeley.edu/subway/phylogen.html>

<http://www.dbbm.fiocruz.br/james/GlossaryA.html>

Wikipedia has a useful entry (+ further links) here <http://en.wikipedia.org/wiki/Phylogeny>.

2. You should discuss one or more models and methods now used for establishing an organism’s place in the tree of life and/or for understanding the evolution of a particular species (homo sapiens for instance). For comparing organisms that diverged relatively recently (say, up to a few 100 million years ago!) genetic sequencing works well; on bigger

Theme 3: Evolution

time scales, the analysis of the 20 amino acids in proteins is more effective (the so-called 'logdet method' is one of the buzz-words to follow up here).

3. In order to put your work in context, you should include enough background to make the mathematical models understandable to your target audience (comprising fellow MMath students no special knowledge of the subject). So you might have to touch on a wide range of disciplines, from paleontology and organic chemistry to genetic structure and data analysis.

4. It is a big subject, so you will clearly have to limit the scope of your report by careful selection of the material you discover. Your work will be judged more on the integrity and coherence of your story than on its comprehensiveness. One carefully-chosen aspect of the subject with real mathematical content, fully motivated and put in context, is preferred to a broad sweep of the field.

Important: *This brief will be strictly adhered to when assessing your work. If you wish to deviate from it, you must apply to the Course Organiser (either in person or by sending an email to ma469@warwick.ac.uk).*

Further Information

You will need to become a Jack or Jill of all trades to navigate your way through the different strands of this subject and you will need to learn more than you eventually need in order to make an intelligent selection of material for your story. To begin you could do worse that take a quick trip through Warwick's online course of Bioinformatics at

<http://www.warwick.ac.uk/telri/Bioinfo/home.htm>

It concentrates on genes and proteins and ways of analysing them of using a variety of computer software. It will give you a taste for both the theory and practice in an area that provides some of the raw data for pylogenetics. In this area, there's a recent book about modelling cell biology called *Computational Cell Biology* by C. Fall and others, published this year (ISBN 0387953698). (I should say that I haven't been able to look at many of the books I mention in this theme description.)

As usual there is a vast amount of material on the Web. The selection of sites I mention below offer you links to many more, and as you build up knowledge of the technical terminology, you can use Google to expand your resources still further.

Here are a few URLs of sites that address aspects of phylogenetics:

<http://www.ucmp.berkeley.edu/subway/phylogen.html>

<http://tolweb.org/tree/phylogeny.html>

There are several general books on *Mathematical Biology* (J.D.Murray's volume of that title, for instance, published by Springer-Verlag in 2001, ISBNs 0387952233). They are compilations from many areas and only a few chapters are relevant to your needs.

Chapters 14,15, 16, 37, 38 and 44 of the online Biology book at

<http://www.emc.maricopa.edu/faculty/farabee/BIOBK/BioBookTOC.html>

are relevant, albeit elementary.

Finally, and somewhat tangentially, there is a nice site on the evolution (or rather, lack of it) of the horseshoe crab at <http://www.horseshoecrab.org/evo/evo.html>.

Books

Here are a number of books which I have come across in my travels round the Internet and elsewhere. From their titles, abstracts, or reviews, they look relevant to this theme, but I have not examined them and cannot vouch for their credentials.

1. "Principles of Population Genetics (3rd Edition)" by D. L. Hartl and A. G. Clark, Sinauer Associates Incorporated, 1998. Paperback - 480 pages. ISBN: 0878933069.
2. "An Introduction to Population Genetics Theory" by J. F. Crow and M. Kimura, Burgess Intl Group, 1970. ISBN: 0808729012.
3. "Evolutionary Quantitative genetics" by D. A. Roff, Kluwer Academic Publishers, 1997. Paperback 448 pages. ISBN: 041212971X.
4. "Introduction to Quantitative Genetics (4th Edition)" by D. S. Falconer and T. F. C. Mackay, Longman, 1995. Paperback 479 pages. ISBN 0582243025.
5. "An Introduction to Genetic Algorithms" by Melanie Mitchell, MIT Press Reprint edition 1998. 224 pages, ISBN: 0262631857
6. "Adaptation in Natural and Artificial Systems" by J.H.Holland, MIT Press Reprint edition (1998). Paperback 228 pages. ISBN 0262581116.
7. "Genetic Algorithms in Search, Optimization, and Machine Learning" by D.E.Goldberg, Addison-Wesley, 1989. Hardcover 412 pages. ISBN 0201157675
8. "Molecular Evolution, a phylogenetic approach" by Roderic D.M. Page and E.Holmes. Blackwell Science (UK), 1998. Paperback 352 pages. ISBN: 0865428891
9. "Molecular Evolution and Phylogenetics" by Masatoshi Nei and Sudhir Kumar. Oxford University Press Inc (USA), 2000. Paperback 336 pages. ISBN: 0195135857
10. "Inferring Phylogenies" by Joe Felsenstein. Sinauer Associates Incorporated, 2002. 580 pages, ISBN: 0878931775.
11. The chapter: *Phylogenetic Inference* in "Molecular Systematics, 2nd edition" by David M. Hillis, Craig Moritz (editors). Sinauer Associates Incorporated, 1996. 620 ISBN: 0878932828.
12. "Tangled Trees -- *Phylogeny, Cospeciation, and Coevolution*" by Roderic D.M. Page (Editor). University of Chicago Press. Paperback 378 pages. ISBN: 0226644677 (to be published November, 2002)
13. "Biological Sequence Analysis" by Durbin, R., Eddy, S., Krogh A. and Mitcheson, G. Cambridge Univ. Press, 1998. Paperback - 368 pages ISBN: 0521629713 (I have this book in front of me as I type. Chapters 7 and 8 provide a solid mathematical background for phylogenetic analysis.)
14. "Bioinformatics and Genome Analysis" Hans-Werner Mewes (Editor), H. Seidel (Editor), B. Weiss (Editor) Hardcover 320 pages, Springer-Verlag, 2002. ISBN: 3540428933 (Covers the latest research.)
15. "Evolutionary Genetics (2nd Edition)" by John Maynard Smith, Oxford University Press, 1998. Paperback - 344 pages . ISBN 0198502311.
16. "Molecular Evolution" by Wen-Hsiung Li. Sinauer Associates 1997.

Theme 3: Evolution

17. “Molecular Evolution and Phylogenetics” by Masatoshi Nei and Sudhir Kumar, Oxford University Press, 2000. Paperback: 333 pages ISBN 0195135857.
18. “Phylogenetics Trees Made Easy: A How-To Manual for Molecular Biologists” by Barry G. Hall
19. “Mathematics of Evolution and Phylogeny” by Olivier Gascuel (Editor), Oxford University Press, USA, 2005, 448 pages, ISBN: 0198566107.
20. “Analysis of Phylogenetics and Evolution with R” by Emmanuel Paradis, Springer-Verlag New York Inc. 2006 (Paperback 223 pages). ISBN: 0387329145. *Focuses on applications of programming languages (mainly R) to phylogenetic analysis.*

Theme 4: Space Travel

Synopsis

The main thrust of this theme is the mathematics for low-budget flights around the solar system. The mathematics ranges from Newtonian Mechanics to Chaos Theory via numerical solutions to the three-body problem. Rather than waffle on, I can offer no better introduction to this theme than Keith Devlin's column in February 1999 for the Mathematics Association of America entitled "Stardust Equations" at the following URL

[.http://www.maa.org/devlin/devlin_2_99.html](http://www.maa.org/devlin/devlin_2_99.html)

It describes how mathematics saves NASA millions of dollars on those planetary excursions where time is no object. It includes several good links to launch you into a fruitful Internet orbit.

Your Brief

There are 3 main areas to choose from:

- **Slingshot Orbits:** The exploitation of the gravitational fields of the sun and the planets to accelerate spacecraft and send them, by possibly circuitous routes, to their ultimate target(s), be it a comet, a moon, a planet, or some other interesting phenomenon in outer space. The mathematics involved is mainly classical mechanics.
- **Langrangian Chaos:** This approach focuses on chaotic behaviour near stable and unstable Langrangian points (where several gravitational fields cancel out) and takes advantage of the control these points afford to direct spacecraft in widely differing orbits with very little expenditure of energy. Halo orbits around Lagrange points play a role here.
- **Solar Sailing:** A potentially-even-more-economical means of planetary exploration is solar sailing, which uses the "solar wind", in other words, photonic pressure, to propel small spacecraft indefinitely around the solar system. There are a number of concrete proposals but NASA has yet to take up the challenge of this as-yet-untried approach.

I strongly advise you to give a qualitative overview of the whole theme of space mechanics, and then, because different mathematics is involved, to concentrate either on "Langrangian Chaos" or else on the other two titles, possibly in combination. The important thing is to tell a compelling story that is accessible to fellow MMath students who have no special knowledge of this topic. In the scholarly report, your story should include all the relevant background (including politics, economics, and history if you think it appropriate), and you should also prepare your reader mathematically before launching into the serious stuff. If you have an interest in computing, you might also want to say something about the programs used to implement the theory in calculating actual orbits and the timing of the burns.

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Further Information

As usual there is a wealth of useful information on the World Wide Web. I list just a few sites below, divided loosely into the four categories. You will navigate easily to many more Web treasures, but the usual caveat applies: you can't believe everything you read.

1. General Material

<http://ssd.jpl.nasa.gov/>

http://science.nasa.gov/newhome/headlines/ast28jul98_1.htm

<http://solar-center.stanford.edu/>

http://sohowww.nascom.nasa.gov/gif/halo_orbit.gif

<http://history.nasa.gov/SP-401/ch9.htm>

2. Slingshot Methods

<http://www-spof.gsfc.nasa.gov/stargaze/Stostars.htm>

<http://www-spof.gsfc.nasa.gov/stargaze/Sintro.htm>

3. Lagrangian Libration Points

http://en.wikipedia.org/wiki/Lagrangian_point

<http://www-spof.gsfc.nasa.gov/Education/wlagran.html>

<http://www.ieec.fcr.es/libpoint/viewgraphs.html> (conference research papers)

4. Solar Sailing

http://en.wikipedia.org/wiki/Fast_solar_sailing and [..../Solar_Sail](#)

<http://www.nature.com/news/2004/040809/full/040809-5.html> (U of W has an electronic subscription to *Nature* accessed via electronic resources on the Library catalogue)

<http://www.u3p.net/>

<http://www.solarsails.info/>

<http://www.ec-lille.fr/~u3p/Glenans/Glenans4.html> (in French)

Books

Here is a long list of books, most of which I haven't been able to look at. It is clear from the reviews and titles that they vary greatly in level and scope. I suggest you go to the publisher's Web site, or to an Internet bookshop like Amazon, to read more about them.

1. "Orbital Mechanics" by John E. Prussing, Bruce A. Conway, Conway Prussing. Oxford University Press, 1997, Hardcover 208 pages. ISBN 0195078349.
2. "Adventures in Celestial Mechanics: A First Course in the Theory of Orbits" by Victor G. Szebehely. Univ of Texas Press 1989. Hardcover 175 pages. ISBN 0292751052.
3. "Fundamentals of Astrodynamics" by Roger R. Bate Dover Publications, 1971. Paperback 455 pages ISBN 0486600610 (good intro but dated).

4. "Satellite Orbits" by Oliver Montenbruck, Gill Eberhard. Springer-Verlag, 2000. Hardcover 383 pages. ISBN 354067280X.
5. "Spaceflight Dynamics (2nd Edition)" by E. William Wiesel. McGraw-Hill Education (ISE Editions), 1997. Paperback 368 pages. ISBN 0071156313
6. "Fundamentals of Astrodynamics and Applications, Second Edition" (Space Technology Library, Volume 12) by David Anthony Vallado, Wayne D. McClain Kluwer Academic Publishers. Hardcover 980 pages. ISBN 0792369033.

The next few are more technical.

7. "Theory of Orbits Volume 1: Integrable Systems and Non-perturbative Methods" by D. Occaletti, and G. Pucacco, Springer-Verlag, 1996. Hardcover 392 pp. ISBN 3-540-58963-5.
8. "Hamiltonian Dynamics and Celestial Mechanics" by Zhihong Xia (Editor), Donald G. Saari (Editor). American Mathematical Society, 1996. Paperback - 240 pages ISBN 0821805665. (Conference proceedings.)
9. "Theory of Orbits in the Restricted Problem of Three Bodies" by Victor G. Szebehely, Academic Press, 1967. ISBN 0126806500.
10. "Dynamics and Mission Design near Libration Points" (World Scientific Monograph Series in Mathematics) by G Gómez, À Jorba, C Simó and J. Masdemont.
 - Volume I: Fundamentals: The Case of Collinear Libration Points
 - Volume II: Fundamentals: The Case of Triangular Libration Points
 - Volume III: Advanced Methods for Collinear Points
 - Volume IV: Advanced Methods for Triangular Points
11. "Hamiltonian Systems and Celestial Mechanics (HAMSYS-98)" Proceedings of the III International Symposium edited by J Delgado, E A Lacomba, E Pérez- & J Llibre

Finally some offerings on solar sailing!

12. "Solar Sailing: Technology, Dynamics and Mission Applications" by Colin R. McInnes. Springer-Verlag, London, 1999. Hardback 296 pages ISBN 185233102X.
13. "Starsailing: Solar Sails and Interstellar Travel" by Louis Friedman. Wiley, New York, 1988. Paperback 158 pages. ISBN 0471625930.
14. "Space Sailing" by Dorothy M. Souza. Lerner Publications, Minneapolis, 1994. ISBN 0822528509.
15. "Space Sailing" by Jerome Wright. Gordon and Breach Science Publishers, 1992. Paperback 272 pages. ISBN 288124842X .
16. "The Starflight Handbook: a Pioneers Guide to Interstellar Travel" by Eugene F. Mallove and Gregory L. Matloff. John Wiley & Sons (Wiley Science Editions), New York, 1989. Hardcover 288 pages. ISBN 0471619124.

Theme 5: Weather Forecasting

Synopsis

Here are three important inputs to a weather forecast:

- The model
- The data
- The analysis of the data in the light of the model

The **model** is based on understanding meteorological phenomena and how they interact; local and global behaviours are usually handled separately (on so-called micro and meso-scales). The each model accepts certain **data**, parameters that describe the states of things affecting the weather, such as the temperature, movement and chemistry of the sea, land and sky. The **analysis** is the hard part. Glancing at the sky, tapping the barometer, or holding a moist finger to the wind to pronounce on tomorrow's rainfall are no longer good enough. These days, we rely on a **computer program** based on the model to convert a set of recent data into predictions of features like wind velocity, warmth and precipitation at a given place and time; the program should be able to run on an existing machine and produce the desired output in a finite time.

Your Brief

The area is vast and the experts working in it tend to be specialised. It will be a real challenge to extract from the wealth of information available a good story about the role of mathematics in weather forecasting and to make it accessible to your target audience of fellow MMath students who know nothing about the subject. Here are some guidelines

- Learn enough about the basics of meteorology to understand how a weather model might be built.
- Find out about one or more of the current weather-forecasting models and how mathematics is used in formulating and driving it. You might need to be selective here (confining your attention, say, to fluid dynamics, DEs, chaotic behaviour, or algorithms).
- Learn something about the way the model is implemented on computer so that your story contains an overview of the whole process.
- For interest you might add details of how weather data is collected, how well the various models perform, what the future holds, etc.

Important: *This brief will be strictly adhered to when assessing your work. If you wish to deviate from it, you must apply to the Course Organiser (either in person or by sending an email to ma469@warwick.ac.uk).*

Further Information

The Web is well worth exploring. There are a large number of institutions, local, national, academic, that do research into aspects of the environment affecting the weather. The few listed below have links to many more. Finding hard scientific knowledge may be harder.

General Interest (including history)

<http://www.news.harvard.edu/gazette/2002/08.22/13-weather.html>

<http://www.usatoday.com/weather/wworks0.htm>

<http://sln.fi.edu/tfi/hotlists/weather.html>

Weather Forecasting and Research Organizations

<http://www.metoffice.gov.uk/>

<http://www.ecmwf.int/research/>

<http://www.jma.go.jp/jma/indexe.html>

<http://www.soc.soton.ac.uk/JRD/MET/>

http://www.fmi.fi/research_meteorology/meteorology_4.html

Models

http://www.mmm.ucar.edu/ppws/ppws_weather.html

University Meteorology Depts

<http://www.helsinki.fi/lehdet/uh/liite01za.htm>

http://www.atmos.ucla.edu/web/research/climate_wx.html

<http://www.see.leeds.ac.uk/research/icas/index.htm>

<http://www.atm.ch.cam.ac.uk/>

<http://www-vortex.mcs.st-andrews.ac.uk/~dgd/>

Books

1. “Meteorology Today : An Introduction to Weather, Climate and the Environment” by Donald D. Ahrens. Thomson Learning, 2002. Paperback. ISBN 0534397751. (A very elementary introduction.)
2. “Atmospheric Science: An Introductory Survey” by John Wallace and Peter V. Hobbs. Academic Press, 1977. Case bound. ISBN: 0127329501.

It presents the basics of meteorology in more detail than texts aimed at non scientists. A knowledge of calculus and college physics is really necessary to understand the book. It is a good choice for scientists or engineers in other fields who want, or need, to learn the basics of meteorology. The basic formulas in meteorology are derived and explained.

3. “Meteorology for Scientists and Engineers (2nd Edition)” by Roland B. Stull, Brooks Cole, 1999. Paperback 400 pages. ISBN 0534372147.
4. “Atmospheric Modeling, Data Assimilation and Predictability” by [Eugenia Kalnay](#) Cambridge University Press 2002. ISBN: 0521796296

Theme 5: Weather Forecasting

5. "Introduction to Three-Dimensional Climate Modeling" by Warren M. Washington and Claire L. Parkinson. University Science Books, 1986. Clothbound 450 pages. ISBN: 0935702520.
6. "Modeling Dynamic Climate Systems" by W.A. Robinson. Springer-Verlag New York Inc, 2001. Hardcover 200 pages. ISBN: 0387951342.

The dynamics of the atmosphere, ocean, and climate are inherently nonlinear and complex, making computer models ideal for accurate, complete understanding of these systems. In the process of building and using models, the reader of this book will learn how the different components of climate systems function, interact with each other, and vary over time. Topics covered include the stability of climate, earth's energy balance, parcel dynamics in the atmosphere, the mechanisms of heat transport in the climate system, and mechanisms of climate variability. Special attention is given to the effects of climate change. The book is accompanied by a cross-platform CD containing models and a run-time version of STELLA (R) software.

7. "Large-Scale Atmosphere-Ocean Dynamics" (Volume 1: Analytical Methods and Numerical Models; Volume 2: Geometric Methods and Models) edited by John Norbury and Ian Roulstone. Cambridge University Press, 2002. Hardcover 400 pages. ISBN 052180681X

Numerical weather prediction is a problem of mathematical physics. The complex flows in the atmosphere and oceans are believed to be accurately modelled by the Navier-Stokes equations of fluid mechanics together with classical thermodynamics. However, due to the enormous complexity of these equations, meteorologists and oceanographers have constructed approximate models of the dominant, large-scale flows that control the evolution of weather systems and that describe, for example, the dynamics of cyclones and ocean eddies. The simplifications often result in models that are amenable to solution both analytically and numerically. The lectures in these volumes examine and explain why such simplifications to Newton's second law produce accurate, useful models and, just as the meteorologist seeks patterns in the weather, mathematicians seek structure in the governing equations, such as groups of transformations, Hamiltonian structure and stability. This book and its companion show how geometry and analysis facilitate solution strategies.

8. IUTAM Symposium on Advances in Mathematical Modelling of Atmosphere and Ocean Dynamics edited by P.F. Hodnett. Kluwer Academic Publishers, 2001. Hardcover 308 pages. ISBN 0792370759.

This volume contains papers from the five keynote lecturers and 34 other contributed papers presented at the IUTAM Symposium on Advances in Mathematical Modelling of Atmosphere and Ocean Dynamics. The symposium provided a forum where atmosphere and ocean scientists could present their latest research results and learn of progress and promising ideas in these allied disciplines. This was seen to be especially important in view of current efforts to model climate requiring interaction between atmosphere, ocean and land influences. This volume presents a valuable collection of papers, which highlight advances in modelling of atmosphere and ocean dynamics and should further advance interaction between theory and applications in atmosphere/ocean dynamics. The volume should be a valuable reference for researchers involved in atmosphere, ocean and climate modelling.

9. "Glossary of Weather and Climate" edited by Ira W. Geer. The American Meteorological Society, 1996. Paperback 272 pages. ISBN 1878220217.

This is just one of many reference books published by the American Meteorological Society that gives readers a better understanding of the field of meteorology and related sciences. From Ablation to Zulu time, this glossary contains over 3,000 definitions of climatological, meteorological, oceanic and hydrologic terms.

10. "The Essence of Chaos" by Edward N. Lorenz. University of Washington Press, 1996. Paperback. ISBN: 0295975148 (Reprint).

*This is an introduction (for non-scientists) to the topic by the scientist who "discovered" chaos. Lorenz's idea has become key to theories of weather forecasting and to other sciences. (A brief description of how Lorenz began the chain of thinking that led to the chaos theory and its implications for weather forecasting is in the forecasting chapter of *The USA TODAY Weather Book*). In his own book, Lorenz describes his work and its implications. The book is based on a series of lectures Lorenz gave at the University of Washington. Often it's said that one idea from chaos theory is "the flap of a butterfly's wings in Brazil can set off a tornado in Texas." Lorenz's discussion of what this means, and doesn't mean, is enlightening.*

11. "Introduction to Geophysical Fluid Dynamics" by B. Cushman-Roisin. Prentice Hall, 1993. Hardcover: 336 pages. ISBN 0133533018 (Reissue).

This is the first and only introductory level text available on geophysical fluid dynamics. Emphasis is placed on physics, not mathematics and simple and complex laboratory demonstrations are featured in most chapters. Special contemporary topics, of climate dynamics and equatorial dynamics, including the greenhouse effect, global warming and southern oscillation are covered.

Theme 6: The Human Cell

Synopsis

This theme marshals forces from vast areas of contemporary science, above all from biology, mathematics and statistics, and computer science. Our current view on the functions of the cell is based on the double helix of DNA (the human genome) in its nucleus, selected expression of its genes, their transcription of messenger RNA, and finally translation into the proteins that carry out a myriad of different tasks to keep us alive and well. Most appealingly, the study of the human cell promises us understanding and control of pathology, and offers the tantalising prospect of victory over disease, perhaps ultimately the philosopher's stone of everlasting life.

If you are a beginner and want to get an up-to-date understanding of what's happening currently in this exciting field, I suggest you start by reading the nine tutorials on the Science primer Web page of the US *National Center for Biotechnology Information* at

<http://www.ncbi.nlm.nih.gov/About/primer/index.html>,

clicking first on the title *Molecular Genetics*. The essays are well written and fully accessible to the novice; they take about an hour of concentrated reading altogether.

Your Brief

Because of the breadth of the subject area, this is the most open-ended of this year's themes. I propose that you choose one of the three following topics:

- DNA sequencing
- DNA microarrays (or DNA chips, as they are sometimes called)
- Protein analysis

In each case, for your scholarly report you should introduce enough background to make your account accessible to your target audience (fellow MMath students who know no more about the subject than you did when you started). Above all, you must highlight the role mathematics plays in research and applications (for this purpose, mathematics can be interpreted broadly to include statistics and theoretical computer science). And, if you wish, you can add your own twists of history and speculation to make your story interesting and engrossing.

Important: *This brief will be strictly adhered to when assessing your work. If you wish to deviate from it, you must apply to the Course Organiser (either in person or by sending an email to ma469@warwick.ac.uk).*

Further Information

There is a great deal of useful information on the World Wide Web. I list just a few sites below, and from them you will be able to navigate easily to many more. Search engines like *Google* will take you deeper as you learn new key words, acronyms, and phrases to pursue. Of course, you should be selective and canny about what you read and believe. The URLs are divided loosely into the six categories, but there is considerable overlap.

Glossary

http://www.ornl.gov/TechResources/Human_Genome/glossary/glossary.html#dna

General Tutorials

<http://www.warwick.ac.uk/telri/Bioinfo/home.htm> (Warwick's online course)

<http://www.research.ibm.com/journal/sj/402/swope.html> (IBM Research Journal)

<http://csm.jmu.edu/biology/monroejd/amcp/genome2.html>

Institutions

<http://www.research.ibm.com/compsci/compbio/> (IBM research group)

<http://mips.gsf.de/>

<http://www.ncgr.org/>

Sequencing

<http://www.research.ibm.com/journal/sj/402/headgordon.html>

<http://www.nature.com/nature/supplements/collections/humangenome/index.html>

Microarrays

<http://www.ncbi.nlm.nih.gov/About/primer/microarrays.html>

http://ihome.cuhk.edu.hk/~b400559/book_mray.html#Microarray (book list)

<http://research.nhgri.nih.gov/microarray/index.html>

<http://www.nature.com/genomics/post-genomics/microarrays.html>

Proteomics

<http://www.cryst.bbk.ac.uk/education/AminoAcid/overview.html>

<http://www.cse.ucsc.edu/research/compbio/ismb99.handouts/KK185FP.html#c6>

Books

Because the field relatively new and fast-developing, the latest research will not have reached the textbooks yet. You might therefore want to be adventurous and read some articles in the learned journals, such as the online IBM research journal linked to above. In any case, type some key words into the Amazon book search engine, noting that, with your tutor's signature on the form, you can use the Inter-Library Loan scheme to get books and copies of articles that are not in the University's periodicals collection.

I begin with two books with serious mathematical content relevant to the theme:

1. "Biological Sequence Analysis" by R. Durbin, S. Eddy, A. Krogh and G. Mitchinson. Cambridge University Press, 1998. Paperback 356 pages. ISBN 0521629713.
2. "Mathematics of Genome Analysis" by J.K. Percus. Cambridge University Press, 2002. Paperback 139 pages. ISBN 0521585260.

Theme 6: The Human Cell

There now follows a list of books which I haven't been able to look at and therefore cannot vouch for. I suggest you go to the publisher's Web site, or to an Internet bookshop like Amazon, to read more about them.

3. "Discovering Genomics, Proteomics, and Bioinformatics" by A. Malcolm Campbell and Laurie J. Heyer. Addison Wesley & Benjamin Cummings, just published Sept 2002. Paperback 410 pages. ISBN 0805347224.

A first textbook for upper-level undergraduate and first-year graduate students which combines integrated Web exercises with a problem-solving approach to train students in basic hands-on genomic analysis. Features include an inquiry approach to give students hands-on practice and build on problem-solving skills, students learn to use databases and how to extract pertinent information. "Math Minutes" supply brief tutorials that reveal the math behind the biology.

4. "DNA Microarrays and Gene Expression" by Pierre Baldi and G. Wesley Hatfield. Cambridge University Press, 2002. Hardcover 230 pages. ISBN 0521800226.
5. "Introduction to Proteomics: Tools for the New Biology" edited by D.C. Liebler. Humana Press Inc, 2002. Paperback 210 pages. ISBN 0896039927.
An introduction to proteomics, which is currently the hottest thing in the biological sciences as it enables scientists to study the protein complement of the genome. It refers to studying the functions of many proteins at one time, as opposed to how research was conducted in the mid-90's where the technology did not enable the study of the entire protein system. From this, one is able to develop a systematic overview of the way a genome functions and the role it plays in health and disease.
6. "Protein Evolution" by Laszlo Patthy. Blackwell Science, 1999. Paperback 240 pages. ISBN 0632047747.
7. "Protein Structure, Stability and Folding" edited by Kenneth P Murphy. Humana Press Inc, 2001. Hardcover 264 pages. ISBN 0896036820.
8. "Protein Structure Prediction: Methods and Protocols" by D Webster. Humana Press Inc, 2000. Hardcover 300 pages. ISBN 0896036375.
9. "DNA Microarrays" edited by David Bowtell and Joseph Sambrook. Cold Spring Harbor Laboratory Press, 2002. Paperback 375 pages. ISBN 0879696257.
10. "Microarray Gene Expression Data Analysis – A Beginner's Guide" by Helen Causton, Alvis Brazma, John Quackenbush. Blackwell Science, 2003 (paperback, 176 pages). ISBN 1405106824.
11. "Beyond the Genome-The Proteomics Revolution" by Fred Askari and Emilia Askari. Prometheus Books. 2003 (hardcover, 300 pages). ISBN 1591020190.

Theme 7: Virtual Reality

Synopsis

This theme could be subtitled “Putting the moves on Tony Hawk and Lara Croft”. Anyone who has played a computer game involving 3-D action, seen a film at an iMAX cinema, or visited a Web site with ‘walk-round’ viewing will know how convincing these simulated worlds can be. They are forms of virtual reality (VR), the science of artificially replicating sensory experiences with the help of electronic and physical devices.

Nowadays pilots spend a large part of their training in VR simulators (including soon, no doubt, encounters with virtual terrorists); architects who design in digital space can roam around their virtual buildings before a brick is laid; doctors can probe freely inside their patients’ virtual insides.

Your Brief

1. To find out about the various forms of virtual reality that have been developed (including so-called ‘augmented reality’) and to describe them in a systematic way.
2. To describe and evaluate the uses of virtual reality in contemporary life.
3. To investigate which aspects of virtual reality require significant mathematical support, to describe and justify the mathematics involved, and to explain how it is applied.

Important: This brief will be strictly adhered to when assessing your work. If you wish to deviate from it, you must apply to the Course Organiser (either in person or by sending an email to ma469@warwick.ac.uk).

Further Information

In 3-D computer graphics, objects are typically stored and manipulated in three dimensions and are only projected into a viewable plane (for example, the monitor screen) at a late stage of the process. Two major aspects of representing images for virtual reality are:

1. photo-reality, that is to say, the faithful reproduction of light, shade and texture in each projected image, taking both direct and reflected light into account;
2. animation of the scene, creating the impression of fluent movement of the visualised objects and allowing for changes in the observer’s viewing position .

Related issues include:

- linear approximation of curves by polygons and smooth surfaces by discrete polyhedra;
- higher-dimensional approximations and vector scalable graphics;
- rendering, that is to say, converting the wire-frame approximation into the pixels to display a smooth solid image, a multiple process involving transformation into the display plane and algorithms for shading and the removal of hidden surfaces;
- the incorporation of the senses of touch and hearing to enhance the simulated experience.

Theme 7: Virtual Reality

In practice, viable solutions are constrained by processor power, storage, and the degree of accuracy required for the output, which is high for computer-aided design (CAD) for instance. Investment by software developers and end-users in a given technique that is practical and effective can produce inertia to change.

The University of Warwick has a virtual reality lab, called the ‘Cybersphere’, created by Vinesh Raja in Manufacturing Engineering. He has several interesting research problems to solve; one involves minimising distortion when projecting onto a cylindrical surface and marrying up images from multiple projectors. If you would like to see his installation and find out more about his research, please send an email request to Vinesh.Raja@warwick.ac.uk suggesting times when you are free, copying your email to ma469@warwick.ac.uk.

Sources

1. A good book for the theory and practice is *Computer Graphics* (2nd Edition) by Foley, van Dam, Feiner, and Hughes, Addison-Wesley (1996); ISBN 0201848406. (Main Library call number QA 79.2.F6.)

At the moment there are only two copies in the Main Library and they are in demand by Computer Science students. Look particularly at Chapter 5 on Geometric Transformations, which covers a lot of basic linear algebra, and at the appendix entitled “Mathematics for Computer Graphics”.

2. *3-D Computer Graphics* (3rd Edition) by Alan Watt, Addison-Wesley (2000); ISBN: 0201398559 (Main Library classmark QA 79.2.W2.)

This third edition is a readable and comprehensive introduction to most aspects of computer graphics for VR, and (exceptionally) includes a chapter on animation. It deals lightly with the theory and in more detail with the practice.

3. *Advanced Animation and Rendering Techniques* by Alan Watt and Mark Watt, Addison-Wesley (1992); ISBN: 0201544121. (Main Library classmark QA 79.2.W2.)

The main source for animation and rendering.

4. *Mathematical Elements for Computer Graphics* (2nd Edition) by D.F. Rogers and J.A. Adams. McGraw-Hill (1990); ISBN: 0070535302. (Main Library call number QA 79.2.R6.)

A broad coverage of the linear algebra and 2 & 3-dimension geometry behind computer graphics with a number of appendices on implementation. Its dated first chapter shows how fast computer graphics has moved in the past decade.

5. *Computer Graphics* by F.S. Hill Jr., Prentice-Hall (1990); ISBN: 0023548606. (Main Library classmark QA 79.2.H4.)

A long leisurely account of the practical techniques and algorithms for 2- and 3-dimensional computer graphics and of how to write programs to implement them. Although the book 15 years old, most of the techniques it describes are still used today

6. *Realistic Ray Tracing* by Peter Shirley, A.K. Peters Ltd (2000); ISBN: 1568811101. (Main Library call number QA 79.2.S4.)

This book is a short introduction to a rendering technique called ‘ray tracing’ and how it can be implemented in computer code. Linear algebra and probability theory play important roles.

7. *Mathematics for Computer Graphics* by S.G. Hoggar, Cambridge University Press (1992); ISBN: 0521375746. (Main Library call number QA 79.2.H6.)

This book focuses on the mathematics behind various aspects of computer graphics (including data compression) without saying much about how it is applied. Fractals are covered in some detail. He has a new book due out in December 2005 entitled *Mathematics of Digital Images: Creation, Compression, Restoration, Recognition*, to be published by CUP (ISBN: 0521780292)

8. *The Geometry of Computer Graphics* by W.F. Taylor, Wadsworth and Brooks (1991); ISBN: 0534171001. (Main Library call number QA 79.2.T2.)

A primer on the analytic and projective geometry needed to understand and write computer programs that display basic 2- and 3-dimensional shapes.

9. *Quaternions and Rotation Sequences: A Primer with Applications to Orbits, Aerospace and Virtual Reality* by Jack B. Kuipers, Princeton University Press, 400 pages, (2002); ISBN: 0691102988. (Not yet in Main Library).

In this book, Jack Kuipers introduces quaternions for scientists and engineers who have not encountered them before and shows how they can be used in a variety of practical situations, including VR.

The Internet: As usual there is an embarrassment of riches. Here is a random selection.

- I suggest you first try the on-line essay by Jerry Isdale called *What Is Virtual Reality?* which can be found at

<http://www.isdale.com/jerry/VR/WhatIsVR.html>

- There lots of VR links at <http://www.vruniverse.com>
- An idea of the mathematics is needed for computer graphics can be found at <http://www-static.cc.gatech.edu/~turk/>
- Some VR scenarios can be found at <http://www.activeworlds.com>
- Jim Vallino's comprehensive introduction to the area known as 'augmented reality' can be found at <http://www.se.rit.edu/~jrv/research/ar/> along with an impressive collection of Web links.