

PAST-FUTURE INTERFERENCE IN ϕ -DECAYS
INTO ENTANGLED KAON PAIRS

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Abstract

Quantum interference effects between past and future events (neutral kaon transitions, including CP violating decays) are discussed for entangled kaon pairs produced through $\phi \rightarrow K^0 \bar{K}^0$ in ϕ -factories. Contrasting with the exciting conclusions of other recent analyses, we predict the inexistence of such an observable effect.

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1. Introduction

The possibility of faster-than-light communication and the related, and even more intriguing, prospective that future events could have an influence over present ones, have been discussed several times in physics. For a manifestation, if any, of such a highly speculative violation of causality, the concurrence of several very special and counter-intuitive circumstances should (at least) be required. The experimental success and the well-known non-locality of quantum mechanics – as clearly manifested in Einstein-Podolsky-Rosen (EPR) experiences – is sometimes considered as a rather promising one. A second, favourable circumstance could be related to the non-invariance of time-reversal, as observed (and theoretically well described) in neutral-kaon decays. Both of these two unusual circumstances concur when dealing with ϕ -meson transitions into (EPR) entangled states containing two neutral-kaons which subsequently decay through time-reversal- or CP-violating channels. A high-luminosity ϕ -factory – such as DAΦNE, under construction in Frascati [1] – is therefore an exceptionally appropriate machine for this kind of studies. In this ϕ -factory context, Datta, Home and Raychaudhuri [2], [3] proposed some time ago the intriguing possibility of detecting such a curious non-local and faster-than-light propagating effect on the statistical (i.e., observable) level. Their result was criticized by several authors [4] on general grounds and by Ghirardi et al. [5] in a more detailed way, but the controversy appears to be still open [3].

More recently, Srivastava and Widom [6] have presented a detailed and interesting discussion on a new experiment (somehow related to that considered by Datta et al [2]) leading again to an intriguingly positive result. A relevant feature of this new proposal is that it fits perfectly well with the DAΦNE configuration and, on the theoretical side, only well tested and extensively studied equations (specially by workers preparing DAΦNE experiments) are used. Indeed, one starts considering ϕ -decays at $t = 0$ into entangled neutral-kaon pairs,

$$|\Phi(t = 0)\rangle = \frac{1}{\sqrt{2}} \left[|K^0\rangle | \bar{K}^0\rangle - | \bar{K}^0\rangle | K^0\rangle \right], \quad (1)$$

where the relative minus sign comes from the well-known negative charge-conjugation of the initial ϕ . The kaons fly apart in opposite directions along a given axis, thus defining a left and a right beam. Kaons along the left beam travel in free-space from the source, at $t = 0$, up to a nearby detector of neutral-kaon decays into a given channel (say, c-channel) placed at a "time (of flight) distance" t ; the number of c-channel decays per unit of time, dP_c/dt , at t is measured there. Kaons in the right beam fly freely from the common source, $t = 0$, up to the edge of a distant absorber, placed at a "time-distance" $T > t$. The essential claim in ref. [6] (quite in line with that in refs. [2], [3]) is that measurements performed on the left beam at a "time-distance" t can be statistically modified by the presence or not of the kaon absorber located at a larger "time-distance" T , $T > t$, on the other beam. In other words, that $dP_c(t, T)/dt$ depends not only on t (as it obviously does), but *also* on T , *i.e.*, on "where" on the other side (future) absorption starts (or immediately takes

places if an "ideal" absorber is used). The analysis by Srivastava et al. is somehow simplified by the use of an "ideal" (or drastic) absorber and by neglecting (irrelevant) kaon decay phases. But, most interestingly, the analysis holds for any space-time separation of events, including c-channel decays at t belonging to the absolute past of the respective absorption events at T . In this sense, one is dealing with the Einstein-Tolman-Podolsky effect [7] rather than with the more familiar EPR-effect discussed by Datta et al. [2]. The unexpectedly predicted T -dependence is then enhanced by performing a time- t integration from t_i to t_f under specific T -independent conditions. This final step, which is not essential for our main discussion (i.e., if dP_c/dt is T -dependent or not), enhances the effect up to about 10^{-4} , thus allowing for possible detection at DAΦNE.

Our purpose in this note is to present a detailed and accurate rediscussion of the whole situation taking all the phases into account and substituting the ideal absorber at T by a more general and realistic one absorbing homogeneously from T to infinity. To this aim, the introduction of three different basis for the neutral-kaon system is required. These standard basis are discussed in the next paragraph along the lines of ref. [8]. Then we proceed to explicitly calculate dP_c/dt for a general, homogeneous absorber and a final section is devoted to a discussion including the particular case of "ideal" absorption considered in [6].

2. Basis for neutral kaons

i) The first, "strong-interaction" basis contains the two eigenstates $|K^0\rangle$ and $|\bar{K}^0\rangle$, with strangeness $S = +1$ and -1 , and, contrasting with the next two basis, it is the only orthogonal one, $\langle K^0 | \bar{K}^0 \rangle = 0$. This is the suitable basis to discuss S -conserving strong interactions, but not to consider simple neutral kaon evolution in free space or inside matter; in both of these later cases, conversions or $K^0 - \bar{K}^0$ oscillations take place.

ii) The "free-space" basis is defined by the K -short and K -long states

$$\begin{aligned} |K_S\rangle &= (1 + |r|^2)^{-1/2} [|K^0\rangle + r | \bar{K}^0 \rangle] \\ |K_L\rangle &= (1 + |r|^2)^{-1/2} [|K^0\rangle - r | \bar{K}^0 \rangle], \end{aligned} \quad (2)$$

where $r \equiv (1 - \epsilon)/(1 + \epsilon)$ and ϵ is the usual CP-violation parameter. $|K_S\rangle$ and $|K_L\rangle$ are the normalized eigenvectors of the effective weak hamiltonian

$$H = \begin{pmatrix} \lambda_+ & \lambda_-/r \\ \lambda_-r & \lambda_+ \end{pmatrix}, \quad (3)$$

with eigenvalues

$$\begin{aligned} \lambda_S &= \lambda_+ + \lambda_- = m_S - (i/2)\Gamma_S \\ \lambda_L &= \lambda_+ - \lambda_- = m_L - (i/2)\Gamma_L, \end{aligned} \quad (4)$$

where $m_{S,L}$ and $\Gamma_{S,L}$ are the physical masses and decay widths of $K_{S,L}$, and PCT-invariance requires the diagonal elements of H be equal. This is the appropriate basis to discuss propagation in free space. Indeed, K_S and K_L states do not convert into each other and show simple exponential decay in time

$$|K_S(t)\rangle = e^{-i\lambda_S t} |K_S\rangle, \quad |K_L(t)\rangle = e^{-i\lambda_L t} |K_L\rangle \quad (5)$$

The non-orthogonality of this basis is given by

$$\delta \equiv \langle K_L | K_S \rangle = \langle K_S | K_L \rangle = \frac{1 - |r|^2}{1 + |r|^2} = \frac{\epsilon + \epsilon^*}{1 + |\epsilon|^2} \quad (6)$$

and simple unitarity arguments lead to the so-called Bell-Steinberger relation [8]

$$\delta = \left(\sum_f A_{K_S \rightarrow f}^* A_{K_L \rightarrow f} \right) / (\Gamma + i\Delta m) \quad (7)$$

with $\Gamma + i\Delta m \equiv i(\lambda_L - \lambda_S^*) = (\Gamma_L + \Gamma_S)/2 + i(m_L - m_S)$ and the sum extending over all possible decay amplitudes $A_{K_{L,S} \rightarrow f}$.

iii) The "inside-matter" basis is given by the K'_S, K'_L normalized states

$$\begin{aligned} |K'_S\rangle &= (1 + |r\bar{\rho}|^2)^{-1/2} [|K^0\rangle + r\bar{\rho} | \bar{K}^0 \rangle] \\ |K'_L\rangle &= (1 + |r/\bar{\rho}|^2)^{-1/2} [|K^0\rangle - (r/\bar{\rho}) | \bar{K}^0 \rangle], \end{aligned} \quad (8)$$

where we have introduced the regeneration parameter ρ [8], and $\bar{\rho}$,

$$\bar{\rho} \equiv \sqrt{1 + 4\rho^2} + 2\rho, \quad \rho \equiv \frac{\pi\nu}{m_K} \frac{f - \bar{f}}{\lambda_S - \lambda_L}. \quad (9)$$

The K'_S, K'_L states diagonalize the effective weak plus strong interaction hamiltonian

$$H' = H - \frac{2\pi\nu}{m_K} \begin{pmatrix} f & 0 \\ 0 & \bar{f} \end{pmatrix}, \quad (10)$$

where f and \bar{f} are the forward scattering amplitudes for K^0 and \bar{K}^0 on nucleons and ν is the nucleonic density in the homogeneous absorber. The corresponding eigenvalues are

$$\begin{aligned} \lambda'_S &= \lambda_+ - \frac{\pi\nu}{m_K} (f + \bar{f}) + \lambda_- \sqrt{1 + 4\rho^2}, \\ \lambda'_L &= \lambda_+ - \frac{\pi\nu}{m_K} (f + \bar{f}) - \lambda_- \sqrt{1 + 4\rho^2}. \end{aligned} \quad (11)$$

The K'_S, K'_L eigenstates are appropriate to discuss propagation inside a homogeneous medium showing no-reconversion into each other and simple exponential extinction in time

$$|K'_S(t)\rangle = e^{-i\lambda'_S t} |K'_S\rangle, \quad |K'_L(t)\rangle = e^{-i\lambda'_L t} |K'_L\rangle \quad (12)$$

due to both weak decays and strong interaction scattering out of the beam. The non-orthogonality now reads

$$\begin{aligned}\delta' &\equiv \langle K'_L | K'_S \rangle = \langle K'_S | K'_L \rangle^* \\ &= (1 + |r\bar{\rho}|^2)^{-1/2} (1 + |r/\bar{\rho}|^2)^{-1/2} (1 - |r|^2 (\rho/\bar{\rho}))\end{aligned}\quad (13)$$

and unitarity requires

$$\delta' = \left(\sum_f A_{K'_S \rightarrow f}^* A_{K'_L \rightarrow f} \right) / (\Gamma' + i\Delta m') \quad (14)$$

with $\Gamma' + i\Delta m' = i(\lambda'_L - \lambda'^*_S)$ and the sum extending both to weak decays (as in δ , eq (7)) and to strong interaction collisions.

To clarify the relationships among these three basis, two limiting cases of absorbers are worth considering. For a very soft, low-density absorber ($\nu, \rho \rightarrow 0, \bar{\rho} \rightarrow 1$) one obviously has $|K'_{S,L}\rangle \rightarrow |K_{S,L}\rangle$, as seen from eqs (2) and (8). For a drastic, high-density ("ideal", in ref. [6]) absorber, strong interactions dominate over weak decays (i.e., $\nu, |r|, |\bar{\rho}| \rightarrow \infty$), and one now has $|K'_S\rangle \rightarrow |\bar{K}^0\rangle$ and $|K'_L\rangle \rightarrow |K^0\rangle$, as expected and immediately seen from eqs (8).

3. Computing dP_c/dt

Entangled kaon pairs coming from a ϕ -decay at $t = 0$ are described by eq (1) or, equivalently, by

$$|\Phi(t=0)\rangle = \frac{1}{\sqrt{2}} \frac{1 + |r|^2}{2r} [|K_L\rangle |K_S\rangle - |K_S\rangle |K_L\rangle], \quad (15)$$

with future time evolution in free space given by eqs (5). The decay rate into a given c-channel, at time t , on the left beam of the previously described configuration, is the sum of two contributions: $dP_{c,\bar{\rho}=1}(t, T)/dt$ and $dP_{c,\bar{\rho}\neq 1}(t, T)/dt$. The former is the probability rate for c-channel decay (at t), with an accompanying weak decay taking place along the right beam between time 0 and T ; the second contribution corresponds to the c-channel decay rate with accompanying absorption after T (decay or scattering inside the long absorber). Both separated contributions are expected to depend on t (when c-decay occurs) and T (when the absorber is reached). Indeed, the use of standard formulae leads unambiguously to

$$\begin{aligned}dP_{c,\bar{\rho}=1}(t, T)/dt = \\ \frac{1}{8} \left(\frac{1}{|r|} + |r| \right)^2 \left[\Gamma_L^c (1 - e^{-\Gamma_S T}) e^{-\Gamma_L t} + \Gamma_S^c (1 - e^{-\Gamma_L T}) e^{-\Gamma_S t} + \right. \\ \left. 2\sqrt{\Gamma_L^c \Gamma_S^c} e^{-\Gamma t} \frac{1 - |r|^2}{1 + |r|^2} \left(e^{-\Gamma T} \cos(\varphi + \Delta m(T - t)) - \cos(\varphi - \Delta m t) \right) \right] \quad (16)\end{aligned}$$

where $\Gamma_{S,L}^c$ are the c-channel $K_{S,L}$ partial widths and φ is the phase of the ratio of the corresponding amplitudes, $A_{K_L \rightarrow c}/A_{K_S \rightarrow c}$ ($\simeq \epsilon = e^{i\varphi} |\epsilon|$, $\varphi \simeq \pi/4$, for the typical channels $c = \pi^+\pi^-$, $\pi^0\pi^0$, neglecting ϵ' -effects).

The computation of the second contribution is somewhat more subtle. One has to use the $K_{S,L}$, free-space basis and require no decay along the right beam before the edge of the absorber is reached at time T . At this point, the free-space propagating $K_{S,L}$ states have to be written in terms of the $K'_{S,L}$ basis, being the appropriate one to study inside matter propagation. Finally, $K'_{S,L}$ absorption takes places between T and infinity as described by eq (12). All this implies

$$dP_{c,\bar{\rho} \neq 1}(t, T)/dt = \frac{1}{8} \left(\frac{1}{|r|^2} + 1 \right) \frac{1}{1 + \bar{\rho}^2} \int_0^\infty dt' \sum_f \quad (17)$$

$$\left| A_{K_L \rightarrow c} e^{-i\lambda_L t} e^{-i\lambda_S T} \times \right.$$

$$\left[(1 + |r\bar{\rho}|^2)^{1/2} (1 + \bar{\rho}) A_{K'_S \rightarrow f} e^{-i\lambda'_S t'} + (1 + |r/\bar{\rho}|^2)^{1/2} \bar{\rho} (\bar{\rho} - 1) A_{K'_L \rightarrow f} e^{-i\lambda'_L t'} \right]$$

$$- A_{K_S \rightarrow c} e^{-i\lambda_S t} e^{-i\lambda_L T} \times$$

$$\left. \left[(1 + |r\bar{\rho}|^2)^{1/2} (1 - \bar{\rho}) A_{K'_S \rightarrow f} e^{-i\lambda'_S t'} + (1 + |r/\bar{\rho}|^2)^{1/2} \bar{\rho} (\bar{\rho} + 1) A_{K'_L \rightarrow f} e^{-i\lambda'_L t'} \right] \right|^2,$$

where the origin of integration over t' has been shifted from $T \rightarrow 0$ ($t' \rightarrow t' - T$) to simplify the notation. Integrating up to infinity eliminates any dependence on $\bar{\rho}$, i.e., on the kind of absorber used, and finally leads to

$$dP_{c,\bar{\rho} \neq 1}(t, T)/dt =$$

$$\frac{1}{8} \left(\frac{1}{|r|} + |r| \right)^2 \left[\Gamma_L^c e^{-\Gamma_L t} e^{-\Gamma_S T} + \Gamma_S^c e^{-\Gamma_S t} e^{-\Gamma_L T} \right.$$

$$\left. - 2\sqrt{\Gamma_L^c \Gamma_S^c} e^{-\Gamma t} e^{-\Gamma T} \frac{1 - |r|^2}{1 + |r|^2} \cos(\varphi + \Delta m(T - t)) \right]. \quad (18)$$

As expected, the two separated contributions (16) and (18) depend on both t and T . However, their sum, which corresponds to the the reading of the c-channel decay detector, turns out to be

$$\frac{dP_c(t, T)}{dt} = \frac{1}{8} \left(\frac{1}{|r|} + |r| \right)^2 \left[\Gamma_L^c e^{-\Gamma_L t} + \Gamma_S^c e^{-\Gamma_S t} - 2\sqrt{\Gamma_L^c \Gamma_S^c} e^{-\Gamma t} \delta \cos(\varphi - \Delta m t) \right] \quad (19)$$

with no T -dependence.

4. Discussion

Unfortunately our results disagree with the much more interesting ones obtained by Srivastava et al. [6] in essentially the same context. Indeed, our T -independence

in eq (19) will be preserved if additional t -integrations between t_i and t_f with T -independent cuts are performed as in ref. [6]. To understand the origin of the discrepancy, we particularize eq (17) to the "ideal" absorber case, i.e., to the case of immediate strong absorption at T (no t' -integration in eq (17) is thus required) with identification of a K^0 versus a \bar{K}^0 (the only two terms surviving in the summation). Eq (17) now becomes

$$dP_{c,ideal}(t,T)/dt = \frac{1}{8} \left(\frac{1}{|r|} + |r| \right)^2 \quad (20)$$

$$\left[\left| A_{K_L \rightarrow c} e^{-i\lambda_L t} e^{-i\lambda_S T} \langle K^0 | K_S \rangle - A_{K_S \rightarrow c} e^{-i\lambda_S t} e^{-i\lambda_L T} \langle K^0 | K_L \rangle \right|^2 \right. \\ \left. + \left| A_{K_L \rightarrow c} e^{-i\lambda_L t} e^{-i\lambda_S T} \langle \bar{K}^0 | K_S \rangle - A_{K_S \rightarrow c} e^{-i\lambda_S t} e^{-i\lambda_L T} \langle \bar{K}^0 | K_L \rangle \right|^2 \right],$$

leading again to the r.h.s. of eq (18) if eqs (2) are used to obtain the four $\langle K^0, \bar{K}^0 | K_{S,L} \rangle$ projections with their corresponding phases. If the sign difference in the $\pm r$ term in eq (2) is (injustifiably, in our opinion) ignored we reproduce all the results derived in ref. [6], where the use of projection operators was avoided.

Although in our discussion we have always assumed $t < T$, our final T -independent result (19) obviously holds for $T \leq t$ as well, i.e., T -reversal non-invariance is of no help here. In this sense, we fully agree with the general theorems or conclusions proposed in refs. [4], [5], namely, that the results (at the statistical or observable level) obtained in one of the beams in an EPR-like experiment cannot be modified by acting along the other beam. This general conclusion is usually deduced from the basic formalism of Quantum Mechanics which is not entirely free from subtleties and controversies, as exemplified in refs. [2]- [5] and [9]. We feel that the possible significance of our paper lies in the fact that – as in the interesting analysis by Srivastava and Widom, that triggered our present reconsiderations – only standard formulae and an explicit and widely accepted procedure have been used.

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