

# PEDESTRIAN NOTES ON QUANTUM MECHANICS

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“Get your facts straight, and then you can distort them as much as you please.”

Mark Twain

I present an elementary essay on some issues related to foundations of nonrelativistic quantum mechanics, which is written in the spirit of extreme simplicity, making it an easy-to-read paper. Moreover, one can find a useful collection of ideas and opinions expressed by many well-known authors in this vast research field.

## I. INDEFINABLES

Physics is first of all the science of measurement. As Lord Kelvin put it

I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it.

According to Kelvin a collection of thoughts cannot advance to the stage of Science without numbers. Any observable of interest in physics should be measurable or expressed in terms of measurable quantities. Length and time are two of the indefinables of classical mechanics, since on an intuitive base there are no simpler or more fundamental quantities in terms of which length and time may be expressed. The problem of space-time picture of the physical world is connected with the rigour of exact description of nature requiring, say, differential equations, by means of which we are able to gauge the intuitive space-time scales of any motion. The full number of indefinables in mechanics is three, as all its quantities could be expressed by only three indefinables. The third mechanical indefinable is usually the mass, but also force may be chosen [1]. Human beings in their everyday lives are continuously “measuring” the mechanical indefinables, as well as other indefinables of physics, e.g., the temperature, by means of their physiological senses. Of course, it is a very rough “measurement”, because it could be expressed in words, not in numbers. Words and numbers are complementary units of knowledge. Pure numbers do not tell us much about Nature unless we assign them some significance. As a good example consider the number 3.52. Just a (real) number as any other. But now write it as  $2\pi/e^\gamma$ . For some physicists it has already

a meaning. Finally, let us write down the full chain, i.e.,  $2\Delta_0/T_c = 2\pi/e^\gamma = 3.52$ . It tells us that 3.52 is the BCS value for the ratio between the gap at zero temperature and the critical temperature for the transition to the superconductor phase. One gets 3.52 only in BCS theory.

Because of its beauty, I am tempted to give a second example which has been quoted by Noyes [2] from the books of Stillman Drake on Galileo. So, what about the number 1.1107... Nothing special at first glimpse. But now let us give a first significate:  $1.1107... = \pi/2\sqrt{2}$ . Geometrically it is the ratio between the quarterperimeter of the circle and the side of the square inscribed into that circle. Geometrical (i.e., spatial) measurements and thinking were much developed by Old Greeks. But Galileo was the first to give 1.1107... as the ratio of two times, namely the time  $t_p$  it takes for a pendulum of a specific length  $l$  to swing to the vertical through a small arc to the time  $t_f$  it takes a body to fall from rest through a distance equal to the length of the pendulum. Galileo’s measurement was  $942/850 = 1.108$ , but he was not aware he measured  $\pi/2\sqrt{2}$ . However he gave a remarkable formulation of the “law of gravity”. The Galilean gravitation states that the ratio of the pendulum time to the falling time as specified above is the constant 1.108, “anywhere that bodies fall and pendulums oscillate”.

To obtain the number of indefinables (NOI) a community of physicists should adopt rules of their measurements, especially since NOI is not a fixed number, and new types of experiments might augment NOI. The rule of classical indefinables is to choose a durable standard of unit for each of them and to have a good dividing engine. This has been achieved rather easy for the meter of length but not so easy in the case of the meter of time (second). In the latter case the great difficulty has been for a long time the missing of an accurate dividing machine. Large errors were continuously accumulating

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over historical epochs, and people in the fields of Religion and Politics were forced to apply corrections at some times. Atomic standards (lasers) have been introduced since early sixties having a natural atomic time- dividing machine ( the atomic frequency). However, even the very precise atomic standards display statistical results [3], and there are also reported abnormalities during solar eclipses [4]. At present, interferometers could be used for dividing purposes too, and computing machines are usually attached to measuring devices for a more rapid conversion of the physical interactions into real numbers. As regarding computers, there should be a continuous effort to study their rate of producing numbers which is not depending only on the used algorithm, and to have more involved definitions of computable numbers (so-called Turing Problem [5]).

Establishing measurement rules for indefinables is extremely important for the conceptual constructions in the realm of physics. It is not at all an easy job in the case of quantum indefinables. Since the microscopic world is described by another kind of mechanics, the celebrated quantum mechanics, it may be that new indefinables come into play. One new indefinable is the wavefunction, also called the state vector [6], or, more realistically, the wavepacket. These are concepts essentially of mathematical origin, and hence *a priori* calculable ones. In a certain sense they correspond to the fundamental indefinable of euclidean geometry- the point. The geometrical point has neither dimensions nor any attached units, but we can assign numbers (coordinates) to it. In this way one may come to the conclusion that quantum mechanics is more a mathematical theory rather than a physical one. It is a “wave” mechanics allowing a corpuscular duality. The measurement problem will be *ab initio* extremely delicate for quantum mechanics, just because it is a theory containing unmeasurable (mathematical) indefinables. To the mathematical indefinables one could always assign generalized probabilistic meanings depending substantially on the measuring scheme. The mathematical and psycho/philosophical literature is extremely rich in various axiomatic schemes in probability theory and its mental implications [7]. Let us mention the so-called *belief functions* on which Halpern and Fagin have recently elaborated [8]. These are functions that allocate a number between 0 and 1 to every subset of a given set of objects. Such functions have been introduced by Dempster in 1967 [9], and it would be quite interesting to have a quantum mechanics based on them, e.g., to write down a belief density matrix.

It became common lore to say that the measurement process is a more or less instantaneous effect inducing a reduction/collapse of the wavepacket, which is interrupting its quantum unitary evolution as described by the Schrödinger equation. As far as the meaning of measurement is not quite clear whenever one is dealing with probabilistic concepts, the whole quantum theory is sub-

ject to severe questions of interpretation and therefore is open to deep philosophical problems. The axioms of the standard probability theory are not fixed for ever. It is a fundamental scientific goal to exploit various modifications of the probabilistic axioms rendering possible new interpretations of quantum mechanics [10].

## II. THE CONCEPT OF MASSIVENESS

Let us start this section with an excerpt from Glimm [11]

Between quantum length scales (atomic diameters of about  $10^{-10}$  m) and the earth’s diameter ( $10^6$  m) there are about 16 length scales. Most of technology and much of science occurs in this range. Between the Planck length and the diameter of the universe there are 70 length scales. 70, 16, or even 2 is a very large number. Most theories become intractable when they require coupling between even two adjacent length scales. Computational resources are generally not sufficient to resolve multiple length scales in 3 dimensional problems and even in many two-dimensional problems.

At the present time, *technology* is penetrating into the nanometer scale [12], and even atomic scale [13]. By *technology* one should understand: i) machine tools (i.e., processing equipment), ii) measuring instruments (inspection equipment), iii) super-inspection factors (e.g., well-qualified human beings). We have already machine tools at the nanometer scale [14], and one can process shapes down there at only one order of magnitude away from the atomic scale.

Already at the end of 1959 Feynman [15] delivered a remarkable talk on manipulating and controlling things on a small scale. As a matter of fact, human beings are closer to atomic scales than, say, galactic scales, and besides, to fabricate small things is an absolute technological requirement. A list of reasons why we want to make things small was provided by Pease [16] of which we cite: it is *fun*, smaller devices work *faster*, smaller devices consume *less power*, smallness is intrinsically *good*, it is scientifically *important*. One can add to the list of Pease *small is beautiful* introduced by Fubini and Molinari [17].

Nonetheless, even within mesoscopic world the measurement problem will preserve its features. We shall continue to establish correlations between a “system” observable and an “apparatus” observable, i.e., we shall do our measurements in the same common manner [18]. The apparatus should be massive with respect to the system (massive not necessarily meaning of macroscopic size), and should have a “pointer”, which cannot be

but in localized quantum states. According to DeWitt, *massiveness* of the pointer compared to the measured system is absolutely necessary to get experimental results/outcomes. However, the concept of *massiveness* is not elaborated by DeWitt (unless to say that  $M_{ap} \gg m_s$ ). At the nanometer scale, mesoscopic tips are by now in much use making possible the measurement of Van der Waals forces between the tip and the surface under investigation at distances smaller than 100 nm. That means forces in the range  $10^{-6} - 10^{-7}$  N are measured either when the tip is moved or the surface is slowly approaching the tip [19].

Massiveness is related to localization, and may be very helpful in distinguishing amongst various theories of quantum evolution. The fundamental goal of this family of theories is to explain in a unifying way microscopic (quantum) objects and macroscopic ones. Since it is reasonable to think that massiveness is indeed related to the localization features of the system, it would be interesting to study in detail the conditions under which various systems make the transition to massiveness. By this transition which, to this day, is one of the most unclear in quantum mechanics [20], one should not mean in a compulsory manner the various semiclassical ( $\hbar \rightarrow 0$ ; or better  $\hbar/m \rightarrow 0$ ; notice K.R.W. Jones [21] who showed that one can also keep  $\hbar$  fixed) approximations to quantum mechanics or the transition to macrophysics  $N \rightarrow \infty$ . It is merely a transition in the sense of the Born-Oppenheimer approximation. This approximation, going back to 1927, refers to molecular wave functions and is essential in the interpretation of molecular spectra. It is a perturbation expansion in a small parameter defined as the fourth root of the electron mass divided by the mass of the nuclei,  $\kappa = (m/M)^{1/4}$ . Denote by  $H$  a common Hamiltonian for a system of heavy and light particles and by  $H_{CM}$  the hamiltonian of the center of mass motion. The problem is to find out the manner in which the eigenvalues  $W_i$ , and eigenfunctions  $\psi$  of  $H' = H - H_{CM}$  depend on  $\kappa$  [22]. The main hypothesis of Born-Oppenheimer is the existence of an equilibrium position  $X_0$  of the heavy particles such that the eigenvalues  $W_i$  and the scaled eigenfunctions,  $\phi(\xi, x) = x^{3/2(N-1)}\psi(x\xi + X_0, x)$ , are analytic in  $\kappa^2$  and  $\kappa$ , respectively, for fixed  $m$  in the neighbourhood of zero.

Very useful would be to consider the transitions to macroscopic world and to massiveness as problems with multiple time scales, which are pervading many areas in applied mathematics [23].

Let us end this section with the relationship between *big* and *small* in quantum mechanics. For this, I shall present excerpts from Dirac's *Principles* [24]. In the first chapter, "The Principle of Superposition", Dirac states

So long as *big* and *small* are merely relative concepts, it is no help to explain the big in terms of the small. It is therefore necessary

to modify classical ideas in such a way as to give an absolute meaning to size.

On the same page one can read

We may define an object to be big when the disturbance accompanying our observation of it may be neglected, and small when the disturbance cannot be neglected. This definition is in close agreement with the common meanings of big and small....In order to give an absolute meaning to size, such as is required for any theory of the ultimate structure of matter, we have to assume that *there is a limit to the fineness of our powers of observation and the smallness of accompanying disturbance - a limit which is inherent in the nature of things and can never be surpassed by improved technique or increased skill on the part of the observer.*

### III. QUANTUM MECHANICS AND DIFFUSIONS

Words like particle and motion could be considered also in the class of undefined (primary) concepts [25]. Electrons, neutrons, neutrinos and other 'ons and 'inos could be only names that we accept because of their intuitive power. All the so-called elementary particles which have been introduced in the last one hundred years can be considered as particular forms of propagating (transport) processes and energy carriers [26]. Indeed, Schrödinger equation is the basic equation of quantum world, but diffusion equations are, no doubt, more general equations. We say 'no doubt' just because already in 1933, Fuerth [27] showed that Schrödinger equation could be written as a diffusion equation with an imaginary diffusion coefficient,  $D_{qm} = i\hbar/2m$ . This imaginary diffusivity is vexing and many stopped the analogy at this point. On the other hand, negative diffusivity is more natural and one may encounter it in multicomponent systems, implying local increase in the energy of the system as discussed by Ghez [28]. Let us consider a one-dimensional Schrödinger equation  $i\hbar \frac{\partial \psi}{\partial t} = H\psi$  where  $H = \frac{p^2}{2m} + V$  and  $p = -i\hbar \frac{\partial}{\partial x}$ . The diffusive character of such an equation for  $\psi$  is obvious if we take into account a source term related to the potential energy, and the momentum playing the role of flux. For historical reasons the diffusion interpretations ( they may come in three classes: in configuration space, in phase space, and in imaginary time) were not favored during a long lapse of time, though today mainly because of the impetus provided by *quantum optics*, we became accustomed with such methods as quantum-jump [29] and quantum-state diffusion [30] to simulate dissipation processes. Indeed, Schrödinger obtained his wave mechanics by means of a more intuitive analogy in which

he put together the Hamilton-Jacobi theory, relating geometrical optics and particle dynamics, with de Broglie's matter waves. One could say that what Schrödinger did was to randomize a purely classical theory by means of de Broglie hypothesis. It was a way of randomizing within the classical formalism, but, more generally, one should be aware of the multitude of randomization procedures [31].

The apparent difficulty of imaginary diffusivity is not essential when interpreting it in the proper way. The result can well be a more general theory. The picture of the *World* is that of an infinite number of clusters in the sense of percolation theory. Classes of clusters could be defined in terms of their relative diffusivities and fluxes. Some of them are "static" relative to other more kinetic ones [32]. In this diffuson context, the imaginary character of the diffusion coefficient for quantum particles is related to the passage from a parabolic differential equation to a hyperbolic one [33].

Even a presentation of the one dimensional diffusion equation, first in the discretized form and then in the continuous limit, on the lines of the *Primer* of Ghez [28] is very helpful to understand the diffusive aspects of the Schrödinger equation, and I recommend the reader to look in that book. The toy model of Ghez is a pedagogical isotropic one-dimensional random walk, in which one considers points on a line with an arbitrary fixed origin. For the passage to the continuum limit one must introduce a jump distance between the points and a continuous particle distribution, depending not only on time but also on the space variable such that to coincide at the discrete sites with the discrete particle distribution.

There are also papers dealing with the connection between a classical Markov process of diffusion type and the quantum mechanical form of the Hamiltonian for a classical charged particle in an electromagnetic field [34]. These two problems are equivalent as far as one is concerned with the expectation values for the particle energies in the two cases. Consider a continuity equation of the type  $\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho v)$  where  $v = v_0 - D\nabla \ln \rho$ . Such a continuity equation is in fact a Fokker-Planck equation for the probability density  $\rho$  for the position vector of a particle following a Markov process of diffusion type with diffusion coefficient  $D$ . The expectation value for the energy of the particle  $\langle E \rangle = \int [mv_0^2/2 + eV]\rho d^3x$  can also be written  $\langle E \rangle = \int [mv^2/2 + D^2m/2(\nabla \ln \rho)^2 + eV]\rho d^3x$ , and the connection with the electromagnetic phenomena can be established by means of the celebrated Helmholtz theorem for a vector (considering the velocity as a vector, thus no type of spin)  $v = \alpha \nabla \phi + \beta A$ , where  $\phi$  and  $A$  are defined in the usual way up to a gauge transformation, and  $\alpha$  and  $\beta$  are constants which should be chosen in an appropriate way to achieve the correspondence. There would be interesting to study the passage ways from microscopic to macroscopic description of electromagnetic fields [35] in this framework. The traditional

one, going back to Lorentz, and which is applicable to common molecular media, is by averaging the differential equations for microscopic quantities by integration over some macroscopic volume. This is the most trivial procedure for going to the macroscopic approximation. There are other approaches, e.g., the topological one as discussed by Brusin [36].

#### IV. QUANTUM MECHANICS AND LOCALIZATIONS

The collapse postulate of quantum mechanics is one of the most debatable points in the conceptual base of this theory, being at the same time the main desideratum for a modified quantum dynamics [37]. The collapse of the state vector is required by the formal quantum theory of measurement. One must assure somehow the decoherence of the macroscopic states of the apparatus in order to have a definite outcome for any experiment involving quantum particles. We still do not know if this decoherence is dependent on the particular interaction and hence on the particular type of measurement or is a universal feature of the transition from microscopic to macroscopic behaviour. The first hypothesis is called environmental (Zeh-Joos) localization [38]. On the other hand, the universal localization, also known as spontaneous (or GRW) localization is due to Ghirardi, Rimini and Weber [39]. It is difficult to decide between the two models. In our opinion, they are not completely opposite ideas. The dynamical (environmental) localization may be specific to the particular experiment, while the spontaneous localization might be thought of as related to the transition to massiveness, which one would like to see as universal. In this way having different purposes, the two standpoints are not contradictory. At the same time GRW localization could be considered only as a special type of environmental localization at the scale given by the parameters of the model. The point is that these parameters have been raised to the level of fundamental constants of Nature by the authors. Anyway, one must spell out explicit conditions allowing to pass from a regime of continuous spontaneous (or dynamic) localization to a discontinuous regime characteristic to the GRW localization. We recall that in the GRW approach the N-particle wave function of the non-relativistic Schrödinger quantum mechanics (NRQM) is coupled to a normalized Gaussian jump factor  $J_{GRW}(x) = K \exp(-x^2/2a^2)$ . The frequency of the jumps and the localization constant are considered as two new fundamental constants of Nature of the following orders of magnitude  $\nu_{GRW} \sim 10^{-15} s^{-1} = 10^{-8} \text{ year}^{-1}$  and  $a \sim 10^{-5} \text{ cm}$ .

The spontaneous localization implied by the GRW model might be tested experimentally by means of mesoscopic phenomena, e.g., by looking for instabilities of the mesoscopic growing (thread-like, filamentary) patterns.

Recently, Kasumov, Kislov, and Khodos [40] observed the displacements of the free ends of threads of amorphous hydrocarbons of 200-500 Å in width and 0.2-2.0 μm in length relative to a fixed reference point on the screen of a transmission electron microscope. The minimal displacements were of about 5 Å, and the observations were made in a regime of stationarity of the threads, i.e., very low density of the beam current ( $\sim 0.1$  pA/cm<sup>2</sup>). KKK observed random jumps of the free ends of the carbon threads of 10-30 Å in length and of frequency of  $\sim 1$  Hz. They discussed possible reasons to induce vibrations, and came to the conclusion that no classical external forces could explain the jumps. They attributed them to the “quantum potential”, and to localizations of GRW type, but the range of the observed parameters do not correspond to that of the GRW ones.

What I would say is that the jumps or mesoscopic fluctuations of the carbon threads are a kind of mesoscopic Brownian motion which damps in time, being different from the microscopic quantum fluctuations which never damps out.

*Moreover, if one takes into account the recent work of Sumpter and Noid [41] the KKK results can be classified as red herrings. Sumpter and Noid assigned the onset of positional instabilities in samples of carbon nanotubes to nonlinear resonances controlled by their geometry, i.e., the contour length around the end of the tube and the length of the tube along its axis. It is quite probable that the same mechanism applies also to microtubules in biology. For the connection between “quantum jumps” and nonlinear resonances in classical phase space see Holthaus and Just [42]*

I would like to point out that the GRW-type localization corresponds to a weak coupling limit of Hamiltonian systems with coherent/squeezing interaction with the environment [43]. Indeed Gaussian localization is specific to coherent and squeezed states in the configuration representation. An immediate scope is to generalize this type of localization to relativistic quantum mechanics (RQM), and to quantum field theories (QFT). In NRQM one is dealing with spatial probabilities, that is with probabilities associated with a spatial domain ( $\Delta X$ ) at a moment of time  $T$ . To go to RQM, one must extend the spatial probability to a spacetime domain as in [44].

## V. NONLINEAR WAVEFUNCTION COLLAPSE ?

The quantum wavefunction varies in time in a continuous way, following the deterministic Schrödinger evolution. When an observer wishes to measure a physical quantity of a quantum system, the wavefunction corresponding to that physical quantity is exposed to an apparatus specially designed to do this. The general effect of the apparatus, usually macroscopic with respect to the physical system, is to induce a discontinuous change

of the wave function from a superposition of states into just one state. This general effect is known in the quantum formalism as the collapse of the wavefunction. The open question is to find out the general mechanism of the collapse of the quantum wavefunction. In the literature one can find many interesting ideas on this problem. As a quite acceptable interpretation of the collapsing phenomenon we mention here the old ideas of Schrödinger, who tried to relate the modulus of the wavefunction to a materialistic and realistic density of electronic matter, and *not* to probabilities. For a recent discussion of this viewpoint the reader is referred to a paper of Barut [45]. This model for the modulus of the wavefunction can be elaborated further by making use of progress due to Chew [46].

In the following, we would like to comment on some phenomenological features of physical collapses from other areas of physics in the hope to gain more insights in the possible physical picture of quantum mechanical collapses of admittedly fundamental origin. Our standpoint is that the present status of the wave function reduction phenomenon is too formal, even though one may find an abundant literature with interesting presentations of the topic [47]. It is fair to say that we have no generally accepted physical mechanism of the reduction process for the time being. In the literature, one can find only extreme descriptions, claiming for a strong nonlinear process in which gravity [48] and/or quantum gravity [49] is thought to play an important role. On the other hand, collapsing phenomena, presumably displaying similar patterns can be encountered in several other fields of physics, in the case when nonlinear effects are not balancing any more the dispersive spreading of waves (solitons). Of course, in such cases one is already outside the restricted regime of linear dissipation implied by standard quantum mechanics. Moreover, one can avoid thermodynamical arguments against nonlinear variants of Schrödinger equation [50] by making use of more general entropies [51].

A relevant example of nonlinear collapse is the Langmuir collapse in plasma physics. Langmuir collapses belong to the class of wave collapses, a well-defined topic in nonlinear physics [52]. The collapse of Langmuir wave packets in two or more dimensions was first predicted by Zakharov [53], and it is observed in the laboratory. It is a strong non-linear collapse occurring in strong Langmuir turbulence, which consists of many locally coherent wave packets interacting with a background of long-wavelength incoherent turbulence [54]. Langmuir collapses are governed by a non-linear Schrödinger equation of the type  $i\psi_t + 1/2\Delta\psi + |\psi|^s\psi = 0$  which, as it is well-known, allows singularity formation in a finite time  $t = t_0$ , for  $sd \geq 4$ , ( $d$  is the dimension of space). The phenomenology of the Langmuir turbulence is extremely interesting. Wave packets are observed to “nucleate” in existing density depressions. The nucleation of new wave-packets takes

place by the trapping of energy from long-wavelength background turbulence into localized eigenstates of relaxing density wells. Since the collapse transfers energy to short scales, where there is strong damping, a process called “burnout” occur in which energy is transferred to the electrons and the collapse is stopped. In this way the Langmuir field is dissipated, the density cavity relaxes and can serve as a nucleation site for a new wave packet. Perhaps an equivalent physical picture as that of the turbulent wave collapse might be made available with some modifications for wavefunction collapse in a nonlinear scheme of quantum mechanics (e.g., a dust plasma model).

## VI. REMARKS ON VARIOUS OTHER TOPICS

### A. Friction modifications of quantum mechanics

Modifications of quantum mechanics may be thought of in terms of friction terms for the more general situation of open quantum systems. The problem of the ways of including various types of friction in the quantum mechanical framework has been an active field for decades. Many authors considered the dissipation in the form of friction as a means to reconcile quantum mechanics and general relativity, and also as able to cast light on the transition between classical and quantum physics. Even though the dissipation of energy seems to be more appropriately described in terms of a density operator approach, there has been always a steady activity towards understanding friction at the level of wave functions [55].

In this area, the damped harmonic oscillator is considered to be “the primary textbook example of the quantum theory of irreversible processes”, to quote Milburn and Walls [56].

Some time ago, Ellis, Mavromatos and Nanopoulos [57] studied string theory models from the frictional point of view. They gave reasons to believe that the light particles in string theories obey an effective quantum mechanics modified by the inclusion of a quantum-gravitational friction term, induced by the couplings of the massive string states. According to these authors the string frictional term has a formal similarity to simple models of environmental quantum friction.

Finally, Beciu [58] sketched a proof showing that a friction term for a cosmological fluid still retaining the symmetries of a perfect fluid at the level of the stress tensor is equivalent to an inflaton field.

### B. Wave-particle dualities

Historically speaking, the wave-particle dualities were established before the advent of the quantum differen-

tial equations. We say dualities and not duality because, not only for historical reasons, one must distinguish the duality of photons from that of massive particles, say electrons.

The wave-particle duality of light is defined by the Einstein relation  $E = h\nu = \frac{hc}{\lambda}$ . This duality of light was used by Einstein to explain the photoelectric effect, by the Nobel-prize formula for the kinetic energy of the emitted electrons  $\frac{1}{2}mv^2 = E - E_0$  where  $E = h\nu$  is the quantized energy of the incident photons and  $E_0 = e\phi$  is the threshold energy with  $\phi$  the work function.

The duality of massive particles, on the other hand, was established by de Broglie two decades after Einstein’s duality. The wavelength and the momentum of an electron (and of any other massive particle) is given by  $\lambda = \frac{h}{p}$ .

It is worth noting the fact that the two dualities are related to each other through the photoelectric effect,  $\frac{\hbar^2 k^2}{2m} = \frac{hc}{\lambda} - e\phi$ .

Usually, the textbooks and the literature present the wave-particle dualities as a logical result of Young slit experiments. As a rule, a more or less detailed discussion of the complementarity principle is accompanying the discussion of the Young experiment. Interesting ideas concerning the slit complementarity and duality have been put forth by Wootters and Zurek [59], Bartell [60] and Bardou [61]. These authors made attempts to transcend the rather dogmatic presentation of this fundamental topic. Bartell introduced the idea of intermediate particle-wave behavior. Most probably, we need generalizations of the concepts of wave and particle, of their interactions, and a deep scrutiny of the effects of the type of experiment.

In the last couple of years, the investigation of particle-wave dualities became a very active one, mainly because of the rapid progress of some new technologies. Perhaps, one of the most interesting experiments is that performed by Mizobuchi and Ohtaké [62], which is just a repetition of the old double prism experiment done by Bose as long ago as 1897, however not with microwaves but, following a suggestion of Ghose, Home, and Agarwal [63], with single photon states. An *and*-logic for the wave-particle duality at the single-photon level has been claimed.

An open problem in detecting photons is the precise meaning of the photon in the detection process. The point is that we are detecting signals, and these signals depend on the experimental detection schemes. The signals will give some pulses in the detectors. Thus the full detection process is governed by some electronic relationships in the signal-pulse-detector system [64].

Understanding better the manifestations of wave-particle dualities for light can be highly relevant in photonics and optical computing [65].

At this point, let me quote from the recent paper entitled “Anti-photon” of W.E. Lamb, Jr. [66]

... there is no such thing as a photon. Only a

comedy of errors and historical accidents led to its popularity among physicists and optical scientists.

Then, of course, the wave-particle duality for light will be losing its physical picture but will gain in mathematical rigor.

### C. The problem of the constancy of the Planck constant

It was remarked by Barut [67] that the free electromagnetic field has no scale. There are only frequencies. Moreover, Planck originally derived his formula from the properties of the oscillator on the boundary of the black-body cavity, not from the quantization of the field. The common practice of quantization of the fields came later. Therefore, we believe that even today careful experimental checks of the constancy of the Planck constant should be made, and in fact have been made in some laboratories [68]. Barut showed that a formulation of quantum mechanics without the fundamental constants  $\hbar$ ,  $m$  and  $e$  is possible [67] [69]. It looks like a pure wave theory in terms of frequencies alone, and it might be used more profitably in experiments where one measures frequency differences. In this case, the energy becomes a secondary concept, and different quantum systems are characterized by an intrinsic proper frequency  $\omega_0$ . On the other hand, one can consider quantum dynamics with two Planck constants, like did Diósi [70].

As soon as we depart from the assumption of the constancy of the Planck constant by merely considering a variable Planck parameter ( $\mathcal{H}$ ), but nonetheless preserving the constancy of  $\mathcal{H}/m$  we may consider some kind of quantization at large scales, planetary or even galactic ones. In fact there is a quite vast literature on megaquantum effects. We draw attention to the fact that such effects are related to the interpretation of  $\mathcal{H}/m$  as a pseudo-Planck constant which is associated to some gravitational systems (e.g., the Solar System [71], quasars [72]).

A viewpoint to be recorded was put forward by Landsman [73]. He claimed that only dimensionless combinations of  $\hbar$  and a parameter characteristic of the physical system under study are variable in Nature. The references [71] [72] seem to confirm this idea.

We would like to point briefly on the possible effect of the spatial scale of the measurement scheme on the numerical value of fundamental constants. We shall use as an example the fine-structure constant  $\alpha = e^2/\hbar c$ . At the present time, we know a very precise macroscopic phenomenon, namely the quantum Hall effect, from which the fine-structure constant can be obtained from the quantized Hall resistance. (I consider quantized Hall resistance a more precise experiment as compared to that

involving the proton gyromagnetic ratio, proton magnetic moment and Josephson frequency-to-voltage ratio). The numerical value obtained from the quantum Hall effect is [74]:  $\alpha^{-1} = 137.0359943(127)$ . On the other hand, the standard atomic measurement (coming from the anomalous magnetic moment of the electron) gives  $\alpha^{-1} = 137.0359884(79)$ . The two values differ only at the level of 0.1 ppm. The QED corrections are confined to distances of the order of the Compton wavelength of the electron, whereas the primary interaction in the quantum Hall effect is between the electrons in the metal and those circulating in the coils which produce the magnetic field. The spatial scale in this case is of the order of a few cm. It would be extremely interesting to relate the very small differences in the numerical values of the fundamental constants to the spatial scale of the phenomena used to measure them. Presumably, there might be correlations between the last different digits of the numerical values of the fundamental constants and the spatial scale of the measuring device used to determine that value. At least some self-similar correlations are to be expected.

### D. Quantum mechanics and cosmology

The previous subsection already introduced us into the much more ambitious program of describing the universe as a whole in quantum mechanical terms. The difficult problem of interpretation is not so much with respect to considering the Hilbert space of the Universe. It is related to the fundamental fact that there can be no *a priori* division into *observer* and *observed*. In other words, there is no Feynman's "rest of the Universe".

A generalization of the Copenhagen interpretation such as to be applied to cosmology was first provided by Everett [75] in 1957. His theory of "many worlds" has been replaced at the present time by theories of "many histories" (time-ordered sequences of projection operators [76]), but the essential ideas remained those of 1957. As a matter of fact, what Everett has done may be entailed in the process of probabilistic modeling, i.e., the organization of the space of wave function(s) as a probability space [77].

Everett showed how in his interpretation it is possible to consider the observer as part of the system (the universe) and how its fundamental activities- measuring, recording, and calculating probabilities- could be described by quantum mechanics. As incomplete points in Everett's interpretation, which has been much clarified subsequently, one should mention the origin of the classical domain we see all around us, and a more detailed explanation of the process of "branching" that replaces the notion of measurement.

The main concept that has emerged in this area is that of decoherence functionals, and the main debated topic is

that of connecting this concept to the probability interpretation. Recently, Isham and collaborators presented a classification of the decoherence functionals based on a histories analog of Gleason's theorem [78]. To be noticed are the "negotiations" on the border between quantum and classical in the decoherence framework published in *Physics Today* of April 1993.

### E. Quantum jumps

The interesting topic of quantum jumps [79] has to do with the rare but strong fluctuations that may show up in any stochastic process, be it classical or quantum. The mathematical theory of large deviation estimations has been already elaborated in considerable extent [80]. All quantum mechanical equations have solutions to which probability representations may be given [81]. The mathematical problem is to find out probability measures of Poisson processes with jump trajectories, which are similar to the Feynman-Kac transformation of probability measures for processes with continuous trajectories. For relativistic equations we have usually Poisson probability representations, whereas for nonrelativistic equations diffusions in imaginary time have been worked out, but also Poisson representations are possible. One can establish the scale at which the transition from the covariant hyperbolic Dirac dynamics to the non-covariant parabolic dynamics of the Schrödinger equation occurs [82].

As a further argument that quantum jumps, i.e., "discontinuities in time of the wavefunction" in the terminology of Zeh [83], are related to rare fluctuations of stochastic nature, we remark that they are observed even in single quantum systems [84].

### F. Analogies to quantum mechanics

Thinking by analogy is considered to be a clear indication of superior reasoning and of human intelligence [85]. In physics there are a vast amount of analogies of much help in the progress of many different branches of this science. Many analogies are not complete and it is precisely this point to induce into error all those residing too much on this beautiful aspect of human thinking. One should keep in mind the danger of extrapolating the analogies beyond their natural limits, which should be carefully estimated, and also the risk of using them in the wrong way.

Coming to quantum mechanics, we would like to recall two quite attractive analogies. The first one is the electric network discussed by Cowan [86] long ago. The Cowan networks have the distribution of the electric energy density in three dimensional space similar to that of probability density waves corresponding to a spinless particle in any potential field.

The second analogy has been recently discussed by J.L. Rosner [87] who showed that the so-called Smith Chart method used for antenna impedance matching corresponds in quantum mechanics to a simple conformal transformation of the logarithmic derivative of the wavefunction. The Smith Chart is a convenient graphical representation for analysing transmission lines [88], and clearly may help understanding from a different point of view the tunneling processes.

### G. Human brain and quantum computers/brains

The flux of literature tells us that *quantum computers* are at good moments of the *gate* phase and of exploratory discussions of various physical setups from the quantum computational standpoint. This exciting topic has been started about two decades ago (though one can think of Szilard, von Neumann, and Brillouin as well) and might turn into a really major general discipline.

Apparently the functioning of the human brain is not based on quantum effects. The membrane voltages of the neurons do not imply the Planck constant, and the important physical processes are essentially the mesoscopic transport ones. A great advantage of the human brain is a quite flexible microtubule architecture due to a remarkable phenomenon, the so-called *dynamic instability* [89]. The origin of this important phenomenon is debatable, and after having read the note of Sumter and Noid (J. Chem. Phys. of April 22, 1995) I think that a nonlinear resonance mechanism should be considered as a good proposal. Many brain mysteries are hidden in the microtubule assembly characterizing any individual biological brain, and there is much unexplored physics.

The mesoscopic functioning of the human brain does not imply that an almost quantum (e.g., nanoscopic) brain cannot be fabricated. For example, Josephson junctions may be the component units of such a brain since the relationship between the applied voltage and the emitted frequency involves Planck's constant.

In his paper "Is quantum mechanics useful?" [90], Professor Landauer remarked that *technologies differ in their explicit utilization of quantum mechanical behaviour*. The important technological task in considering quantum computers is to print the bit on as small a material structure as physically possible in order to diminish the energy dissipation in the copying process, and to substantially reduce the switching time from one bit to another. Actually, the real technological effort is evolving at the intricate nanometer scale, which clearly will be essential for the general human progress. The emphasis on the devices is this time both to understand what they measure and mostly to estimate their computing capabilities. As mentioned by Feynman [91] the present transistor systems dissipate  $10^{10}$  kT. He considered bits written "ridiculously", as he said, on a single atom. At

present we know this is not ridiculous since we already are talking about atomic transistors [13].

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### • I. Indefinables

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An interesting paper on the scales of length and time is: A.M. Wolsky, "The scales of length and time in classical and modern physics", *Am. J. Phys.* **39**, 529 (1971)

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Berlin, 1993); A.N. Kolmogorov, *Foundations of the theory of probability* (Chelsea Press, New York, 1950); Yu.A. Rozanov, *Stationary random processes* (Holden-Day, San Francisco, 1967); *Lect. Notes in Math* **1303**, *Quantum probability and applications III*, eds. L. Accardi and W. von Waldenfels (Springer, Berlin, 1988); D.N. Page, "Probabilities don't matter", gr-qc/9411004 (lecture at the 7th Marcel Grossmann Meeting (1994). Let us mention also the topic of "fuzzy" sets (and logic) started in 1965 by L.A. Zadeh who introduced the concepts of linguistic variable and approximate reasoning. "Fuzzy" literature is today at the level of more than 7000 publications.

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- [9] A.P. Dempster, "Upper and lower probabilities induced by a multivalued mapping", *Ann. Math. Stat.* **38**, 325 (1967)
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### • II. The concept of massiveness

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On the other hand, Nature is doing by herself morphogenetic movements. Take any common animal on the Earth. In a short time lag before being born, it evolves from a cell to a complex biological system. Many biologists consider the morphogenesis within Animal World to reside in some special molecules endowed with cell-adhesion properties. See, e.g., G.M. Edelman, "Cell-adhesion molecules:

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The rapid morphogenetic period is followed by a somewhat longer growth period which is governed by allotropic transformation formulas. Towards the end of the normal life-cycle of a biological system one must take into account an allotropic scaling down of the biological material.

Physicists believe also that morphogenesis has molecular grounds; see A.P. Balachandran et al., “Molecules as quantum shapes and how they violate P and T”, *Int. J. Mod. Phys. A* **7**, 2087 (1992)

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J.Luczka et al., “Randomly interrupted diffusion”, *Phys. Lett. A* **167**, 475 (1992); The authors of this paper consider processes driven by Gaussian white noise, which is interrupted randomly by a two-state 0,1 Markov stochastic process. They construct also an infinitesimal generator of the evolution operator for single-event probability distributions, which describes spatial and temporal non-local evolution of the system under consideration. The two-state Markov process could be substituted by the GRW-type localization, as a simulation of the GRW collapses.

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