

REMARKS ON THE FREQUENCY SPECTRA OF SOME FUNDAMENTAL QUANTUM EFFECTS

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Summary: - Short remarks on the problem of assigning frequency spectra to Casimir, sonoluminescence, Hawking, Unruh, and quantum optical squeezing effects are presented.

Introduction

Recent years witnessed great progress toward understanding in a quantum field approach the following fundamental quantum effects: Casimir, sonoluminescence, Hawking, Unruh, and squeezing ones. These effects have been treated in many ways, various authors emphasizing either peculiar aspects or similarities between them. A large body of knowledge was accumulated over the years in their regard but nevertheless they continue to be highly challenging topics. Many well-known authors retain special interest in these paradigm-like effects and their treatment is pervading a good deal of the modern literature in theoretical physics.

A unifying basis of all these fundamental effects may occur when approaching them either from the standpoint of quantum vacuum energy, i.e., the energy of zero-point fluctuations [1], or as quantum Brownian noises [2]. Such treatments are very helpful for those who would like to stress the similarity of such effects. It is by now well settled in the literature that the vacuum energy may have in certain conditions a thermal-like representation. This is a most interesting fact by itself, and a glimpse to the active developments of more precise scientific terminology for the general concept of *thermality* will convince the reader on the progress achieved since the times of Wien and Planck. Let us point out for example that at least three kinds of reservoirs have been recognized, depending on the noise they produce. One may speak of reservoirs with coloured noise, with phase-dependent noise and of squeezed-vacuum reservoirs. It is true however that the majority of the analytical treatments are based on the white-noise assumption, which is appropriate whenever the correlation time of the reservoir is small as compared to the dynamical time scale. This is essentially the case in all the practical situations.

The heat-bath features of vacuum fluctuations within the realm of the mentioned effects show similarities of course, but also differences, and it is the main purpose of the present work to draw attention, at a heuristic level, on such dichotomies that in our opinion may prove quite useful for further progress in the understanding of those effects. For example, the analogy between Casimir effect and Hawking effect has been under the focus of a number of authors. One of the most detailed analysis

of this analogy has been provided by Nugayev [3], and is based on the approach according to which the blackbody radiation in the Hawking case is created in the close vicinity of the event horizons. On the other hand, Grishchuk and Sidorov [4], dealing with the squeezing of the gravitational waves, have briefly considered the squeezing produced by a Schwarzschild black hole. They showed that the Bogolubov-Hawking transformations can be interpreted as a two-mode type of squeezing, a conclusion that applies to Rindler motion as well. Thus, it appears that there are definite connections among the various standpoints. A good procedure for studying the interplay of the effects is to examine the assignment of a frequency spectrum in each case because in this way one may hope to better characterize the differences as well as the similarities between them. In our opinion, the frequency spectrum is an extremely appropriate means to disentangle the dichotomy *similar-different* for such effects. Frequency spectra are the basic means to investigate stationary noises and even nonstationary ones, though the situation is a bit delicate in the latter case for which tomographical methods may be the best [5]. As is known the integral of the frequency spectrum is called the noise power. This is the basic concept used both in theoretical and engineering considerations. Let us consider the spectrum of Schottky noise, which is of primary importance for stochastic cooling in storage rings [6]. This noise expresses the fluctuation of electrostatic potential near the beam. Its spectrum is thus directly related to the charge-density fluctuation in the beam which in turn is characterized by static and dynamic form factors. But by means of the fluctuation-dissipation theorem the dynamic form factor is related to the dielectric response function. All of these relationships are very useful when dealing with noises, either classical or quantum ones. Especially in the latter case it is known that the Fourier transform of the Kubo-Martin-Schwinger condition [7] is actually the thermal fluctuation-dissipation relation.

Our remarks in the following refer to the Casimir effect, sonoluminescence, Hawking radiation and Unruh radiation, quantum optical squeezing, and also to frequency measurements of uniformly accelerating observers, in this order, ending up with conclusions.

1. Frequency Spectrum for Casimir Effect

Although the total zero-point energy of the electromagnetic field contained in a cavity bounded by conducting walls is divergent, its variation due to the displacement of the boundaries is finite and corresponds to a weak but measurable attraction between the walls (for a review see [8]). This was first remarked by Casimir in 1948. In the case of two parallel, conducting plates separated by a distance d , Casimir obtained the simplest expression for the Casimir energy, i.e., the interaction energy density per unit area as follows

$$\rho_C(d) = -\frac{\pi^2}{720} \frac{\hbar c}{d^3}. \quad (c.1)$$

Ford [9] was the first to pose the problem of the frequency spectrum in the case of Casimir effect. He defined the spectrum by means of suited spectral weight

functions distorting the original spectrum of quantum fluctuations. In this way, such functions are able to reveal the contribution of each frequency interval to the finite Casimir energy. His motivation was based on the example of the energy density of a massless scalar field in $S^1 \times R$, the two-dimensional flat spacetime with spatial periodicity of length L . A work of Brevik and Nielsen [10] on the Casimir energy of a piecewise string is also very close to this simple model. The Casimir energy density can be expressed as follows

$$\rho = -\pi^{-1} \int_0^\infty \frac{\omega d\omega}{e^{L\omega} - 1} = -\frac{\pi}{6L^2}. \quad (c.2)$$

As remarked by Ford a suggestive interpretation is that of an integral over a thermal spectrum with a temperature equal to L^{-1} . However, many different integrals over ω could yield the same result. More examples of energy-momentum tensors that can be written as integrals with a thermal denominator but different phase-space numerators are given in [11]. As a toy model for the effects of the spectral weighting Ford used the vacuum energy density of a scalar field in $S^1 \times R$

$$\rho_W = (2L)^{-1} \sum_{n=-\infty}^{n=+\infty} \omega_n W(\omega_n). \quad (c.3)$$

If the spectral weight function $W(\omega)$ vanishes sufficiently rapidly for $\omega \rightarrow \infty$ then the weighted vacuum energy density is finite. Ford found complex behavior of the spectrum, with discontinuities and oscillations. Also, according to Hacyan et al. [12] the weight function depends significantly on the specific experiment.

2. Spectrum of sonoluminescence

Sonoluminescence (SL) is an intriguing phenomenon known since 1934 and consisting in picosecond flashes of light that are synchronously generated by the extremely nonlinear *cavitation collapse* of, e.g., water bubbles which are trapped at the velocity node of a resonant sound field in water. Recent experiments on water SL have been performed by Hiller, Putterman and Barber [13]. These authors claim that SL is radiation of black-body type at a temperature as high as 25,000 K. There are many other experiments and the whole topic is in a very active period. The last scientific works of Schwinger attributed SL to a dynamical Casimir effect [14], but recent debate do not favor this interpretation. The SL phenomenon can be extrapolated to other fields of physics, for example, one may think of a SL-type phenomenon as responsible for the gamma bursts in astrophysics.

3. Spectrum of Hawking Radiation and of Unruh Radiation

A paper of H. Ooguri [15] under the title ‘‘Spectrum of Hawking radiation and the Huygens principle’’ has as purpose to discuss a result of Takagi [16] who proved that the vacuum spectrum detected by a uniformly accelerating detector in the case of a free massless scalar field in Minkowski spacetime (i.e., the Unruh effect), and similar spectra in de Sitter and Schwarzschild spacetimes are of Bose-Einstein type

in even spacetime dimensions and of Fermi-Dirac type in odd number of spacetime dimensions. According to Ooguri this is a consequence of the fact that massless field theories in odd dimensions do not satisfy the quantum version of the Huygens principle, i.e., the expectation value of the commutator of massless fields does not vanish in the timelike region. Takagi proved that the vacuum power spectrum of a massless scalar field in n spacetime dimensions could be written in the following form

$$F_n(\omega) = \frac{\pi}{\omega} \frac{D_n^M(\omega) d_n(\omega)}{e^{\omega/T} - (-1)^n}. \quad (h.1)$$

$D_n^M(\omega)$ is the Minkowski density of states, i.e., that of massless Minkowski quasi-particles in n - dimensional Minkowski spacetime which is given by

$$D_n^M(\omega) = \frac{2^{2-n} \pi^{(1-n)/2}}{\Gamma((n-1)/2)} |\omega|^{n-2} \quad (h.2)$$

and $d_n(\omega) = D_n^R(\omega)/D_n^M$ if n is even and the same ratio multiplied by $\coth(\omega/2T)$ if n is odd. The D_n^R is the Rindler density of states of Rindler quasiparticles. This result can be extended to Hawking effect as well. As emphasized by Takagi the statistics -changing phenomenon refers to the distribution function characterizing the power spectrum of the noise and not to the change in the basic algebra obeyed by the operators. It is related to special relationships between the Green's functions in successive dimensions. Indeed, it is well known [17] that the Wronskian condition on the coefficients of Bogolubov transformations ensures in both Lorentzian and Euclidean spacetimes the preservation of the formal commutation relations at any time. This has also to do with methods of constructing wavefronts of wave equations in curved spaces or spaces with special causal structures, in other words, with mathematical aspects of Huygens' principle [18]. Further progress in this field can be sought in terms of Radon-type transforms [19].

4. Spectrum of Squeezing

Since, as was mentioned in the introduction, the Hawking effect, as well as the Unruh effect, might be interpreted as two-mode squeezing effects, we present the laboratory squeezing case in view of further possible relationships [20]. For a general non-squeezing context, it is recommendable for the reader to take a look in an old paper of Eberly and Wódkiewicz [21].

The spectrum of squeezing in laboratory quantum optics has been discussed in some detail by Carmichael [22], who first defined it as the ratio of the homodyne spectrum and the shot spectrum in the zero-frequency limit where the shot-spectrum is flat. It is a photocurrent spectrum and for a direct comparison with frequency spectra of the other sections one should pass to the photocount regime. The spectrum of squeezing worked out by Carmichael is given by his formula 3.26 in [22]

$$S(\omega, \theta) = 8\eta r(2\gamma_1) \int_0^\infty d\tau \cos\omega\tau \langle: \Delta X_\theta(0) \Delta X_\theta(\tau) :> , \quad (s.1)$$

where the quantity in the brackets denotes the normally ordered, time-ordered correlation function for the intracavity field quadrature X_θ , with the phase θ controlled by the phase of the local-oscillator; the $2\gamma_1$ factor gives the rate for photon escape through the mirror and is a damping constant in the master operator equation of the cavity; the parameter r is the reflection coefficient at the beam splitter while η is a detection efficiency. The formula (s.1) is a quite standard one for a power spectrum with the normally ordered treatment of the autocorrelation function. According to Carmichael, the signature of squeezing at a certain frequency ω is the phase-dependence of the power spectrum between a *negative* minimum value and a positive maximum one at a phase in quadrature with respect to the first one.

We recall here that the detection of an amplitude component of a field can be implemented by means of a homodyne detector. The procedure is as follows. The signal beam is combined by means of a beam splitter with an intense *local oscillator* (LO) field operating at the same frequency. The combined field is then directed to a photodetector and the amplitude component of the field is revealed as the beating between the two input fields. To avoid noise from intensity fluctuations of the LO, the balanced configuration is usually chosen, by using two photodetectors with equal gains and a 50-50 (%) beam splitter. The difference photocurrent I_D between the two photodetectors is not influenced by the fluctuations of the LO intensity. On the contrary, I_D measures the interference between the signal beam and the LO, the interference being constructive at one photodetector and destructive at the second one [23].

5. Frequency Measurements of Uniformly Accelerating Observers

Measuring frequencies in noninertial frames is by far a non-trivial issue. Recently, Moreau [24] commented on the nonlocality in the frequency measurements of uniformly accelerating observers. The extension of special relativity to accelerated frames is based on the standard assumption of locality, that is the equivalence between an accelerated observer and an instantaneously comoving inertial observer. However the measurement of the frequency of a wave associated with a particle performed by an accelerating observer is an example of a nonlocal observation [25], complicating a lot the physical discussion.

Conclusion

The problem of assigning frequency spectra to various noises of classical and quantum origin is of the first importance. Here, I addressed this problem for a number of effects of fundamental character in theoretical physics, merely as an introduction to other people's works. In this 'noise' approach, the *thesis* is to consider whatever the effects as noises and see what information comes out from their noise spectra.

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References

1. Isaak Pomeranchuk said “The vacuum is filled with the most profound physical content !”, quoted by Okun L.B., “Vacua, Vacuum: the physics of nothing”, CERN/TH.7460/94 (1994).
2. See, e.g., Hu B.L. and Matacz A., *Phys. Rev. D* **49** (1994) 6612.
3. Nugayev R.M., *Commun. Math. Phys.* **111** (1987) 579; *Phys. Rev. D* **43** (1991) 1195.
4. Grishchuk L.P. and Sidorov Yu.V., *Phys. Rev. D* **42** (1990) 3413; See also, Rosu H., *Mod. Phys. Lett. A* **8** (1993) 3429.
5. Man’ko V.I. and Vilela Mendes R., “Non-commutative time-frequency tomography”, *IEEE Trans. Signal Proc.* (1999) (physics/9712022).
6. Totsuji H., *Phys. Rev. A* **46** (1992) 2106.
7. Bratteli O. and Robinson D.W., *Operator Algebras and Quantum Statistical Mechanics I, II* (Springer, New York, 1981).
8. Plunien G., Müller B., and Greiner W., *Phys. Rep.* **134** (1986) 87.
9. Ford L.H., *Phys. Rev. D* **38** (1988) 528; *Phys. Rev. A* **48** (1993) 2962.
10. Brevik I. and Nielsen H.B., *Phys. Rev. D* **41** (1990) 1185.
11. Cocho G. et al., *Int. J. Theor. Phys.* **28** (1989) 699.
12. Hacyan S. et al., *J. Phys. A* **23** (1990) 2401.
13. Hiller R., Puterman S.J., and Barber B.P., *Phys. Rev. Lett.* **69** (1992) 1182.
14. Schwinger J., *Proc. Natl. Acad. Sci. USA* **89** (1992) 1118, 4091.
15. Ooguri H., *Phys. Rev. D* **33** (1986) 3573.
16. Takagi S., *Prog. Theor. Phys.* **72** (1984) 505; *Prog. Theor. Phys.* **74** (1985) 142, 501; Stephens C.R., University of Maryland report, 1985 (unpublished).
17. Kandrup H.E., *Phys. Rev. D* **42** (1990) 3401.
18. Ibragimov N.Kh. and Oganesyanyan A.O., *Usp. Mat. Nauk* **46:3** (1991) 111 [*Russian Math. Surveys* **46:3** (1991) 137.]
19. Carrion P.M. and Foster D.J., *J. Acoust. Soc. Am.* **86** (1989) 1093; Beylkin G., *J. Math. Phys.* **26** (1985) 99.
20. Grishchuk L.P., Haus H.A., Bergman K., *Phys. Rev. D* **46** (1992) 1440.
21. Eberly J.H. and Wódkiewicz K., *J. Opt. Soc. Am.* **67** (1977) 1252.
22. Carmichael H.J., *J. Opt. Soc. Am. B* **4** (1987) 1588; See also: Gea-Banacloche J. et al., *Phys. Rev. A* **41** (1990) 369, 381; Yurke B., *Phys. Rev. A* **29** (1984) 408; Collet M.J. and Gardiner C.W., *ibid.* **30** (1984) 1386.
23. D’Ariano G.M., *Nuovo Cim. B* **107** (1992) 643.
24. Moreau W., *Am. J. Phys.* **60** (1992) 561.
25. Mashhoon B., *Phys. Lett. A* **122** (1987) 67; *Phys. Lett. A* **139** (1989) 103; See also, Gupta M.S., *Am. J. Phys.* **43** (1975) 1087.