

String Theory, Black Holes and Klein's Lemma¹

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Abstract

In this lecture, I will discuss the connection between entropy, black holes and the measurement problem of quantum mechanics. I will show how the ideas of Oskar Klein play an important role in a possible solution of the information paradox: black hole complementarity.

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1 Introduction

The problem that I want to discuss in this lecture has ancient roots. Through the ages it has taken different guises. A hundred years ago many asked the question: “What is the fundamental meaning of thermodynamics?” Today, as we will see, some wonder whether the unitarity of quantum mechanics is broken. Oskar Klein also had his thoughts on the subject, and this has been the motivation behind my lecture. His famous lemma, which I will come back to, contains ideas of extreme importance to my main theme.

A centerpiece in these discussions is the second law of thermodynamics. Although formulated in the 19th century, its equivalent, described in different words can be found much earlier. In fact, we need to go back to 100 B.C. when Lucretius, in his “De Rerum Natura” (“On the Nature of Things”), wrote:

*“Again, perceivest not
How stones are also conquered by Time?-
Not how the lofty towers ruin down,
And boulders crumble?- Not how shrines of gods
And idols crack outworn?- Nor how indeed
The holy Influence hath yet no power
There to postpone the Terminals of Fate,
Or headway make 'gainst Nature's fixed decrees?”*

Clearly, the idea that the decay of the worlds is a law of nature is not new. Even the idea that this change may be the cause of time can be found a long way back in history. Indeed, in 400 A.D. Augustine wrote in his “Confessions”:

“In you my mind , I say, I measure times. What I measure is the effect, itself present, which passing things create in you, and which remains when they have passed away. In measuring times, I do not measure the things which passed in the act of creating that effect, but rather the effect itself. Therefore either times are that effect, or I do not measure times at all.”

But it was not until Ludvig Boltzmann in the 19th century, that these ideas could be put on a more firm footing through the discovery of statistical

mechanics. The question of whether Boltzmann succeeded in giving a complete account of the second law is a subtle one. Furthermore, as we will see, the need for contemplating these questions can be found in many unexpected areas of physics. It relates to issues like the arrow of time, the measurement problem of quantum mechanics, and, which is the main topic of my talk, to the black-hole information paradox.

The precise question that I want to address is: “Is thermodynamics always ‘just’ statistical mechanics, or is it, sometimes, something more?”. This question was discussed intensely by Boltzmann and his opponents. An anti-atomist like Ernst Mach (perhaps better known for his thoughts on relativity) certainly thought there were something more to thermodynamics than statistical mechanics. It was considered by Mach and his followers to be a great loss if the second law of thermodynamics were to be explained by pure microphysics. Clearly they were mistaken in their views. Atoms do exist and thermodynamics is transcended by statistical mechanics. But is this always true? Could there be situations where, perhaps, there is no microscopical description?

Modern physics can supply a candidate; evaporating black holes. It was discovered in the seventies [1] that black holes radiate due to quantum effects. Hawking noted that a temperature,

$$T = \frac{\hbar c^3}{8\pi GM}, \tag{1}$$

could be associated with the radiation. Furthermore, the black holes could also be assigned an entropy [2] given by

$$S = \frac{4\pi kGM^2}{c\hbar} = \frac{kc^3 A}{4G\hbar}. \tag{2}$$

M denotes the mass of the black hole and A the area of the event horizon. It was soon realized that the phenomenon of Hawking radiation leads to a surprising paradox. Let me briefly explain why this is so.

When a black hole is formed, information about from what it was formed is hidden from view behind the event horizon. This is no mystery. Even if the information is not accessible to us, we can still assume that it is safely stored inside the black hole. Eventually, it seems, the black hole will completely evaporate due to the Hawking radiation. The only thing left will be the

radiation itself. Will this radiation carry a memory of what formed the black hole? According to Hawking's original calculations, the answer is no. If this is so, the evaporation process has destroyed information and produced "true" entropy. After all, since the black hole has disappeared, we can no longer claim that information, although inaccessible, still exists behind an event horizon. Formulated in the language of quantum mechanics it means that unitarity is broken. We lose the ability to predict, given an initial state, the precise final state. We can only give probabilities for various wave functions. In other words, the Schrödinger equation no longer works [3].

This is a serious problem. If the conclusion is correct the consequences are far reaching. Not only would non-unitary processes be important for exotic phenomena like macroscopic black hole evaporation, they would also have consequences in more or less daily life. Virtual black hole formation and evaporation would spoil unitarity even when there are no real black holes around. Clearly one must carefully explore possible loopholes in the argument.

I will consider three different ways of attacking the problem in the black hole context. I will give a personal review of the progress made and which are the implications, if any.

First I will consider the problem from a mathematical model point of view. That is, investigate the possibility of finding a model of an evaporating quantum black-hole where the consequences can be calculated! Clearly this will meet with only partial success, hence I will proceed by considering a less rigorous approach. One can try to *invent* scenarios which in a logical way provide some solution of the problem. Whether these scenarios are realized in a model will be left for future work. Finally, one can try to imagine possible uses for true entropy. Do we need it in any way? Does it help explaining other phenomena? Does it enrich physics? In many ways, the discussion today resembles the one held a hundred years ago, but we have yet to see the outcome this time around.

But first we must recall some basic thermodynamics. This will be necessary for the following discussion, and we will also see that Oskar Klein plays an important role.

2 Entropy, the Arrow of Time and All That

The main contribution of Boltzmann was his discovery of a microscopic definition of entropy. He found that

$$S = -k \int \rho \log \rho, \quad (3)$$

where ρ is the probability density on phase space. It follows from this definition that the entropy is larger if ρ is smeared out. Think of a gas. There are fewer ways to distribute the molecules in the corner of a container than in the whole container. Given a macroscopic state, the entropy is higher if there are more microscopic realizations.

But how can we understand the approach to thermal equilibrium? Why is the entropy always increasing? The microscopic definition of entropy given by Boltzmann does not, by itself, explain the second law of thermodynamics. On the contrary, the time evolution of ρ is that of an incompressible fluid, and the entropy remains constant. To improve on this situation, Boltzmann considered a gas of *colliding particles*. The collisions are described by his famous *Stoßzahlansatz*. Initially the positions and the velocities of the particles are assumed to be known with some accuracy. As the gas evolves and the particles collide some of the information about positions and velocities is transferred into information about correlations of the particles. Thanks to the Liouville theorem the phase-space volume must remain constant. But in practise we do not care about correlations when we study a gas of particles. If we ignore the correlations, the phase-space volume will grow and hence the entropy increase. More precisely, instead of using the full $6N$ dimensional phase space (called Γ -space by Boltzmann), where the system of N particles corresponds to a single point, he considered a 6 dimensional phase space with N points (called μ -space). While the entropy remains constant using ρ_Γ , it will increase if ρ_μ is used.

If one is interested in crystals and other regular systems, however, it is not appropriate to ignore correlations. In these cases we must follow Gibbs, and work with the Γ -space directly. Then, again, how can entropy increase? Luckily, even the entropy of Gibbs can be made to increase after some convenient coarse-graining. Let us assume that we can study the phase space only with some finite resolution, and that the entropy is computed after the phase space distribution has been smoothed. The smoothing will enlarge the

phase space volume, hence the entropy. The difference will be insignificant if there is no structure at the resolution scale. If there is a lot of structure on small scales however, the change will be considerable. Therefore, if a smooth phase space volume evolves through time into a complicated shape with a lot of structure on small scales it follows that entropy will increase. Entropy increase is in general related to information loss of the above type. Relevant information is transformed into irrelevant information as far as the macroscopic description is concerned.

Unfortunately the argument can be run backwards. A typical microscopic representative of a given macroscopic state will evolve towards larger entropy in either direction of time. We are forced to conclude that entropy will not only be larger in the future, it must also have been larger in the past. But this does not agree with experiment! We have therefore failed to derive the arrow of time. The solution of this paradox is to realize that the *real* microscopic representatives are *not* typical as far as backwards evolution is concerned. The world seems to be the product of very special initial conditions. On the other hand, entropy increase also means that there are no *final* conditions. These would appear as inexplicable conspiracies. Boltzmann were aware of these problems. He imagined that the initial conditions were a product of some very unlikely chance fluctuations. In the long run, there would be no preferred direction of time. A nice discussion of these and other related issues can be found in [9].

So far we have only discussed classical physics. How do we formulate these questions in a quantum framework? This question was addressed by Oskar Klein in his seminal paper: “*Zur quantenmechanischen begründung des zweiten Hauptsatzes der Wärmelehre*” [10]. He found a fundamentally new way for information to be lost hence entropy to increase, special to quantum mechanics.

This result is called Klein’s lemma. Let me illustrate his reasoning using a simple example. Consider the pure density matrix

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad (4)$$

with entropy $S = 0$. The precise form of the density matrix depends on the wave-function basis in which it is expressed. For instance, we might choose

another basis such that it becomes

$$\rho = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}. \quad (5)$$

Let us now ignore the off-diagonal elements in the new basis. We then get

$$\rho \rightarrow \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}. \quad (6)$$

If we now attempt to calculate the entropy we find $S = k \log 2 > 0$. If we can argue, in some specific basis, that the off diagonal terms are irrelevant we have obtained an increase of entropy. The main idea is that relevant information can, through some process akin to the Stoßzahlansatz of Boltzmann, be transferred into irrelevant quantum phase information. Averaging over the phases makes the off-diagonal terms vanish. As we will see later, this particular way for the entropy to increase is of fundamental importance. Hence, I will have reason to come back to Klein's lemma further on.

3 Ways to find out

3.1 Calculations

To fully understand the physics of quantum black holes, it is necessary to have a theory of quantum gravity. It has been incredibly difficult to find such a theory. The only candidate we have today where at least some calculations can be made is string theory. Clearly it would be desirable to take a theory of a four dimensional string (such exist) and investigate what happens when a black hole forms and evaporates. A full quantum calculation would presumably teach us a lot. Unfortunately this is beyond our present mathematical ability. Instead one is, in reality, forced to guess and approximate. And at the end of the day there are a lot of different opinions and wild disagreement. Perhaps one should consider a much simplified model which can be solved exactly? Such models can be found in two dimensions. Since gravity is renormalizable in two dimensions we can consider models both with and without strings. Lately, following the work [11], much attention has been given to the two dimensional space-time action

$$S = \frac{1}{4\pi} \int d^2x \sqrt{-g} e^{-2\phi} (R + 4(\nabla\phi)^2 + \lambda^2). \quad (7)$$

It can be derived from string theory by demanding world-sheet conformal invariance at the quantum level. Only solutions of the equations of motion derived from the above action obey this requirement.

The expression (7), and the presence of a dilaton, can be motivated also from a purely field theoretic point of view. Unlike other dimensions, the pure Einstein action is a topological invariant in two dimensions, the Euler characteristic. Hence we must complicate life a bit – the introduction of the dilaton is natural.

The simplest solution of the equations of motion is the linear dilaton vacuum given by

$$\begin{aligned} ds^2 &= -dt^2 + dx^2 \\ \phi &= -\lambda x. \end{aligned} \tag{8}$$

Another solution is the black hole [12] given by

$$\begin{aligned} -(1 - \frac{M}{\lambda} e^{-2\lambda x}) dt^2 + \frac{1}{1 - \frac{M}{\lambda} e^{-2\lambda x}} dx^2 \\ \phi = -\lambda x. \end{aligned} \tag{9}$$

The black hole has been studied extensively. It can be shown to Hawking radiate with a temperature $T = \frac{\lambda}{2\pi}$. This is most easily seen by going to Euclidean signature and making sure that there is no conical singularity on the horizon. In two dimensions it is also possible to use the well developed techniques of conformal field theory. In particular, it is easy to evaluate the expectation value of the energy momentum tensor. Naively it is infinite. However, in what we choose to call the vacuum it must, by definition, be zero. This is fixed by normal ordering where the infinity is subtracted. The subtraction depends on which coordinate system is used. In other words, different observers using different coordinate systems have different opinions of what the vacuum is. This is the cause of the Unruh effect [13] where accelerated observers see thermal radiation.

Let me now make a small digression to better understand the above and to hear what Klein might have to say. Hawking radiation from a black hole and the Unruh radiation are, presumably, equivalent phenomena related by the equivalence principle. A careless application of the equivalence principle would, however, imply the existence of radiation not only for an observer hovering above a black hole, but also above, let's say, a neutron star. This

is however not correct. Such an observer would not see any radiation. The vacuum state over a not fully collapsed object is different from that above a black hole due to different boundary conditions [14]. We understand from this that the vacuum is an object with much structure. Continuing the historical comparisons it is very similar to the concept of the ether of the previous century. Let me quote Klein [15]:

“... the practical situation of general relativity theory taken together with our present knowledge regarding matter and the vacuum would rather point to the assumption that the world as we know it is to be regarded as a weak excitation of the vacuum state, which state, in spite of its relative character, may be compared to the absolute space of Newton.”

Let me now come back to the two dimensional black hole! The energy momentum tensor will necessarily react back on the geometry. Among other things, the mass of the black hole must decrease! Some of these effects can be studied by adding a correction to the action and then solve the modified, semi-classical, equations of motion. Attempts have then been made to follow the evaporation process towards the end [16, 17, 18]. Unfortunately, no definite conclusions can be drawn. However, some of the more elaborate calculations [19, 20, 21], give remarkable hints indicative of the scenarios that I will discuss in the next section.

Models of strings moving in two dimensional space-times have been studied using other much more powerful methods; the matrix models. Matrix models represent the world-sheet of a string through triangulations. These triangulations are then given by Feynman-diagrams of matrix fields. For a review and list of references, see [22]. Remarkably, these techniques allow for the exact evaluation of correlation functions in the linear dilaton theory.

There have also been attempts to give a matrix model description of the black hole [23, 24, 25]. Clearly it would be wonderful to have such a tool! Even in these models exact correlation functions can be calculated. In principle these results should tell us a lot about the information paradox, if only we had the means to interpret the mathematics.

Let us now leave these mathematical considerations and move into much less restricted areas.

3.2 Speculations

I now want to describe the black-hole information paradox in more detail. We will see that the key-question is: “Do you lose your memory if you travel into a black hole?”. Clearly, this should not be the case according to the equivalence principle. For a large black hole the tidal forces are weak, even close to the horizon. There is nothing exceptional, locally, at the point of no return. A black-hole explorer could safely travel through the horizon and into the black hole. How would an observer remaining outside interpret this journey? If we believe in the unitarity of quantum mechanics and that all information about what went inside should be possible to recover just by looking closely (no true entropy!), we must conclude that information about the infalling observer (including quantum phases) are contained in the Hawking radiation.

But quantum mechanics strikes back! It is impossible, according to quantum mechanics, to make perfect copies and keep the original [26]. No unitary process can duplicate an arbitrary unknown initial state. For some thoughts on the subject see [27]. So, *if* the Hawking radiation really do contain all information about the matter that formed the black hole, then no information can pass into the black hole. You would in other words lose your memory if you attempted to travel into a black hole! Clearly we have run into a paradox.

Is there a way out? Recently a possible resolution of the paradox has emerged. This is “Black Hole Complementarity”. It says that:

“You shall not speak about the inside and the outside of a black hole at the same time. The notebooks of an inner and an outer observer can never be compared and hence need not agree.”

Black hole complementarity can be realized in two ways, each related to an uncertainty principle. The first one is *quantum mechanical complementarity*. It uses the fact that questions about physics far out from the black hole and questions about physics close to the horizon might be complementary in a quantum sense. Mathematically speaking, the corresponding operators do not commute [28]. This is easy to see. It is well known that the expectation values of the energy-momentum tensor is not very big at the horizon. This I discussed briefly in the previous subsection. Hence, it is concluded, the back reaction is not a serious problem and the semi-classical approach of Hawking

basically correct. However, this is the picture appropriate to an *infalling* observer. Let us now consider a measurement far out where a Hawking photon is detected. Once detected, it is justified to trace it back in time to see where it came from. Due to the large blueshift, it is found to have had extremely high energy

very close to the horizon. The back-reaction will be considerable and the standard picture breaks down. In the next section I will discuss a framework which is useful in this context.

The second possible reason for complementarity is *string theoretical complementarity*. It comes from the fact that a string looks larger when redshifted! [29, 30, 31] The reason is that the size of the string depends on the cut-off used for its vibrational modes. The higher the cut-off (i.e. the closer we examine the string) the larger it looks. Hence, an observer far out looking at a string near the horizon would, through the redshift, use a smaller cut-off (in the string frame) than an observer travelling along with the string. Since everything is supposed to be composed of strings the outer observer's view of the black hole would be blurred. He would not be able to make accurate statements about the space time structure near the horizon. Indeed, he might not be able to claim that there is matter falling into the black hole at all!

3.3 Dreams

Let me now discuss the problem from a different point of view. Is true entropy in any way needed? Does it fit naturally into some part of physics? Let us consider this question in the context of the measurement problem of quantum mechanics. This has been a very popular subject and a starting point for various claims about quantum mechanics being incomplete.

The heart of the mystery is the collapse of the wave-function. Let us look at this more closely. There seems to be two different time evolutions in quantum mechanics. The *unitary* evolution between measurements, and the *non-unitary* collapse at a measurement.

The first is described by the Schrödinger equation. The collapse, however, is not. The collapse proceeds in two steps. The first one involves the disappearance of superpositions. In other words, the off-diagonal elements

of the density matrix become zero:

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \rightarrow \begin{pmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{pmatrix}. \quad (10)$$

This means that the very quantum mechanical object on the left, describing a superposition, is transformed into a statistical sum of different possibilities with probabilities on the right. This is needed in order to make predictions. The world we directly observe is always classical. We can never even *imagine* anything else. When we read our measurement apparatus we never, directly, confront superpositions or amplitudes. No one has ever directly seen a superposition. We only have classical readings of measuring devices which we interpret. At most, we need to consider probabilities. The existence of a quantum world may only be inferred from our measurements. The only questions we can ask are classical, or, more precisely, based on classical logic. The second step of the collapse, then, is the realization of a particular alternative according to the probabilities supplied by the first step.

The standard Copenhagen interpretation of quantum mechanics postulates a classical world to which we can relate. It would be desirable to *derive* its existence through more basic means. So, what about (10) above? It is clearly impossible to describe such processes within unitary quantum mechanics. Also, the unitary evolution is certainly deterministic while the world is not. In fact, this is the main message of quantum mechanics. The collapse of the wave function seems to be an extremely important part of the quantum mechanical world. Hence it would be desirable to give it a more precise description. Is it a physical process? It is clearly tempting to imagine non-unitary phenomena responsible not only for the appearance of the classical world but, perhaps, also associated with black-holes.

However, I will argue that this need not be the case, and I'm sure Oskar Klein would have agreed. He would have considered, as I have explained earlier, the vanishing of the off-diagonal terms as being due to the interaction with the environment. In fact, (10) can be shown to be almost unavoidable in the physical world. For instance, it can be calculated that the moon is decohered and put to a classical well defined position through scattering of photons from the microwave background in just a tiny fraction of a second.

The best way to systematize how decoherence helps in producing the classical world is the *consistent history approach*. This is not the place to

give a detailed explanation, instead I refer to the many excellent review papers which exist [32].

A common complaint is that it is not correct to think of the mixed density matrix as just statistical, even though it *looks* that way. When the environment is included, we *still* have superpositions. It is our limited knowledge that gives the illusion of a mixed state. It is true that the first step alone provides us with the means of making statistical predictions, but the second step is needed to realize a particular outcome. After all, in the real world just *one* thing happens. Otherwise the statistical predictions would be meaningless. While the first step can be described, as we have seen, within unitary quantum mechanics, this seems impossible for the second step. From many points of view it is really this second step of the collapse that is the most mysterious.

However, it is important to realize that only the first step of the collapse *needs* a physical description. If the first step can be managed, then we can do without the Copenhagen way of arbitrarily separating the world into quantum and classical. The theory will do it for us. It will even be possible to discuss the quantum mechanics of the whole universe as a closed system. The understanding of decoherence achieved during the last few years solves this problem. What about the second step? In this case there is no hindrance in managing *without* a physical description. Quantum mechanics is a probabilistic theory. We should be satisfied when we have obtained the probabilities and not ask for more. One should not confuse the theoretical *representation* of the world with the physical world itself. Step one of the collapse is fully describable within the mathematical representation, the second step is not, or at least, need not be for consistency. The Copenhagen interpretation did not allow even a description of the first step. This was clearly a limitation. Excluding the *second* step is no limitation however, but rather a success and the way quantum mechanics avoids determinism. This point of view has been nicely explained in [33]. Hence there are no definite reasons to invoke non-unitarity. It has no role within our *representation* of the world.

It is interesting to speculate further on the laws of consistent histories and the emergence of a classical world. What if there were several *different* possibilities? The same wave function of the universe but with several parallel classical worlds? This has been discussed in [34]. Well, I can give a suggestion for why this could be important: “Black Hole Complementarity”.

According to the quantum mechanical version of black hole complementarity, two observers, one close to the horizon and the other one far away, will be making non-commuting observations. As the first observer is approaching the horizon, their respective set of consistent histories will gradually become mutually inconsistent. This will be true even at a macroscopic level, the two observers can not even agree upon the mere existence of the other observer! We find that the measurement problem of quantum mechanics, when properly understood, hints at a solution of the information paradox without non-unitarity rather than, as one naively would expect, gives support for extensions of quantum mechanics.

To conclude, it is hard to find a real need for true entropy. The measurement problem has received a, for practical purposes, satisfactory treatment. We have still to discover strong reasons to believe that quantum mechanics needs modification. But, perhaps, some other day we will. After all, black hole complementarity is just a scenario and we do not know to what extent it is realized in Nature.

4 Conclusions

I have considered three different approaches to the question of true entropy. The first two focused on black holes. Models of quantum black holes do exist, at least within string theory. However, the mathematics is formidable. It is very difficult to draw any definite conclusions.

Another approach is to forget about mathematical rigor and search for consistent scenarios only. Remarkably, as have been realized recently, there is an interesting possibility referred to as “Black Hole Complementarity”. It suggests that true entropy is avoidable in black hole evaporation.

Then I considered the question: what if there were true entropy, would it be of any use? To this end I considered the measurement problem of quantum mechanics but, alas, no application could be found. However, an interesting connection with black hole complementarity was noted.

To fully appreciate the black hole information paradox and its possible resolution through black hole complementarity, it is necessary to understand the measurement problem of quantum mechanics. The connection, as I have argued, is not that they imply non-unitary physics – they do not – instead they both illuminate the subtlety of quantum mechanics and the emergence

of the classical world.

Oskar Klein identified, more than 60 years ago, one of the deepest problems of physics. We are still waiting for the full answer.

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