

Influence of intrinsic decoherence on quantum teleportation via two-qubit Heisenberg XYZ chain

Zhenghong He^{a,*}, Zuhong Xiong^a, Yanli Zhang^b

^a School of Physics, Southwest University, Chongqing 400715, PR China

^b Department of Mathematic and Physics, Shijiazhuang Railway Institute, Hebei 050043, PR China

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Abstract

Quantum teleportation is investigated by using the entangled states of two-qubit Heisenberg XYZ chain in an external uniform magnetic field as resources with intrinsic decoherence taken into account. The influence of intrinsic decoherence on quantum teleportation varies in different initial systems.

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1. Introduction

One-dimensional Heisenberg models have been extensively studied in condensed-matter systems [1]. To realize quantum computation [2] and information processing [3] in these models, several proposals have recently been put forward by employing quantum entanglement. Being the heart of quantum computation and quantum information [4], quantum entanglement plays a crucial role in quantum teleportation. An entangled composite system gives rise to nonlocal correlation between its subsystems that does not exist classically. This nonlocal property enables the uses of local quantum operations and classical communication to teleport an unknown quantum state via a shared pair of entangled particles with fidelity better than any classical communication protocol [5–7]. Although there are already papers showing the results on entanglement teleportation using the thermally mixed entangled states of two-qubit Heisenberg chain as resources, the influence of intrinsic decoherence in these models is ignored [8–12]. Therefore, in this Letter we present our results on entanglement teleportation by using the entangled states of the two-qubit Heisenberg XYZ chain as resources with intrinsic decoherence taken into account.

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2. Two-qubit Heisenberg XYZ chain with intrinsic decoherence [13]

The Hamiltonian of N -qubit anisotropic XYZ Heisenberg model in an external uniform magnetic field B is given by

$$H = \frac{1}{2} \sum_{i=1}^N [J_x \sigma_i^x \sigma_{i+1}^x + J_y \sigma_i^y \sigma_{i+1}^y + J_z \sigma_i^z \sigma_{i+1}^z + B(\sigma_i^z + \sigma_{i+1}^z)], \quad (1)$$

where σ_i^α ($\alpha = x, y, z$) are the Pauli matrices of the i th qubit, J_α ($\alpha = x, y, z$) are the strengths of the Heisenberg interaction and the direction of B is along z axis. For the spin interaction, the Heisenberg chain is said to be antiferromagnetic for $J_\alpha > 0$ and ferromagnetic for $J_\alpha < 0$. When $J_x \neq J_y \neq J_z$, it is called XYZ chain, and the two-qubit Heisenberg XYZ chain in an ex-

* Corresponding author.

E-mail address: hezhenho@swu.edu.cn (Z. He).

ternal magnetic field is expressed as

$$H = J[(\sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+) + \Delta(\sigma_1^+ \sigma_2^+ + \sigma_1^- \sigma_2^-)] + J_z \sigma_1^z \sigma_2^z + B(\sigma_1^z + \sigma_2^z), \quad (2)$$

where $J = (J_x + J_y)$ and $\Delta = (J_x - J_y)/(J_x + J_y)$. The parameter Δ ($0 < \Delta < 1$) measures the anisotropy in the XY plane. $\sigma^\pm = (\sigma^x \pm i\sigma^y)/2$ are raising and lowering operators. If the intrinsic decoherence of the two-qubit Heisenberg XYZ chain is taken into account, the explicit expression of the density matrix of the states is shown as

$$\rho(t) = \sum_{mn} \exp\left[-\frac{\gamma t}{2}(E_m - E_n)^2 - i(E_m - E_n)t\right] \times \langle \phi_m | \rho(0) | \phi_n \rangle \langle \phi_m | \langle \phi_n |, \quad (3)$$

where $E_{m,n}$ and $\phi_{m,n}$ are the eigenvalues and the corresponding eigenvectors of Hamiltonian respectively, and γ is the phase decoherence rate which means intrinsic decoherence. The time evolution of the density matrices of the system in the different initial states is as follows:

(a) Assuming that the system is initially in an entangled state $\rho(0) = |\psi(0)\rangle\langle\psi(0)|$ ($|\psi(0)\rangle = (a|00\rangle + b|11\rangle$), where a and b are parameters, the time evolution of the density matrices of system in the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ will be as follows:

$$\begin{aligned} \rho(t) &= (\sigma - \omega)|00\rangle\langle 00| + \Omega|00\rangle\langle 11| + \Omega^*|11\rangle\langle 00| \\ &\quad + (\sigma + \omega)|11\rangle\langle 11|, \\ \sigma &= \frac{1}{2}a^2 \sin^2(2\theta) - \frac{1}{2}ab \sin(4\theta) + b^2(\sin^4\theta + \cos^4\theta), \\ \omega &= \sin(2\theta) \left[ab \cos(2\theta) + \frac{1}{2}(b^2 - a^2) \sin(2\theta) \right] \\ &\quad \times \exp(-2G^2\gamma) \cos(2Gt), \\ \Omega &= \frac{1}{2}[\sin^2(2\theta) + \cos^2(2\theta) \cos(2Gt) \exp(-2G^2\gamma) \\ &\quad - i \cos(2\theta) \sin(2Gt) \exp(-2G^2\gamma t)]. \end{aligned} \quad (4)$$

(b) Assuming that the qubits 1 and 2 are both initially in the spin-down states, i.e., the system is initially in the unentangled state $\rho(0) = |00\rangle\langle 00|$, where $|0\rangle$ and $|1\rangle$ denote the states of spin-down and spin-up respectively, the density matrices of the system in the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ will be expressed as follows:

$$\begin{aligned} \rho(t) &= (1-u)|00\rangle\langle 00| + v|00\rangle\langle 11| + v^*|00\rangle\langle 11| + u|11\rangle\langle 11|, \\ u &= \frac{1}{2} \sin^2(2\theta) [1 - \cos(2Gt) \exp(-2G^2\gamma t)], \\ v &= \frac{1}{4} \sin^2(4\theta) + \frac{1}{2} \sin(2\theta) \exp(-2G^2\gamma t) \\ &\quad \times [\sin^2(\theta) \exp(2Gti) - \cos^2(\theta) \exp(-2Gti)], \\ \sin\theta &= \frac{G + 2B}{\sqrt{2G(G + 2B)}}, \quad \cos\theta = \frac{J\Delta}{\sqrt{2G(G + 2B)}}, \\ G &= \sqrt{4B^2 + (J\Delta)^2}. \end{aligned} \quad (5)$$

3. The teleportation fidelity of the fully entangled fraction

Standard teleportation P_0 with an arbitrary entangled mixed state resource ρ_{AB} is equivalent to a generalized depolarizing channel $\Lambda_{P_0}^{\rho_{AB}}$ with probabilities given by the maximally entangled components of the resource [5–7]. We consider a qubit in the arbitrary unknown pure state as an input state, that is

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi} \sin\frac{\theta}{2}|1\rangle \quad (0 \leq \theta \leq \pi; 0 \leq \phi \leq 2\pi). \quad (6)$$

The output state is then given by [5–7]

$$\Lambda_P^{\rho(t)}(|\psi\rangle\langle\psi|) = \sum_{j=0}^3 \text{tr}(E^j \rho(t)) \sigma^j |\psi\rangle\langle\psi| \sigma^j, \quad (7)$$

$E^0 = |\psi^-\rangle\langle\psi^-|$, $E^1 = |\phi^-\rangle\langle\phi^-|$, $E^2 = |\phi^+\rangle\langle\phi^+|$, $E^3 = |\psi^+\rangle\langle\psi^+|$, where ψ^\pm , ϕ^\pm are Bell states, and in the standard teleportation protocol P_0 , the maximal teleportation fidelity $\Phi_{\max}[\Lambda_{P_0}^{\rho(t)}]$ achievable is given by

$$\Phi_{\max}[\Lambda_{P_0}^{\rho(t)}] = \frac{2F[\rho(t)] + 1}{3}, \quad (8)$$

where the fully entangled fraction

$$F[\rho(t)] \equiv \max_{i=0,1,2,3} \{ \langle \psi_{\text{Bell}}^i | \rho(t) | \psi_{\text{Bell}}^i \rangle \} \quad (9)$$

when the system is initially in an entangled state $\rho(0) = |\psi(0)\rangle\langle\psi(0)|$, $|\psi(0)\rangle = (a|00\rangle + b|11\rangle)$, let $a = 2\sqrt{2}/3$, $b = 1/3$, we can obtain the fidelity of the fully entangled fraction of the two-qubit Heisenberg XYZ chain as

$$\begin{aligned} F_{1f}[\rho(t)] &= \max\{\beta_0, \beta_1, \beta_2, \beta_3\}, \\ \beta_0 = \beta_3 &= 0, \quad \beta_1 = \sigma - \frac{1}{2}\Omega^* - \frac{1}{2}\Omega, \\ \beta_2 &= \sigma + \frac{1}{2}\Omega^* + \frac{1}{2}\Omega, \end{aligned} \quad (10)$$

$$F_{1f}[\rho(t)] = \begin{cases} \beta_1, & \Omega + \Omega^* < 0, \\ \beta_2, & \Omega + \Omega^* > 0. \end{cases} \quad (11)$$

The calculation results are shown in Figs. 1 and 2, in which the fidelity of the fully entangled fraction F_{1f} are plotted at the two values of the phase decoherence rate γ . One can find that the intrinsic decoherence leads to a suppression of F_{1f} . F_{1f} is the larger one of β_1 and β_2 . As time t approaches infinite, F_{1f} will reach a stable value. The larger the values of phase decoherence rate γ are, the more quickly β_1 and β_2 collapse in short time. Fig. 2 also shows that when the magnet field B approaches 0, F_{1f} are independent of phase decoherence rate γ , i.e., the value of F_{1f} will be 1.0 all the time. One can also find that β_1 and β_2 oscillate with time t and become F_{1f} alternatively. F_{1f} are plotted at a given time in Fig. 3, through which one can obviously find that F_{1f} rapidly decrease to 0.13 as the magnetic field B increases, but it can also be enhanced and weakened in different magnetic field regions, for example, $F_{1f} \sim 0.52$ when $B \sim 0.06$, and $F_{1f} \sim 0.64$ when $B \sim 0.08$.

When the system is initially in the unentangled state we can obtain the fidelity of the fully entangled fraction of the two-

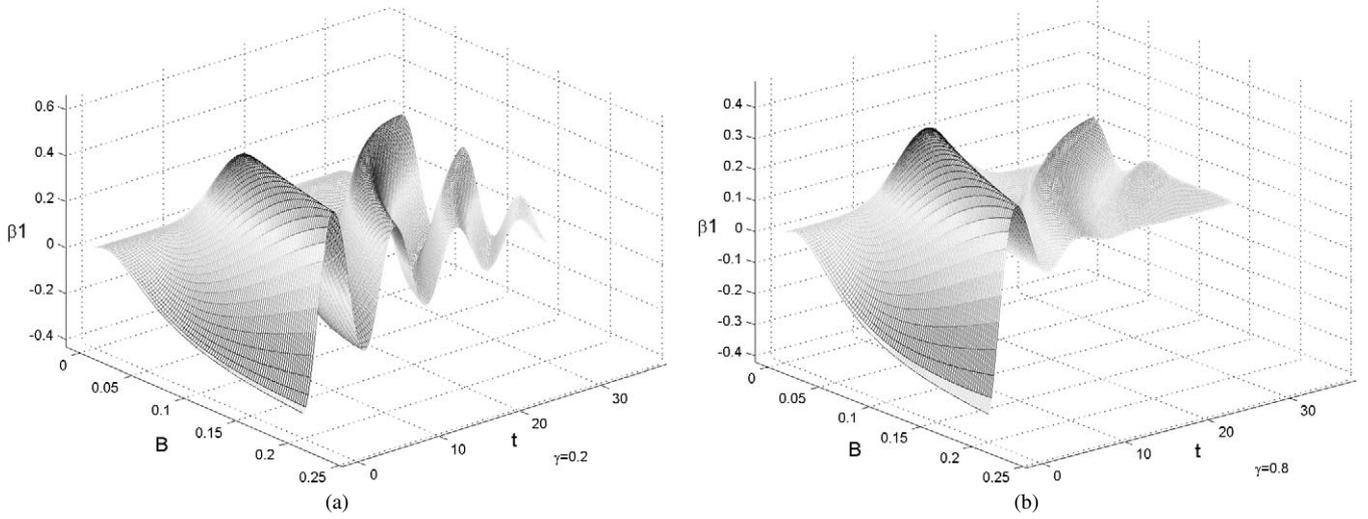


Fig. 1. β_1 versus the magnetic field B and time t . The strength of Heisenberg interaction $J = 1$. The parameter $\Delta = 0.1$, (a) $\gamma = 0.2$, (b) $\gamma = 0.8$.

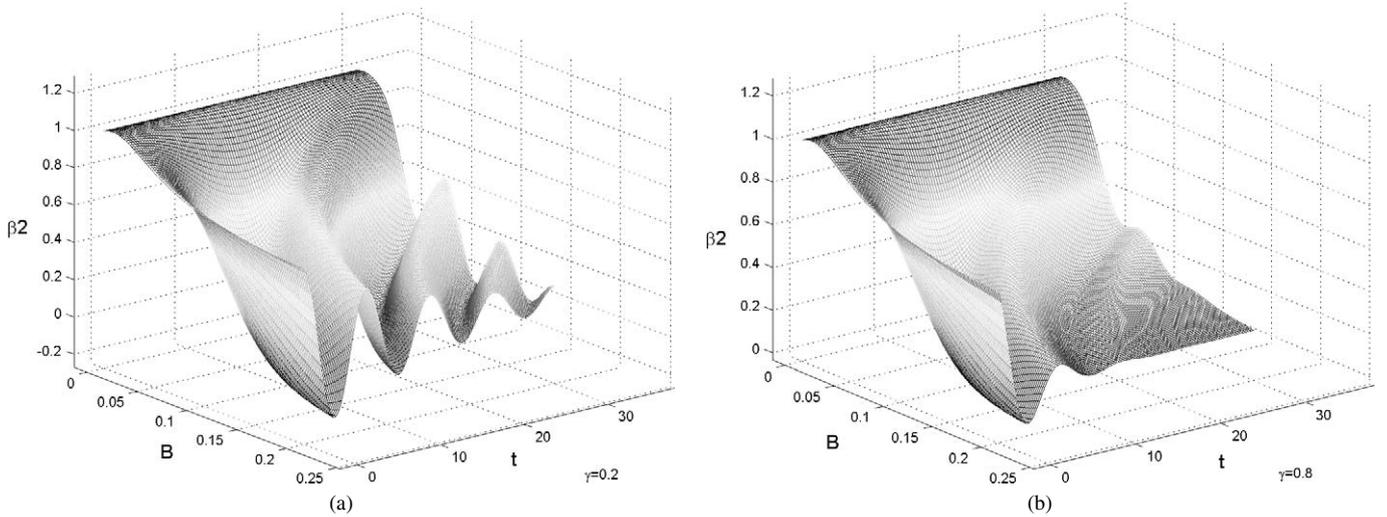


Fig. 2. β_2 versus the magnetic field B and time t . The strength of Heisenberg interaction $J = 1$. The parameter $\Delta = 0.1$, (a) $\gamma = 0.2$, (b) $\gamma = 0.8$.

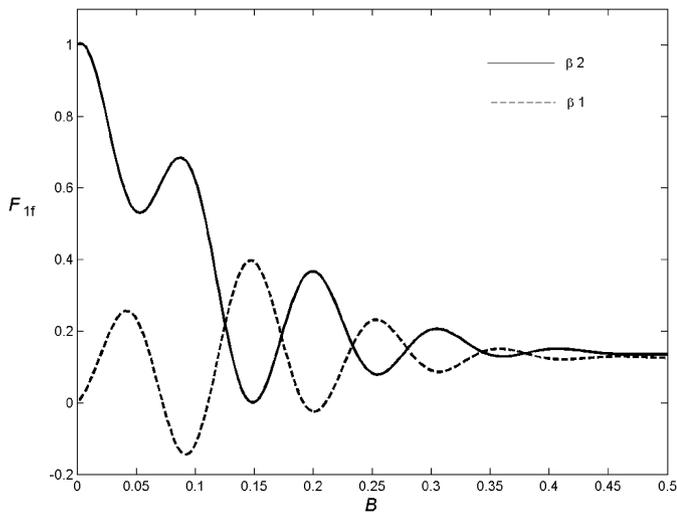


Fig. 3. The fidelity of the fully entangled fraction F_{1f} versus magnetic field B at a given time $t = 15$, $\gamma = 0.2$, the parameter $\Delta = 0.1$, The strength of Heisenberg interaction $J = 1$.

qubit Heisenberg XYZ chain as

$$F_{2f}[\rho(t)] = \max\{\beta_4, \beta_5, \beta_6, \beta_7\},$$

$$\beta_4 = \beta_7 = 0, \quad \beta_5 = \frac{1}{2} - \frac{1}{2}v^* - \frac{1}{2}v,$$

$$\beta_6 = \frac{1}{2} + \frac{1}{2}v^* + \frac{1}{2}v, \tag{12}$$

$$F_{2f}[\rho(t)] = \begin{cases} \beta_6, & v + v^* > 0, \\ \beta_5, & v + v^* < 0. \end{cases} \tag{13}$$

The fidelity of the fully entangled fraction F_{2f} are plotted at the two values of the phase decoherence rate γ in Figs. 4 and 5. F_{2f} are the larger one of β_5 and β_6 . One can find when magnetic field B is less than 0.2, β_5 and β_6 will oscillate with time t , while β_5 and β_6 will decay more quickly with the larger phase decoherence rate γ from Fig. 5. If the magnetic field B is properly increased, the corresponding values of β_5 in Fig. 4(b) will be almost equal to those in Fig. 4(a). When the magnetic field B is larger than 0.2, β_5 is independent of phase decoherence rate γ . This implies that magnetic field B can moderate the

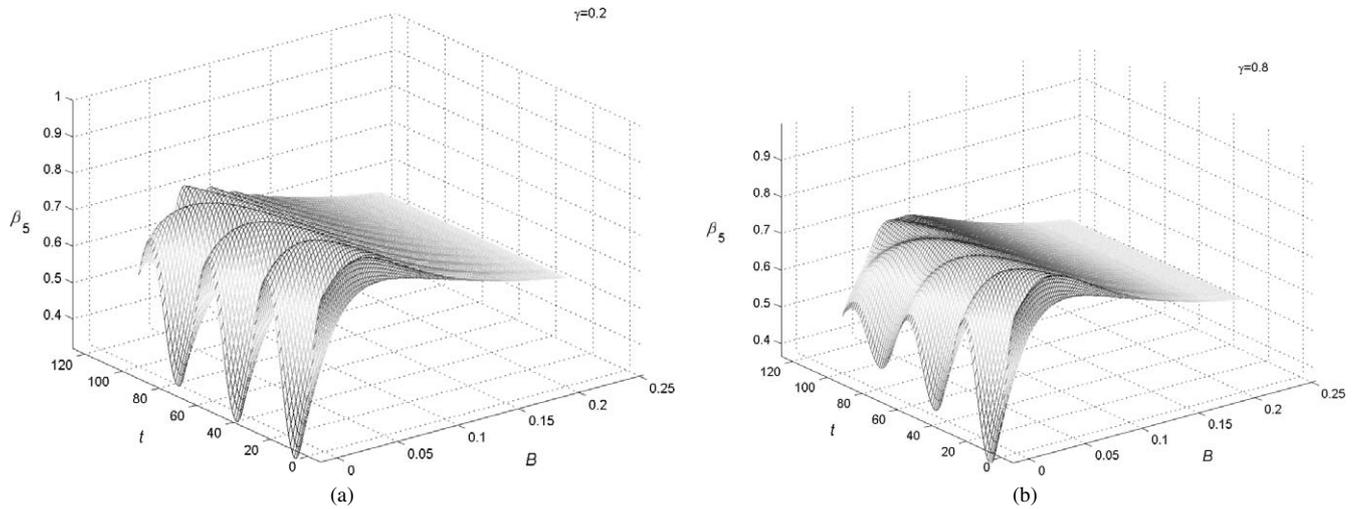


Fig. 4. β_5 versus the magnetic field B and time t . The strength of Heisenberg interaction $J = 1$. The parameter $\Delta = 0.1$, (a) $\gamma = 0.2$, (b) $\gamma = 0.8$.

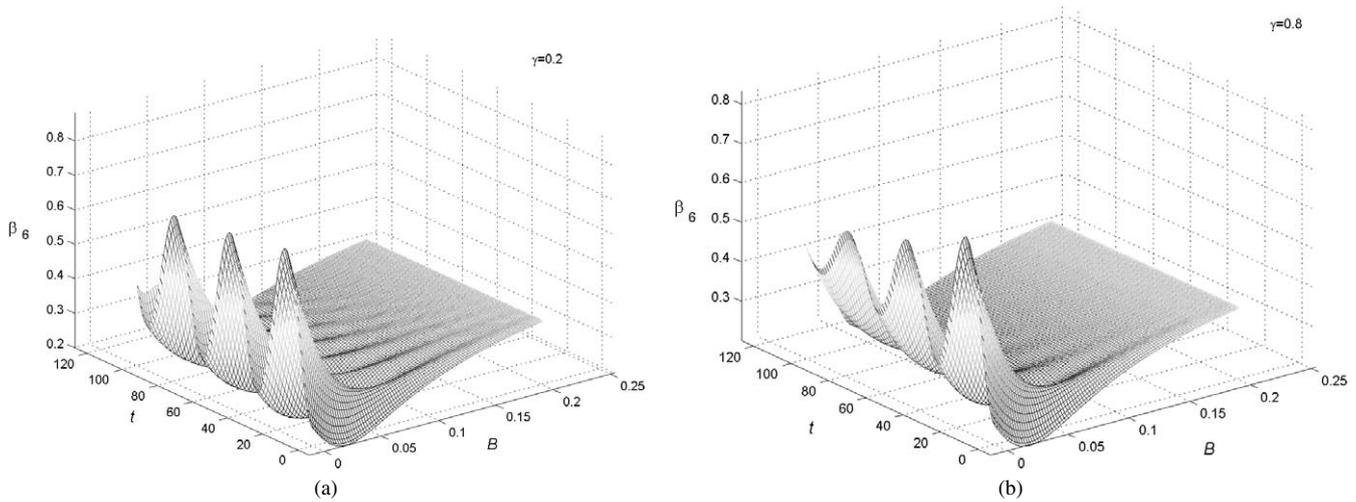


Fig. 5. β_6 versus the magnetic field B and time t . The strength of Heisenberg interaction $J = 1$. The parameter $\Delta = 0.1$, (a) $\gamma = 0.2$, (b) $\gamma = 0.8$.

destruction of intrinsic decoherence. F_{2f} are plotted at a given time in Fig. 6. One can find $F_{2f} = \beta_6$, when B approaches 0; and $F_{2f} = \beta_5$, when $B > 0.02$. One can get the maximum value of F_{2f} . When $B = 0.08$, the maximum $F_{2f} \sim 0.75$. After this peak, F_{2f} will decay monotonously and approach 0.53.

It is known that a bipartite entangled state ρ that has the fidelity of the fully entangled fraction $F_f(\rho) > \frac{1}{2}$ is to be useful for quantum teleportation [5–7]. However, according to our study, the effects of intrinsic decoherence on the fidelity of the fully entangled fraction F_{1f} and F_{2f} are different. But we can adjust the magnetic field B to obtain the ideal values of F_{1f} and F_{2f} in the different initial state. When the initial state is the entangled one, it is possible for the fully entangled fraction $F_{1f}(\rho) > \frac{1}{2}$ during some periods of time and some magnetic field regions from Figs. 1–3. When the initial state is the unentangled one, it is also possible for the fidelity of the fully entangled fraction $F_{2f}(\rho) > \frac{1}{2}$ from Figs. 4–6. Therefore, the system of the two-qubit Heisenberg XYZ chain in an external uniform magnetic field with intrinsic decoherence taken into account is useful for quantum teleportation.

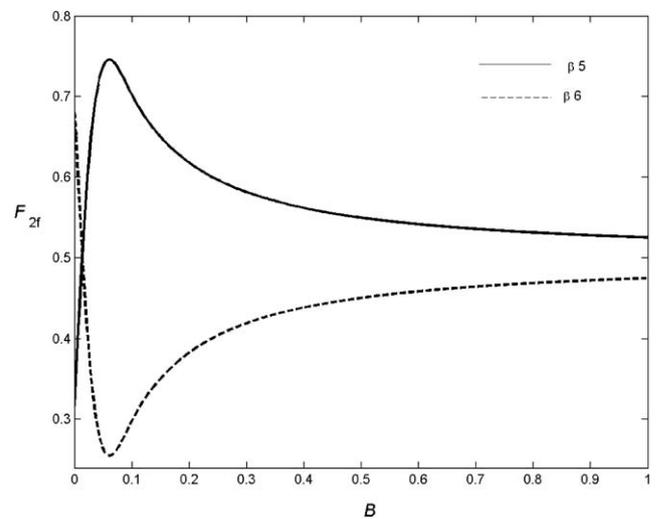


Fig. 6. The fidelity of the fully entangled fraction F_{2f} versus magnetic field B at a given time $t = 15$, $\gamma = 0.2$. The parameter $\Delta = 0.1$, the strength of Heisenberg interaction $J = 1$.

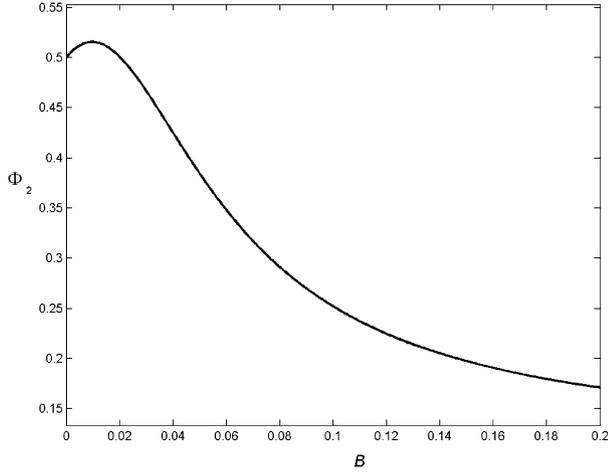


Fig. 7. The average fidelity for the two-qubit Heisenberg chain initially in the entangled state versus magnetic field B , $B \sim 0.018$, $\Phi_2 \sim 0.52$.

4. Average fidelity of quantum teleportation

As the reliability of teleportation is a criterion to judge the quality of the entangled state, we adopt quantitative measures to find the average fidelity of quantum teleportation [5–7], that is, to average the fidelity of all input states.

When the system is initially in the entangled state $\rho(0) = |\psi(0)\rangle\langle\psi(0)|$, $|\psi(0)\rangle = (a|00\rangle + b|11\rangle)$, $a = 2\sqrt{2}/3$, $b = 1/3$, the average fidelity of quantum teleportation is

$$\begin{aligned} \Phi_1[\Lambda_{P_0}^{\rho(t)}] &\equiv \int d\psi \langle\psi| \Lambda_{P_0}^{\rho(t)} (|\psi\rangle\langle\psi|) |\psi\rangle \\ &= \frac{\int_0^\pi \int_0^{2\pi} \Lambda_P^{\rho(t)} (|\psi\rangle\langle\psi|) \sin\theta d\theta d\phi}{4\pi} \\ &= \frac{1}{3}a^2 \sin^2(2\theta) - \frac{1}{3}ab \sin(4\theta) + \frac{2}{3}b^2(\sin^4\theta + \cos^4\theta), \\ a &= 2\sqrt{2}/3, \quad b = 1/3. \end{aligned} \quad (14)$$

When the system is initially in the unentangled state $\rho(0) = |00\rangle\langle 00|$, the average fidelity of quantum teleportation is

$$\begin{aligned} \Phi_2[\Lambda_{P_0}^{\rho(t)}] &\equiv \int d\psi \langle\psi| \Lambda_{P_0}^{\rho(t)} (|\psi\rangle\langle\psi|) |\psi\rangle \\ &= \frac{\int_0^\pi \int_0^{2\pi} \Lambda_P^{\rho(t)} (|\psi\rangle\langle\psi|) \sin\theta d\theta d\phi}{4\pi} \\ &= \frac{1}{3}. \end{aligned} \quad (15)$$

Since the effects of intrinsic decoherence are taken into consideration, the average teleportation fidelity is kept at a relatively low value from Eqs. (14), (15) and Fig. 7.

5. Conclusion

In this Letter the quantum teleportation is investigated by using the entangled states of the two-qubit Heisenberg XYZ chain as resources with intrinsic decoherence taken into account. The results show that this system is useful for quantum teleportation in the case of the entangled initial state and unentangled ones, we can adjust the magnetic field B to reduce the different effects of intrinsic decoherence, and accordingly obtain the ideal fidelity of fully entangled fraction. Whether other means may also be employed to achieve the same goal will be our next research task. Besides, the average fidelity of quantum teleportation in different initial states is also investigated.

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References

- [1] M.C. Arnesen, S. Bose, V. Vedral, Phys. Rev. Lett. 87 (2001) 017901.
- [2] D. Loss, D.P. DiVincenzo, Phys. Rev. A 57 (1998) 120.
- [3] A. Imamoglu, et al., Phys. Rev. Lett. 83 (1999) 4204.
- [4] M.A. Nielsen, I.L. Chuang, Quantum Computation and Quantum Information, Cambridge Univ. Press, Cambridge, 2000.
- [5] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W.K. Wootters, Phys. Rev. Lett. 70 (1993) 1895.
- [6] S. Popescu, Phys. Rev. Lett. 72 (1994) 797.
- [7] M. Horodecki, P. Horodecki, R. Horodecki, Phys. Rev. A 60 (1999) 1888.
- [8] D. Boschi, et al., Phys. Rev. Lett. 80 (1998) 1121.
- [9] Y. Ye, Phys. Lett. A 309 (2003).
- [10] Y. Ye, Phys. Rev. A 66 (2002) 062312.
- [11] Y. Ye, Phys. Rev. A 68 (2003) 022316.
- [12] Y. Ye, et al., J. Phys. A: Math. Gen. 38 (2005).
- [13] B. Shao, T.-H. Zeng, J. Zou, Commun. Theor. Phys. 44 (2005).