

On Quantum Computing with Macroscopic Josephson Qubits

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Abstract — The achievements of quantum computation theory, e.g. Shor’s factoring algorithm, motivate efforts to realize quantum computers. Among systems proposed for quantum computing macroscopic superconducting circuits of Josephson junctions appear promising for integration in electronic circuits and large-scale applications. Recently, a superconducting tunnel junction circuit was designed and a sufficiently high quality factor of quantum coherence has been obtained. This indicates that decoherence need not be among the obstacles in building quantum computers with macroscopic Josephson circuits. In this paper we present the setup of some elementary quantum logic with macroscopic Josephson qubits, strengthened by some simulation work, and then study the feasibility of implementing Shor’s quantum factoring algorithm on them. It is shown that it would be eventually possible to build a 2-Dimensional Josephson qubit array, possibly accompanied by classical computing components, capable of performing useful quantum computations.

I. INTRODUCTION

Current computer systems are based on Boolean logic, which operates on two distinguishable states – False or True, or simply 0 or 1. As the size of microelectronics shrinks, quantum physics becomes increasingly important. Quantum mechanics tells us that if a bit can be in one or the other of two distinguishable states, then it can also exist in coherent superpositions of these states. A single bit of information by such a two-state quantum system is known as a qubit or quantum bit. A qubit exists as a superposition of two distinct states $|0\rangle$ and $|1\rangle$. With two or more qubits, we can consider quantum logical gate operations, which are the building blocks of a quantum computer. Because of the quantum mechanical superpositions of qubits, a quantum computer would have more freedom in computation than classical computers using conventional Boolean operations.

In 1994 Shor discovered a quantum algorithm for factorization that is exponentially faster than any known classical algorithm [1]. It was then realized that quantum computers could perform certain hard tasks that are intractable for any classical computers. The achievements of quantum computation theory motivate efforts to realize quantum computers. Various physical systems were proposed for quantum information processing. Among those macroscopic superconducting circuits of Josephson

junctions appear promising for integration in electronic circuits and large-scale applications. The quantum superposition of two macroscopic persistent-current states on superconducting Josephson circuits have been detected and measured [2]. The proposed Josephson Persistent-Current (PC) qubit, which consists of a micrometer-sized loop with three Josephson junctions, is therefore possible to be brought into quantum coherence to perform quantum computing.

The macroscopic qubits with Josephson junctions can be produced by modern lithography. They can also be easily initiated, precisely manipulated, individually addressed by conventional techniques. However, macroscopic quantum systems may easily suffer from the problem of decoherence, which destroys quantum coherent superpositions by any irreversible interaction with environments. Recently, a superconducting tunnel junction circuit has been designed and measured, and a sufficiently high quality factor of quantum coherence has been obtained [3]. This result shows that decoherence need not be among the obstacles in building quantum computers with macroscopic Josephson circuits [4].

Any quantum computation can be defined as a unitary evolution of quantum network that takes its initial state into some final state [5]. A quantum network is a computing device consisting of quantum logic gates, and each quantum logic gate is a unitary operation on one or more qubits. Quantum computations are then always accomplished by building up quantum logic circuits out of many quantum logic gates. Since a unitary transformation is reversible, quantum gates need to be reversible and cannot be directly deduced from their classical counterparts. In this paper, we discuss the setup of elementary quantum gates with macroscopic Josephson qubits, and study the feasibility of implementing Shor’s quantum factoring algorithm on them.

The structure of the paper is as follows. In section II we present a close look at a Josephson PC qubit and focus on the properties we are interested in. In section III the setup of some elementary quantum logic is discussed and some simulation work is presented. In section IV we present the implementation of Shor’s factoring algorithm on the arrays of superconducting Josephson qubits. Section V concludes the paper.

II. A JOSEPHSON PERSISTENT-CURRENT (PC) QUBIT

A Josephson PC qubit [6] in principle consists of a loop with three Josephson junctions in series that encloses a magnetic flux Φ driven by an external magnet (Fig. 1). In particular when the enclosed magnetic flux is close to half a superconducting flux quantum $\Phi_0 (= h/2e)$, where h is Planck's constant), the loop may have multiple stable persistent current states, and this system behaves as a particle in a double-well potential, where the classical states in each well correspond to persistent currents of opposite sign. The two classical states are coupled via quantum tunneling through the barrier between the wells, and the loop is a macroscopic quantum two level system. This system has two stable states $|0\rangle$ and $|1\rangle$ with opposite circulating persistent currents [2].

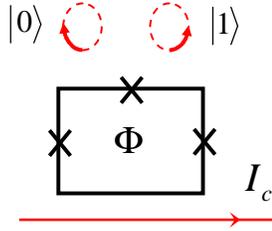


Fig. 1 A Josephson PC qubit

The qubit is operated by resonant microwave modulation of the enclosed magnetic flux by a superconducting control line. The superconducting control line is on top of the qubit, separated by a thin insulator. Measurement can be made with superconducting magnetometers [superconducting quantum interference devices (SQUIDs)]. Two or more qubits can be coupled by means of the flux that the circulating persistent current generates [6].

The superconducting quantum circuit proposed in [3] consists of a Cooper pair box island delimited by two small Josephson junctions in a superconducting loop including a third, much larger Josephson junction. Arbitrary quantum states of the circuit are manipulated by applied microwave pulses and read out by a readout circuit.

III. ELEMENTARY QUANTUM LOGIC

A. A Set of Universal Quantum Gates

As in the case of classical computers, certain sets of quantum gates are universal in the sense of that any quantum computation can be performed by wiring members of them together. We now study such a set of

universal quantum gates, single qubit rotation and the quantum controlled-NOT gate, or CNOT.

An arbitrary single qubit rotation can be written as $e^{-i\sigma} = \cos t\sigma - i\sin t\sigma$ for some Pauli matrix $\sigma = a\sigma_x + b\sigma_y + c\sigma_z$, where $a^2 + b^2 + c^2 = 1$. The PC qubit can be rotated precisely by applying a magnetic pulse for certain duration (Fig. 2, the unitary matrix describing the unitary transformation of states is also shown).

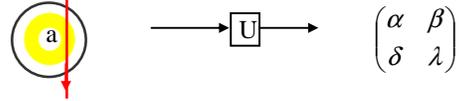


Fig. 2 Single qubit rotation

A controlled NOT is a two-qubit quantum logic gate that flips the value of the second qubit if the value of the first qubit is 1 (Fig. 3). That is, it takes $|00\rangle \rightarrow |00\rangle$, $|01\rangle \rightarrow |01\rangle$, $|10\rangle \rightarrow |11\rangle$, $|11\rangle \rightarrow |10\rangle$. Two qubits can be readily coupled inductively. By exploring the magnetic interference of two qubits, a so-called controlled rotation gate can be made. Then, given a π pulse, a controlled NOT gate can be drawn from the controlled rotation gate. A controlled NOT can be combined with single qubit rotations to give arbitrary quantum logic operations.

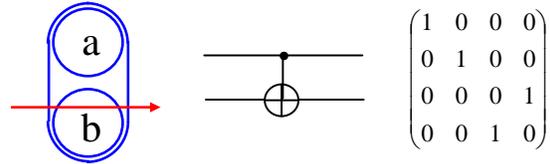


Fig. 3 Controlled NOT (CNOT)

B. Simulation

Quantum-effect devices will probably have more variability in their characteristics than the earlier microstructure counterparts. Therefore the topology of the Josephson PC circuits has to be delicately designed to make them robust in practical implementation. We started simulation with the two qubit coupling. The work was basically done with a package, FastHenry, which computes the frequency dependent self and mutual inductances and resistances between conductors of complex shape. Given the inductances, the coefficients of effective Hamiltonian, which exhibit the interaction between qubits, were approximately calculated with the linear approximation to the persistent currents applied [7].

Two different schemes for two qubit coupling were simulated respectively under certain physical parameters.

TABLE 1. MUTUAL (SELF) INDUCTANCE (PH) IN THE TWO QUBIT COUPLING CIRCUIT

	Transporter	Loop 1	Loop 2	Control line 1	Control line 2
Transporter	30	9.7	9.7	3.1	3.1
Loop 1	9.7	11	-0.069	2.3	0.028
Loop 2	9.7	-0.070	11	0.028	2.3
Control line 1	3.1	2.3	0.026	4.3	0.28
Control line 2	3.1	0.026	2.3	0.28	4.3

The first was proposed by coupling two loops directly through magnetic interference. In the second scheme, the two loops were coupled through a superconductive flux transporter, which was placed on top of them and insulated by a thin layer (Fig. 4). The inductive influence from control current lines, which is on top of the transporter, is also considered. In order to manipulate a PC bit independently, the control line should be strongly coupled with one while weakly coupled with the other. The inductances obtained from the latter simulation are shown in Table 1. The mutual inductance between a PC loop and the control line is 2.3pH, which is about two orders larger than that between the control line and the other loop, 0.026~0.028pH. The mutual inductance between the transporter and the PC loops is 9.7pH, while the mutual inductance between the two loops is about 0.069pH. With the parameters suggested in [2] and the inductance matrix obtained from the simulation, the coefficient of the effective Hamiltonian H , which stands for the interaction between the two PC loops, was calculated to be about 0.08 (in units of Josephson energy E_J), which was 5~6 times larger than that without the transporter. Thus it is shown that the interaction between the PC loops was stronger and they are better coupled to each other with the facilitation of transporter.

With the controlled operation it becomes possible to implement a simple Deutsch-Jozsa (D-J) quantum

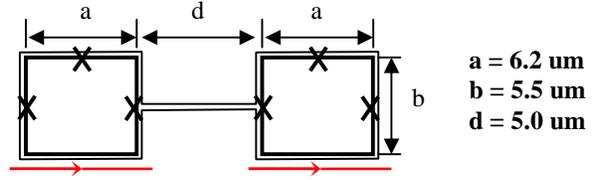


Fig.4 Two qubit coupling

algorithm, which has been demonstrated in NMR quantum computer [8]. The algorithm illustrates that quantum computer can perform useful computation in less steps than any possible classical computers, and its simplest case can be basically implemented on a controlled NOT quantum gate.

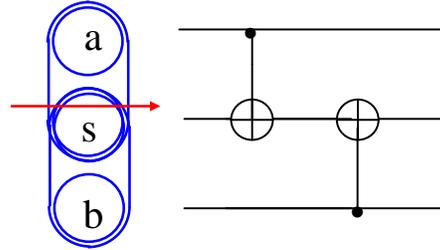


Fig. 5 Quantum sum

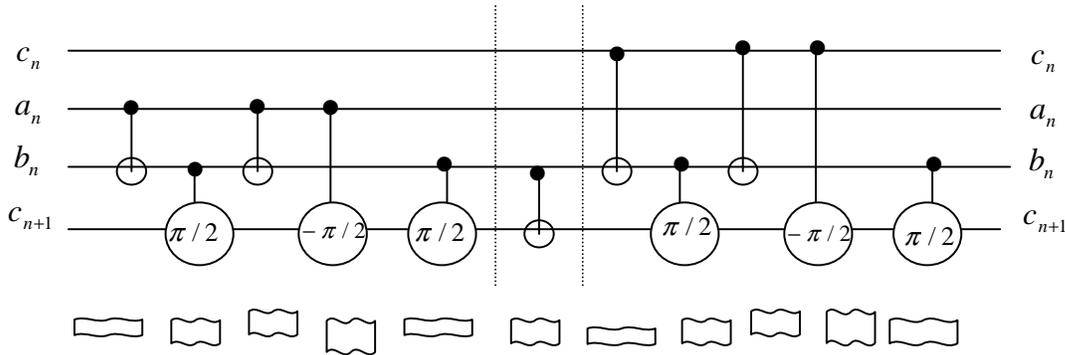


Fig. 6 Quantum carry with RF pulses

C. Quantum Sum and Carry

The addition of two quantum registers $|a\rangle$ and $|b\rangle$ can be written as $|a, b\rangle \rightarrow |a, a+b\rangle$. As the input (a, b) can be reconstructed out of the output $(a, a+b)$ and there is no loss of information in the computation, the calculation can be implemented reversibly. The sum operation can be implemented with two CNOT gates (Fig. 5, time goes from left to right), while the carry has to be obtained in a more complicated way [9] (Fig. 6). A sequence of RF pulses with different frequencies and duration time are injected into the quantum network during operation. During a pulse the involved qubits switch their states.

IV. SHOR'S ALGORITHM

Although there is still a long way to go before quantum computers come into practice, people believe that it can solve problems that are intractable for present classical computers, for example, the prime factorization of large numbers. The best factoring algorithm available for classical computation, for factoring an integer n , is an exponential-time algorithm, which is inefficiently computable with the growing of integer n . Shor discovered a quantum algorithm for factoring, which is polynomial in n , along with some polynomial (in $\log n$) processing time on a classical computer [5].

To factor an odd number n , Shor's algorithm first finds the least integer r such that $x^r \equiv 1 \pmod{n}$, i.e. the period of $f(A)=x^A \pmod{n}$ for A from 0 to $n-1$, where x is random, with $x < n$ and $\gcd(x, n)=1$. Then it finds factors of n by calculating $\gcd(x^{r/2}-1, n)$ and $\gcd(x^{r/2}+1, n)$ if r is even and $x^{r/2} \not\equiv \pm 1 \pmod{n}$, otherwise, it repeats the algorithm. Here, $\gcd(a, b)$ is the greatest common divisor of a and b and it can be effectively computed with Euclid's algorithm on a classical computer.

To perform Shor's factoring algorithm, we start with two quantum registers, one of which is well prepared to be superpositions $|a\rangle$ and the other is in $|0\rangle$ state. We compute $x^a \pmod{n}$ in the second register and keep the first register in the $|a\rangle$ state, i.e., performing the modular exponentiation

$$U_{x,n}|a, 0\rangle \rightarrow |a, x^a \pmod{n}\rangle,$$

which can be basically built on a reversible network of quantum sum and carry [9]. Next, we perform quantum Fourier transformation on the first register to get period r . The discrete Fourier transform is a unitary transformation and can be implemented by a network of quantum CNOT and qubit rotation gates [5]. Finally, we calculate the factors of n with Euclid's algorithm by classical computing. We see that in the realization of Shor's factoring algorithm classical computation is

indispensable; in addition, the experimental realization of a quantum network could be greatly simplified by some classical substitutions.

V. CONCLUSION

We have presented the setup of some elementary quantum logic with macroscopic Josephson qubits, strengthened by some simulation work. The feasibility of building a quantum system to perform Shor's factoring algorithm was explored by implementing the algorithm on Josephson qubit network. Our investigation shows that it would be eventually possible to build a 2-Dimensional Josephson qubit array, possibly accompanied by classical computing components, capable of performing useful quantum computations, e.g. Shor's factoring algorithm. The practical realization would heavily depend on the advancement of physical implementations.

ACKNOWLEDGEMENT

This work is supported by Delft University of Technology, in its DIRC program "Novel Computation Structures Based on Quantum Devices". We would like to thank Hans Mooij, Peter Hadley, Ton Wallast and Patrick DeWilde for discussions.

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