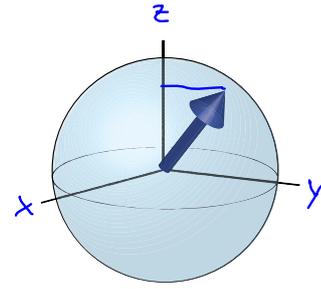


Quantum Measurement

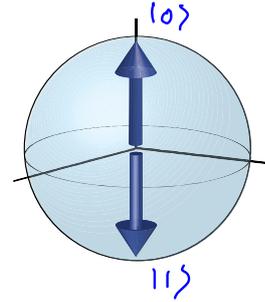
One way to determine the state of a qubit is to measure the projection of its state vector along a given axis, say the z-axis.

On the Bloch sphere this corresponds to the following operation:



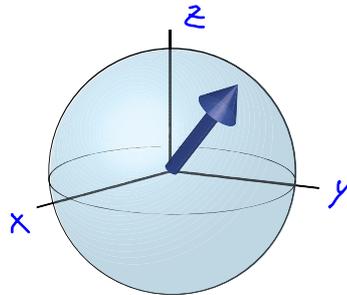
After a projective measurement is completed the qubit will be in either one of its computational basis states.

In a repeated measurement the projected state will be measured with certainty.



phys4.22 Page 1

information content in a single qubit:



- infinite number of qubit states
- but single measurement reveals only 0 or 1 with probabilities $|\alpha|^2$ or $|\beta|^2$
- measurement will collapse state vector on basis state
- to determine α and β an infinite number of measurements has to be made

But, if not measured qubit contains 'hidden' information about α and β .

phys4.22 Page 2

Information content in multiple qubits

- 2^n complex coefficients describe state of a composite quantum system with n qubits!
- Imagine to have 500 qubits, then 2^{500} complex coefficients describe their state.
- How to store this state. 2^{500} is larger than the number of atoms in the universe. It is impossible in classical bits. This is also why it is hard to simulate quantum systems on classical computers.
- A quantum computer would be much more efficient than a classical computer at simulating quantum systems.
- Make use of the information that can be stored in qubits for quantum information processing!

phys4.22 Page 3

Two qubits:

2 classical bits with states:

bit 1	bit 2
0	0
0	1
1	0
1	1

2 qubits with quantum states:

qubit 1	qubit 2
1	00
1	01
1	10
1	11

- 2^n different states (here $n=2$)
- but only one is realized at any given time
- 2^n basis states ($n=2$)
- can be realized simultaneously
- quantum parallelism

2^n complex coefficients describe quantum state

$$|\Psi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$$

normalization condition

$$\sum_{ij} |\alpha_{ij}|^2 = 1$$

phys4.22 Page 4

Composite quantum systems

QM postulate: The state space of a composite systems is the tensor product of the state spaces of the component physical systems. If the component systems have states $|\psi_i\rangle$ the composite system state is

$$|\Psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes \dots \otimes |\psi_n\rangle$$

This is a product state of the individual systems.

example:

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$$

$$|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$$

$$\begin{aligned} \rightarrow |\Psi\rangle &= |\psi_1\rangle \otimes |\psi_2\rangle = |\psi_1, \psi_2\rangle \\ &= \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle \end{aligned}$$

Classical Logic Gates:

		IN	OUT
non trivial single bit logic gate:	NOT	0	1
		1	0
universal two bit logic gate: AND followed by NOT	NAND	00	1
		01	1
		10	1
		11	0

Other gates exist (AND, OR, XOR, NOR) but can all be implemented using NAND gates.

universality of NAND: Any function operating on bits can be computed using NAND gates. Therefore NAND is called a universal gate.

Entanglement:

Definition: An **entangled state** of a composite system is a state that cannot be written as a product state of the component systems.

example: an entangled 2-qubit state (one of the Bell states)

$$|4\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

What is special about this state? Try to write it as a product state!

$$|\psi_1\rangle = \alpha_1 |0\rangle + \beta_1 |1\rangle ; |\psi_2\rangle = \alpha_2 |0\rangle + \beta_2 |1\rangle$$

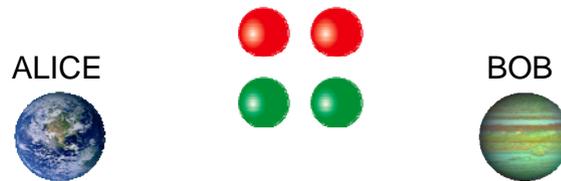
$$|\psi_1 \psi_2\rangle = \alpha_1 \alpha_2 |00\rangle + \alpha_1 \beta_2 |01\rangle + \beta_1 \alpha_2 |10\rangle + \beta_1 \beta_2 |11\rangle$$

$$|4\rangle \stackrel{!}{=} |\psi_1 \psi_2\rangle \Rightarrow \alpha_1 \alpha_2 = \frac{1}{\sqrt{2}} \wedge \beta_1 \beta_2 = \frac{1}{\sqrt{2}} \Rightarrow \alpha_1 \beta_2 \neq 0$$

$\wedge \alpha_2 \beta_1 \neq 0!$

It is not possible! This state is special, it is entangled!

The Strange Properties of Quantum Entanglement



Alice's measurement on her qubit determines the state of Bob's qubit ... instantaneously

realism assumption:

properties of an object exist independently of observation

locality assumption:

results of experiments at A do not influence results at B

Einstein, Podolsky and Rosen (EPR) were uncomfortable with these consequences of quantum mechanics.

But quantum mechanical systems have these properties (as proven by tests of the Bell inequalities).

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in

quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

teleportation of a quantum state



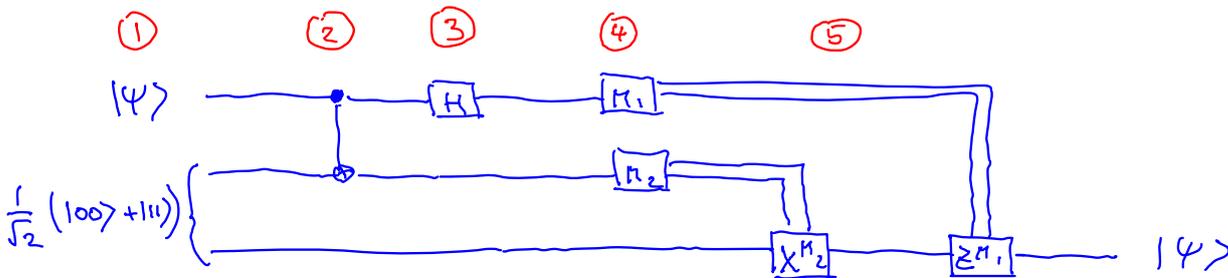
- a way for communicating an unknown quantum state
- a resource for quantum information processing

Quantum Teleportation:

Task: Alice wants to transfer an unknown quantum state ψ to Bob only using one entangled pair of qubits and classical information as a resource.

- note:
- Alice does not know the state to be transmitted
 - Even if she knew it the classical amount of information that she would need to send would be infinite.

The teleportation circuit:



original article:

[Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels](#)
 Charles H. Bennett, Gilles Brassard, Claude Crépeau, Richard Jozsa, Asher Peres, and William K. Wootters
 Phys. Rev. Lett. **70**, 1895 (1993) [PROLA Link]

How does it work?

$$\textcircled{1} \quad |\psi\rangle \otimes \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}} (\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle)$$

CNOT between qubit to be teleported and one bit of the entangled pair:

$$\textcircled{2} \quad \xrightarrow{\text{CNOT}_{12}} \frac{1}{\sqrt{2}} (\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle)$$

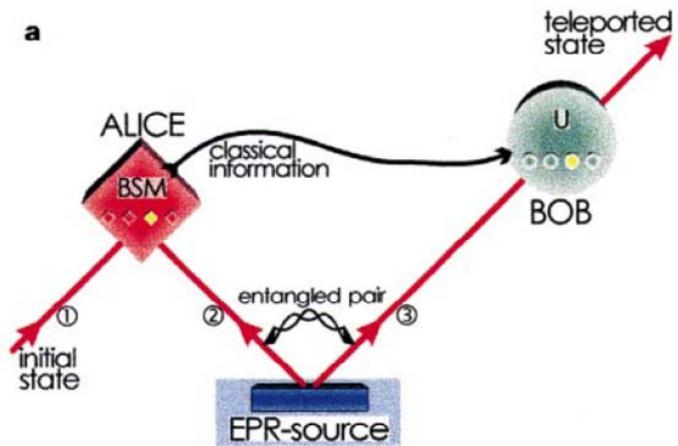
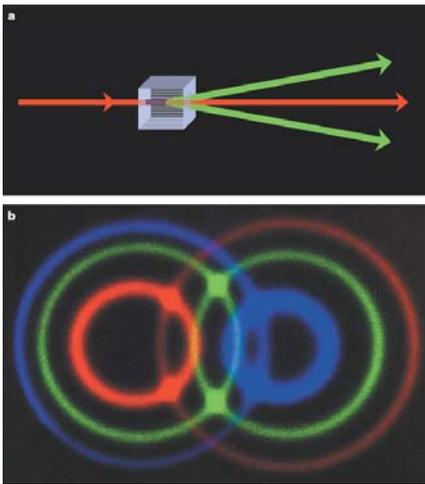
Hadamard on qubit to be teleported:

$$\textcircled{3} \quad \xrightarrow{H_1} \frac{1}{2} \left[(|00\rangle (\alpha|0\rangle + \beta|1\rangle) + |10\rangle (\alpha|0\rangle - \beta|1\rangle) + |01\rangle (\alpha|1\rangle + \beta|0\rangle) + |11\rangle (\alpha|1\rangle - \beta|0\rangle) \right]$$

measurement of qubit 1 and 2, classical information transfer and single bit manipulation on target qubit 3:

$$\textcircled{4} \quad \xrightarrow{M_1, M_2} \begin{array}{l} P_{00} = \frac{1}{4} \quad ; \quad |\psi_3\rangle = \alpha|0\rangle + \beta|1\rangle \xrightarrow{I} |\psi\rangle \\ P_{10} = \frac{1}{4} \quad ; \quad |\psi_3\rangle = \alpha|0\rangle - \beta|1\rangle \xrightarrow{Z} |\psi\rangle \\ P_{01} = \frac{1}{4} \quad ; \quad |\psi_3\rangle = \alpha|1\rangle + \beta|0\rangle \xrightarrow{X} |\psi\rangle \\ P_{11} = \frac{1}{4} \quad ; \quad |\psi_3\rangle = \alpha|1\rangle - \beta|0\rangle \xrightarrow{XZ} |\psi\rangle \end{array}$$

(One) Experimental Realization of Teleportation using Photon Polarization:



- parametric down conversion (PDC)
- source of entangled photons
- qubits are polarization encoded

Experimental quantum teleportation

Dik Bouwmeester, Jian-Wei Pan, Klaus Mattle, Manfred Eibl, Harald Weinfurter, Anton Zeilinger
Nature 390, 575 - 579 (11 Dec 1997) Article

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Deutsch's Problem:

This is the most simple example demonstrating the power of quantum computation.

evaluate if a function f is constant or balanced

$$f: \{0,1\} \rightarrow \{0,1\}$$

classically **two** queries of the function f are required to determine if it is constant or balanced.

x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$
0	0	1	0	1
1	0	1	1	0
	CONSTANT		BALANCED	

$$f(0) \oplus f(1) = 0 \quad \text{or} \quad 1$$

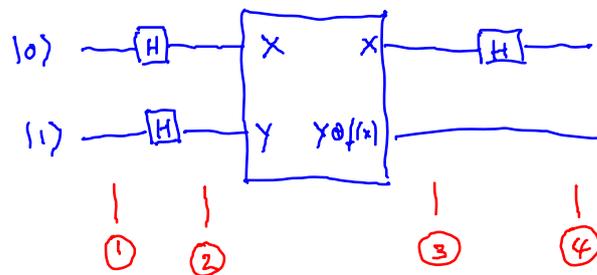
Is there a more efficient way to solve Deutsch's problem on a quantum computer?

Yes! Make use of superposition principle and quantum function evaluation!

Deutsch(-Jozsa) Algorithm

(improved version)

quantum circuit implementation:



execution of algorithm:

$$|\psi\rangle = |0\rangle|1\rangle \xrightarrow{H \otimes H} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \quad (2)$$

$$\xrightarrow{U_f} \begin{cases} \pm \frac{1}{2} (|0\rangle + |1\rangle) (|0\rangle - |1\rangle) & \text{for } f(0) = f(1) \\ \pm \frac{1}{2} (|0\rangle - |1\rangle) (|0\rangle - |1\rangle) & \text{for } f(0) \neq f(1) \end{cases} \quad (3)$$

$$\xrightarrow{H \otimes I} \begin{cases} \pm \frac{1}{\sqrt{2}} |0\rangle (|0\rangle - |1\rangle) & \text{for } f(0) = f(1) \\ \pm \frac{1}{\sqrt{2}} |1\rangle (|0\rangle - |1\rangle) & \text{for } f(0) \neq f(1) \end{cases} \quad (4)$$

$$= \pm |f(0) \oplus f(1)\rangle \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

Measurement of first qubit reveals whether f is balanced or constant.

Notes:

- Deutsch's problem is not the most important one. It has no known (useful) applications.
- BUT it serves as a good example what a quantum computer can do.
- The Deutsch algorithm can be extended to work on an arbitrary number n of bits and determine, if a function is balanced or constant in one evaluation, whereas solving the problem deterministically takes $2^n/2 + 1$ evaluations.
- HOWEVER, on a probabilistic classical computer one could solve the problem with high probability with fewer evaluations.

Experimental Implementations in NMR and ion traps:

Chuang, I. I., Vandersypen, I. M. K., Zhou, X., Leung, D. W. & Lloyd, S.
Experimental realization of a quantum algorithm.
Nature **393**, 143-146 (1998)
[|Article|](#)

Jones, T. F. & Mosca, M.
Implementation of a quantum algorithm to solve Deutsch's problem on a nuclear magnetic resonance quantum computer.
J. Chem. Phys. **109**, 1648-1653 (1998)
[|Article|](#)

Gulde S, Riebe M, Lancaster GPT, et al.
[Implementation of the Deutsch-Jozsa algorithm on an ion-trap quantum computer](#)
NATURE 421 (6918): 48-50 JAN 2 2003

Making predictions about the future can be difficult

"Where a calculator on the Eniac is equipped with 18,000 vacuum tubes and weighs 30 tons, computers in the future may have only 1,000 vacuum tubes and perhaps weigh 1-1/2 tons"

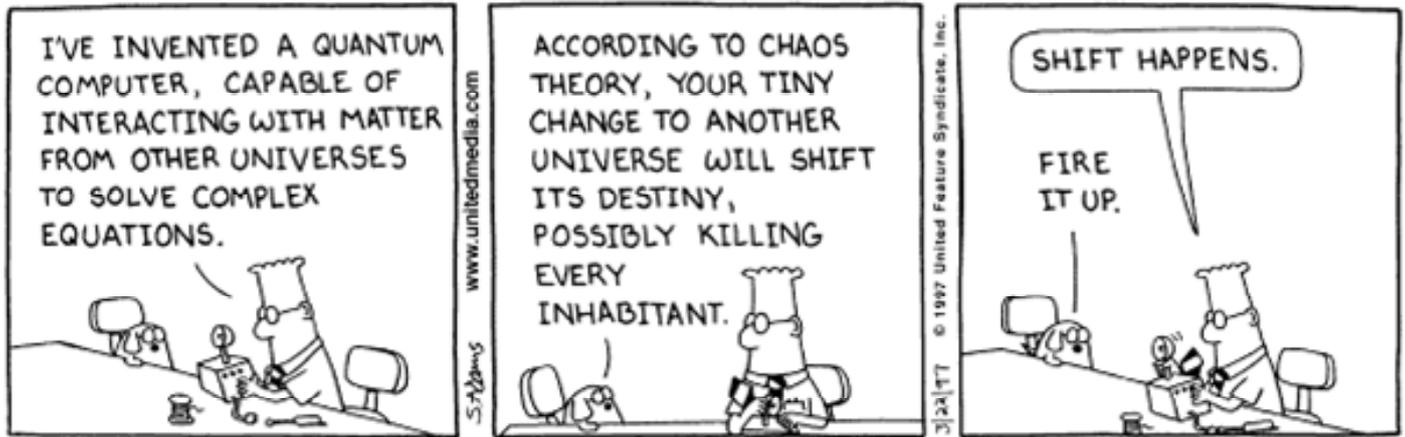
March, 1949 edition of Popular Mechanics

"I think there is a world market for about five computers"

*remark attributed to Thomas J. Watson
(Chairman of the Board of IBM, 1943)*

We are optimistic ...

Dilbert's take on Quantum Computers:



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... but careful too.