

## Qubit Residence Time Measurements with a Bose-Einstein Condensate

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We show that an electrostatic qubit located near a Bose-Einstein condensate trapped in a symmetric double-well potential can be used to measure the duration the qubit has spent in one of its quantum states. The strong, medium, and weak measurement regimes are analyzed. The analogy between the residence and the traversal (tunnelling) times is highlighted.

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With the recent progress in quantum information technology, there often arises a necessity to measure and control the state of a two-level quantum system (qubit). This can be achieved by constructing hybrid devices in which a microscopic irreversible current between two reservoirs is effectively controlled by the qubit's quantum state. Such a device can be realized, for example, by placing an electrostatic qubit close to a point contact (PC) [1,2] or an noninteracting Bose-Einstein condensate trapped in a symmetric optical dipole trap [3]. The two systems have been shown to affect the observed qubit differently: whereas a PC converts any qubit's initial state  $|i\rangle$  into a statistical mixture exponentially in time [4], decoherence of a qubit coupled to a BEC is much slower ( $\sim 1/\sqrt{t}$ ) and strongly dependent on the choice of  $|i\rangle$  [3]. While in a PC set up one measures the current across the contact, a BEC device is best suited for observing the number of atoms which have tunneled into a previously empty reservoir after a time  $T$ . The purpose of this Letter is to demonstrate that a symmetric BEC device whose Rabi oscillations are effectively blocked by the presence of the electron in the first qubit's dot, performs a quantum measurement of the qubit's residence time, i.e., the net duration the electron has spent in the second dot between  $t = 0$  and  $t = T$ . We will show that conceptually the question of residence time is closely related to the traversal (tunnelling) time problem still actively debated in the literature (see, for example, [5,6]). In both cases the time in question is the duration a system spends in a specified subspace of its Hilbert space, the sub-barrier region or the state in one of the quantum dots. Both quantities relate to the total duration of the system's motion, rather than to a single instant, and are conveniently represented by a Feynman functional. One can extend von Neumann's measurement theory to such functionals [7], but, as far as we know, the BEC device proposed in this Letter offers the first practical realization of such a measurement.

The role of a BEC as a measurement tool is best illustrated by considering first somewhat simpler case of a condensate coupled to a two-level fluctuator, i.e., a classical bistable system switching randomly between two positions so that its path  $q(t)$  is a random function taking values of either 0 or 1 [8]. Assuming that the tunnelling rate of the

BEC atoms is enhanced (the barrier is lowered) when  $q = 1$ , we write the Hamiltonian as [9]

$$\hat{H}_{\text{BEC}}(t) = [\Omega + q(t)\delta\Omega](c_L^\dagger c_R + c_R^\dagger c_L), \quad \delta\Omega > 0. \quad (1)$$

The condensate consists of  $N$  atoms initially (at  $t = 0$ ) located in the left well. After a time  $T$ , we wish to count the number of atoms in the right well,  $n$ , in order to obtain information about the noise  $q(t)$  for  $0 \leq t \leq T$ . The probability amplitude for  $n$  atoms to tunnel into the right well is a functional on the fluctuator's path  $q$  given by ( $\hbar = 1$ )

$$G_{n \leftarrow 0}[q(\cdot)] = \langle n | \exp[-i(\Omega T + \delta\Omega\tau)(c_L^\dagger c_R + c_R^\dagger c_L)] | 0 \rangle \quad (2)$$

where  $|n\rangle$  denotes the BEC state with  $n$  atoms in the right well, and  $\tau$  is the duration the fluctuator has spent in the state  $q = 1$ , explicitly given by an expression similar to the traversal time functional  $\tau_{ab}[x(\cdot)]$  of Ref. [5] ( $\delta_{ij}$  is the Kroneker delta)

$$\tau_1[q(\cdot)] \equiv \int_0^T \delta_{q(t)1} dt. \quad (3)$$

For simplicity, we will assume that no tunnelling occurs for  $q = 0$ , i.e.,  $\Omega = 0$ , and that the BEC consists of a large number of identical noninteracting atoms whose Rabi period  $2\pi/\delta\Omega$  is large compared to the observation time  $T$ ,

$$N \rightarrow \infty, \quad \delta\Omega \equiv \alpha/N^{1/2} \rightarrow 0, \quad \delta\Omega T \rightarrow 0. \quad (4)$$

Condition (4) ensures that if the barrier is permanently lowered,  $[q(t) \equiv 1]$ , there is an irreversible macroscopic current into the right reservoir, with the number of tunneled atoms increasing as  $\alpha^2 t^2$ . The energy levels of a noninteracting condensate are obtained by distributing  $N$  atoms between the two single-particle levels corresponding to the symmetric and antisymmetric states of the double-well potential. The two energies are  $-\delta\Omega$  and  $\delta\Omega$ , respectively, and the spectrum of the operator in the exponent of Eq. (2) consists of equidistant levels,  $\epsilon_n = (2n - N)\delta\Omega$ ,  $n = 0, 1, \dots, N$ . Expanding the exponential in Eq. (2) in the basis of the corresponding eigenstates and using the Sterling formula for the factorials yields

$$G_{n \leftarrow 0}(\tau) \approx \alpha^n \tau^n \exp(-\alpha^2 \tau^2 / 2) / (n!)^{1/2} \xrightarrow{n \gg 1} (2\pi n)^{-1/4} \times \exp[-\alpha^2 (\tau - \tau_n)^2], \quad (5)$$

where

$$\tau_n \equiv n^{1/2}/\alpha \quad (6)$$

is the time after which on average  $n$  atoms escape to the right well. If the fluctuator's paths are distributed with a functional density  $W[q(t)]$ , the probability to find  $n$  atoms in the right well,  $|G_{n\leftarrow 0}(\tau)|^2$ , must be averaged further, and we obtain

$$P_{n\leftarrow 0}(T) = \int_0^T |G_{n\leftarrow 0}(\tau)|^2 W(\tau, T) d\tau \quad (7)$$

where the restricted path sum

$$W(\tau, T) \equiv \sum_{\text{paths}} \delta(\tau - \tau_1[q(\cdot)]) W[q(t)] \quad (8)$$

is the fluctuator residence time probability distribution. Thus, finding at  $t = T$  exactly  $n$  tunnelled atoms allows us to conclude that the fluctuator has kept the barrier open for a duration  $\tau_n - 1/\sqrt{2}\alpha < \tau < \tau_n + 1/\sqrt{2}\alpha$ , i.e., that we have measured its residence time to an accuracy  $\Delta\tau = 1/\sqrt{2}\alpha$ .

Next, we replace the fluctuator with a qubit placed near the BEC dipole trap in such a way that the BEC tunnelling rate is enhanced whenever qubit's electron is located in the state  $|1\rangle$ . The Hamiltonian of the system can be written as  $\hat{H} = \hat{H}_q + \hat{H}_{\text{BEC}}$ , where

$$\begin{aligned} \hat{H}_q(\epsilon) &= \epsilon a_1^\dagger a_1 + \omega(a_1^\dagger a_2 + a_2^\dagger a_1) \\ \hat{H}_{\text{BEC}} &= (\Omega + a_1^\dagger a_1 \delta\Omega)(c_L^\dagger c_R + c_R^\dagger c_L), \quad \delta\Omega > 0 \end{aligned} \quad (9)$$

and  $a_{1(2)}^\dagger$  are the creation operators for the qubit's electron in the first (tunnelling enhanced) and the second (tunnelling suppressed) quantum dot, respectively. In the following, we will put the qubit's Rabi frequency to unity,  $\omega = 1$ , and rescale other time and energy parameters accordingly. Like a two-state fluctuator, a qubit can alternate between the two states,  $|1\rangle$  and  $|2\rangle$ , with the important difference that its trajectory  $q(t)$  taking the values 1 or 2 is a virtual (Feynman) path. To such a path, one can assign a probability amplitude  $\Phi[q(t)]$  but not, as above, a probability weight  $W[q(t)]$ . We must, therefore, evaluate the number of tunnelled atoms at  $t = T$  without being able to predict, even with a probability, whether the barrier was up or down at any previous time  $0 \leq t < T$  [10]. For a qubit starting its motion (preselected) in the state  $|i\rangle$  and then at  $t = T$  observed (postselected) in a final state  $|f\rangle$ , this probability amplitude is given by  $\Phi^{f\leftarrow i}[q] = \langle f|q(T)\rangle (-i\omega)^j \times \exp(-i\epsilon\tau) \langle q(0)|i\rangle$ , where  $j$  is the number of times the path crosses from one state to another. Following Feynman and Vernon [11], we can obtain the probability amplitude  $A_{n\leftarrow 0}^{f\leftarrow i}(T)$  for finding  $n$  atoms in the right well given the initial and final states of the qubit by multiplying the amplitude in Eq. (5) by  $\Phi^{f\leftarrow i}[q]$  and summing over all qubit's paths. Assuming, as above,  $\Omega = 0$  and recalling that  $G_{n\leftarrow 0}[q(t)]$  only depends on the path's residence time (3), we write

$$A_{n\leftarrow 0}^{f\leftarrow i}(T) = \int_0^T G_{n\leftarrow 0}(\tau) \Phi^{f\leftarrow i}(\tau, T) d\tau \quad (10)$$

where the restricted path sum (c.f. Refs. [5])

$$\Phi^{f\leftarrow i}(\tau, T) \equiv \sum_{\text{paths}} \delta(\tau - \tau_1[q(\cdot)]) \Phi^{f\leftarrow i}[q(t)] \quad (11)$$

is the qubit's residence time probability amplitude distribution. Thus, the quantum analogue of Eq. (7) is

$$P_{n\leftarrow 0}^{f\leftarrow i}(T) = \left| \int_0^T G_{n\leftarrow 0}(\tau) \Phi^{f\leftarrow i}(\tau, T) d\tau \right|^2. \quad (12)$$

From Eq. (5), it is readily seen that the probability  $P_{n\leftarrow 0}^{f\leftarrow i}$  results from the interference between the paths with  $\tau_n - 1/\alpha \leq \tau \leq \tau_n + 1/\alpha$ , so that by determining  $n$ , we perform a measurement of the qubit's residence time [12] to a *quantum* accuracy  $\Delta^q \tau \equiv 1/\alpha$  [13]. Finally, if the maximum number of atoms which can tunnel over the time  $T$  is large,  $N_{\text{max}} \approx \alpha^2 T^2 \gg 1$ , we can introduce probability density  $w^{f\leftarrow i}(\tau, T)$  for the measured values of  $\tau$ ,  $w^{f\leftarrow i}(\tau, T) \equiv P_{n\leftarrow 0}^{f\leftarrow i}(T) (d\tau_n/dn)^{-1} = 2\alpha\sqrt{n} P_{n\leftarrow 0}^{f\leftarrow i}$ . Explicitly, we have

$$\begin{aligned} w^{f\leftarrow i}(\tau, T) &\approx (2/\pi)^{1/2} \alpha \\ &\times \left| \int_0^T \exp[-\alpha^2(\tau - \tau')^2] \Phi^{f\leftarrow i}(\tau', T) d\tau' \right|^2. \end{aligned} \quad (13)$$

The measurement statistics are determined by the distribution (11), some of whose properties have been discussed in [14]. In particular, it follows from Eq. (11) that

$$\begin{aligned} \Phi^{f\leftarrow i}(\tau, T) &= (2\pi)^{-1} \exp(-i\epsilon\tau) \int \exp(i\lambda\tau) \\ &\times \langle f|\hat{U}(T, \lambda)|i\rangle d\lambda \end{aligned} \quad (14)$$

where  $\hat{U}(T, \lambda)$  is the evolution operator for an asymmetric qubit with the Hamiltonian  $\hat{H}_q(\lambda) \equiv \lambda a_1^\dagger a_1 + (a_1^\dagger a_2 + a_2^\dagger a_1)$ , whose matrix elements  $U_{kk'} \equiv \langle k|\hat{U}(T, \lambda)|k'\rangle$  are given by

$$\begin{aligned} U_{11} &= [\cos(\mathcal{E}T/2) - i\lambda\mathcal{E}^{-1} \sin(\mathcal{E}T/2)] \exp(-i\lambda T/2) \\ &\equiv \exp(-i\lambda T) + u_{11}(\lambda) \\ U_{22} &= [\cos(\mathcal{E}T/2) + i\lambda\mathcal{E}^{-1} \sin(\mathcal{E}T/2)] \exp(-i\lambda T/2) \\ &\equiv 1 + u_{22}(\lambda) \\ U_{12} &= -2i\mathcal{E}^{-1} \sin(\mathcal{E}T/2) \exp(-i\lambda T/2) = U_{21} \end{aligned}$$

where  $\mathcal{E}(\lambda) \equiv (\lambda^2 + 4)^{1/2}$  and  $u_{11(22)}(\lambda) \rightarrow 0$  for  $|\lambda| \rightarrow \infty$ . Inserting the above expressions into Eq. (14) shows that

$$\begin{aligned} \Phi^{1\leftarrow 1}(\tau, T) &= \delta(\tau - T) + \phi^{1\leftarrow 1}(\tau, T) = \Phi^{2\leftarrow 2}(T - \tau, T) \\ \Phi^{2\leftarrow 1}(\tau, T) &= \phi^{2\leftarrow 1}(\tau, T) = \Phi^{1\leftarrow 2}(\tau, T) \end{aligned} \quad (15)$$

where  $\delta(z)$  is the Dirac delta function and  $\phi^{f\leftarrow i}$  are smooth functions of  $\tau$ . For  $T \gg 1$ ,  $\phi^{f\leftarrow i}$  can be evaluated by the

stationary phase method [15]. Considering for simplicity a symmetric qubit,  $\epsilon = 0$ , and introducing a new variable  $\xi \equiv \tau/T - 1/2$ ,  $0 \leq \xi \leq 1$ , we obtain the large-time semiclassical asymptotes valid for  $0 < \tau < T$ ,

$$\phi^{1 \leftarrow 1}(\tau, T) \approx (2/\pi T)^{1/2} (1 + 2\xi)^{1/4} (1 - 2\xi)^{-3/4} \times \cos[(1 - 4\xi^2)^{1/2} T + \pi/4] \quad (16)$$

$$\phi^{1 \leftarrow 2}(\tau, T) \approx -i(2/\pi T)^{1/2} (1 - 4\xi^2)^{1/2} \times \sin[(1 - 4\xi^2)^{1/2} T + \pi/4]. \quad (17)$$

The oscillatory distributions  $\Phi^{1 \leftarrow 1}(\tau, T)$  and  $\Phi^{1 \leftarrow 2}(\tau, T)$  are shown in Fig. 1. It is readily seen that after many Rabi periods of the qubit,  $T \gg 1$ ,  $\Phi^{1 \leftarrow 1}$  develops a stationary region of the width  $\sim T^{1/2}$  centered at  $\tau = T/2$ , which suggests that on average, the qubit shares its time equally between the states  $|1\rangle$  and  $|2\rangle$ . At the same time, the singular term  $\delta(\tau - T)$  appears to imply that the qubit has never left the state  $|1\rangle$ . There is, however, no contradiction, and next we will show that the two conflicting scenarios correspond to two different accuracies of the BEC meter and, therefore, are never observed at the same time. Indeed, for a medium accuracy,  $T^{1/2} < \Delta^q \tau < T$ , the main contribution to integral (13) comes from the stationary regions in Fig. 1 and we have

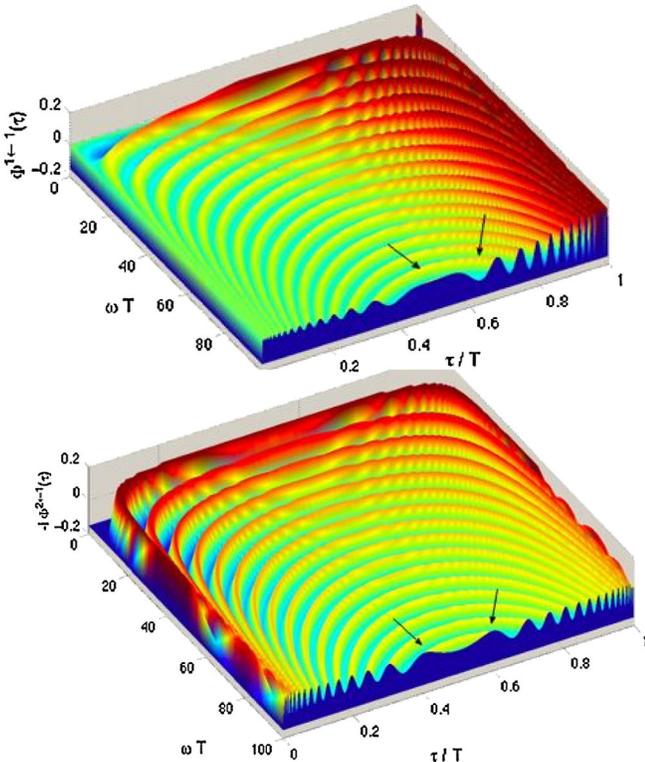


FIG. 1 (color online). Residence time amplitude distributions  $\Phi^{1 \leftarrow 1}(\tau)$  and  $\Phi^{1 \leftarrow 2}(\tau)$  vs  $T$  and  $\tau$  for  $0.02 \leq \tau/T \leq 0.98$ . Arrows indicate the stationary phase region.

$$\begin{aligned} w^{1 \leftarrow 1}(\tau, T) &= w^{2 \leftarrow 2}(\tau, T) \\ &= (2/\pi)^{1/2} \alpha \cos^2(T) \exp[-2\alpha^2(\tau - T/2)^2] \\ w^{1 \leftarrow 2}(\tau, T) &= w^{2 \leftarrow 1}(\tau, T) \\ &= (2/\pi)^{1/2} \alpha \sin^2(T) \exp[-2\alpha^2(\tau - T/2)^2]. \end{aligned} \quad (18)$$

The Gaussian distributions (18), shown in Fig. 2(a) for  $\omega T = 100$  and  $\Delta^q \tau/T = 0.1$  by dashed lines, are consistent with the qubit spending in the state  $|1\rangle$  roughly half of the total time  $T$ . Note that here the contribution from the  $\delta(\tau - T)$  term is cancelled by the oscillations of the regular part of  $\Phi^{1 \leftarrow 1}(\tau, T)$  near  $\tau \approx T$ . To model an actual measurement and check the accuracy of Eqs. (13) and (18), we have divided the time interval  $[0, T]$  into  $N_{\text{bin}} = 100$  equal subintervals  $\delta t = T/N_{\text{bin}}$ , summed the probabilities  $P_{n \leftarrow 0}^{f \leftarrow i}$  in Eq. (12) within each interval, and divided the sum by  $\delta t$ . The results of this binning procedure are shown in Fig. 2(a) by the solid lines. As we increase the coupling strength  $\alpha$ , the integral (13) will still vanish wherever oscillations of  $\Phi$  are fast compared to  $\Delta^q \tau = 1/\alpha$ . Where  $\Delta^q \tau$  is small compared to the oscillation's period, we obtain

$$w^{f \leftarrow i}(\tau, T) \approx (2\pi)^{1/2} \alpha^{-1} |\Phi^{f \leftarrow i}(\tau, T)|^2, \quad 0 < \tau < T. \quad (19)$$

Thus, as the accuracy improves, the measurement will resolve the pattern of  $|\Phi^{f \leftarrow i}(T, \tau)|^2$  in ever greater detail. We also note that the probability densities in Eq. (19) decrease as  $\alpha^{-1}$  as interaction with BEC suppresses qubit's transitions between the states  $|1\rangle$  and  $|2\rangle$ . The approximation (19) and the results of a binning procedure with  $N_{\text{bin}} = 100$  are shown in Figs. 2(b) and 2(c) for  $\Delta^q \tau/T = 0.005$  by the dashed and the solid lines, respectively.

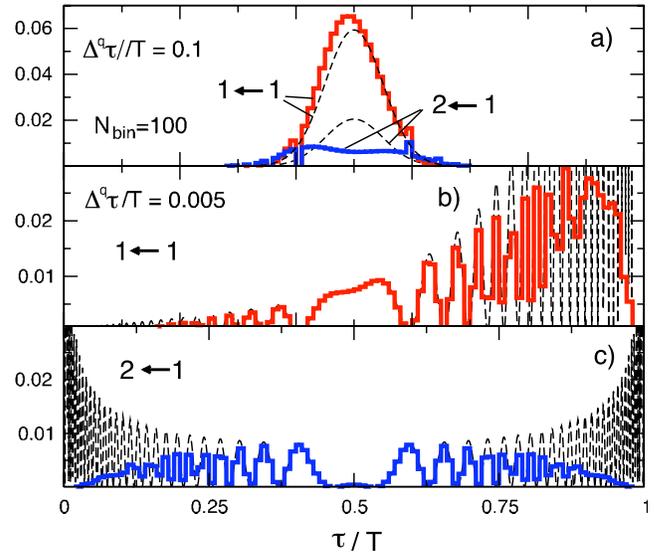


FIG. 2 (color online). (a) Distributions  $w^{f \leftarrow i}(\tau, T)$  for  $\omega T = 100$  and  $\Delta^q \tau/T = 0.1$ : Eq. (18) (dashed line) and the binning procedure with  $N_{\text{bin}} = 100$  (solid line); (b) and (c) Same as (a) but for Eq. (19) (dashed line) and  $\Delta^q \tau/T = 0.005$ .

In the high accuracy limit  $\alpha \rightarrow \infty$ , the probability is conserved owing to the  $\delta(\tau - T)$  and  $\delta(\tau)$  terms present, as seen from Eqs. (15), in  $\Phi^{1\leftarrow 1}(T, \tau)$  and  $\Phi^{2\leftarrow 2}(T, \tau)$ , respectively. Inserting them into Eq. (13) shows that while  $w^{1\leftarrow 2}(\tau, T)$  and  $w^{2\leftarrow 1}(\tau, T)$  vanish,  $w^{1\leftarrow 1}(\tau, T)$  and  $w^{2\leftarrow 2}(\tau, T)$  become

$$\begin{aligned} \lim_{\alpha \rightarrow \infty} w^{1\leftarrow 1}(\tau, T) &= (2/\pi)^{1/2} \alpha \exp[-2\alpha^2(\tau - T)^2] \\ &\approx \delta(\tau - T) \\ \lim_{\alpha \rightarrow \infty} w^{2\leftarrow 2}(\tau, T) &\approx \delta(\tau). \end{aligned} \quad (20)$$

Thus, an accurate measurement on a qubit prepared, say, in the state  $|1\rangle$  would reveal that it has spent there all available time,  $\tau = T$ . This result is correct since the perturbation necessarily produced by such a measurement destroys Rabi oscillations of the qubit. We note further that no atoms will tunnel for a qubit starting in the second state,  $|i\rangle = |2\rangle$ , whereas for  $|i\rangle = |1\rangle$  one obtains a narrow Poisson distribution  $\lim_{\alpha \rightarrow \infty} P_n^{1\leftarrow 1} = (\alpha T)^{2n} \exp(-\alpha^2 T^2)/n!$ .

In order to avoid the back action of the BEC on the qubit's evolution and find the "unperturbed" residence time one may be tempted to decrease the coupling by putting  $\alpha \rightarrow 0$ . Again, it is instructive to analyze first the case of a classical fluctuator. In this weak coupling limit, Eq. (5) yields  $G_{n\leftarrow 0}(\tau) \approx \alpha^n \tau^n / \sqrt{n!}$  and from Eq. (7), we obtain  $P_{n\leftarrow 0}(T) \approx \alpha^{2n} \langle \tau^{2n} \rangle / n!$  where  $\langle \tau^{2n} \rangle \equiv \int_0^T \tau^{2n} W(\tau) d\tau$  is the  $n$ -th even moment of the probability distribution  $W(\tau) \geq 0$ . Thus, from the ratio  $P_{1\leftarrow 0}(T)/P_{0\leftarrow 0}(T) \approx \alpha^2 \langle \tau^2 \rangle$ , we can determine  $\langle \tau^2 \rangle$  and, should the dispersion be small, the mean residence time  $\langle \tau \rangle \approx \sqrt{\langle \tau^2 \rangle}$ .

In the case of a qubit, from Eqs. (5) and (12) in the limit  $\alpha \rightarrow 0$ , we find  $P_n^{f\leftarrow i}/P_0^{f\leftarrow i} \approx \alpha^{2n} |\bar{\tau}^n|^2 / n!$  with  $\bar{\tau}^n \equiv \int \tau^n \Phi^{f\leftarrow i}(\tau, T) d\tau / \int \Phi^{f\leftarrow i}(\tau, T) d\tau = (-i)^n \partial_\lambda \log \langle f | \hat{U}(T, \lambda) | i \rangle |_{\lambda=0}$ . In particular, we have

$$P_{1\leftarrow 0}^{1\leftarrow 1}/P_{0\leftarrow 0}^{1\leftarrow 1} \approx \alpha^2 |\bar{\tau}|^2 = \alpha^2 [T/2 + \tan(T)/2]^2, \quad (21)$$

where  $\bar{\tau}$  is the weak value of the residence time analogous to the Larmor tunnelling time first introduced to quantum scattering by Baz' [16]. It diverges whenever Rabi oscillations put the unperturbed qubit into the state  $|2\rangle$ ,  $T = (k + 1/2)\pi$ ,  $k = 0, 1, \dots$ , may exceed the total duration of motion  $T$ , and cannot be interpreted as a valid residence time. This problem is common to all weak measurements introduced in [17], whose accuracy is so poor that they do not destroy coherence between different values of the measured quantity [18]. The weak residence time  $\bar{\tau}$  in Eq. (21) is the first moment of an alternating amplitude distribution  $\Phi^{1\leftarrow 1}(T, \tau)$  and as such is not directly linked to the physical values  $0 \leq \tau \leq T$  [18].

In summary, we have shown that a hybrid device consisting of an electrostatic qubit coupled to a BEC trapped in a symmetric double-well potential can be used to perform the qubit's residence time measurements. Depending on

the strength of the coupling, the measurement can be "weak" or strong. An accurate (strong) measurement leads to trapping of the qubit's electron in one of the quantum dots thus destroying its Rabi oscillations. Mathematical explanation linking the effect to the presence of singular terms in the residence time amplitude distribution should also apply to a wide range of similar measurements.

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  - [13] While the classical uncertainty  $\Delta\tau$  reflects one's lack of knowledge of the actual value of  $\tau$ ,  $\Delta^q\tau$  arises from the interference between the paths, which leaves the "actual" value of  $\tau$  indeterminate.
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