

# Reading out a quantum bit

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Göran Johansson and Thilo Bauch

In order to get the result of a quantum algorithm we need to read out the state of the qubits in the out register when the algorithm is done. As we will see later one also needs to read out certain qubits during the computation if we want to perform error correction. On the algorithm level a read-out is simply a projective measurement. If we read-out a qubit in the state  $|\Psi\rangle = c_0|0\rangle + c_1|1\rangle$  we should get the answer 0 with probability  $|c_0|^2$  and the answer 1 with probability  $|c_1|^2$ . Furthermore if the result was 0 the qubit should be in the state  $|0\rangle$  after the measurement and similarly in the state  $|1\rangle$  if the result was 1. Thus repeated measurements should always give the same result. A read-out with this property is sometimes called Quantum Nondemolition (QND).

## 1 Measurement and dephasing

Starting with a qubit and a detector which are not coupled to each other

$$\hat{H} = -\frac{\omega_0}{2}\sigma_z + \hat{H}_{\text{det}}.$$

The qubit is initially in the state  $\alpha|0\rangle + \beta|1\rangle$  while the measurement device is initialized in the state  $|e_i\rangle$ , and their combined state is the tensor product (product state)

$$|\Psi(0)\rangle = (\alpha|0\rangle + \beta|1\rangle) \otimes |e_i\rangle.$$

Turning on the interaction between the qubit and the measurement device

$$\hat{H} = -\frac{\omega_0}{2}\sigma_z + \hat{H}_{\text{det}} + \hat{X}_{\text{det}}\sigma_z,$$

where  $\hat{X}_{\text{det}}$  is an operator of the detector. Note that in order to fulfill the requirement on repeated measurements, the detector should not be able to flip the qubit state. This is accomplished by coupling the detector to  $\sigma_z$ , i.e. in the qubit eigenbasis. The detector and the qubit will become entangled, and their state will evolve into

$$|\Psi(t)\rangle = \alpha|0\rangle|e_0(t)\rangle + \beta|1\rangle|e_1(t)\rangle,$$

where  $|e_0(t)\rangle$  and  $|e_1(t)\rangle$  are detector states fulfilling  $|e_0(0)\rangle = |e_1(0)\rangle = |e_i\rangle$ .

The density matrix of the combined system of the qubit and the detector is

$$\begin{aligned} \hat{\rho} &= |\Psi(t)\rangle\langle\Psi(t)| = |\alpha|^2|0\rangle\langle 0| \otimes |e_0(t)\rangle\langle e_0(t)| + |\beta|^2|1\rangle\langle 1| \otimes |e_1(t)\rangle\langle e_1(t)| + \\ &+ \alpha\beta^*|0\rangle\langle 1| \otimes |e_0(t)\rangle\langle e_1(t)| + \alpha^*\beta|1\rangle\langle 0| \otimes |e_1(t)\rangle\langle e_0(t)|. \end{aligned} \quad (1)$$

To get the reduced qubit density matrix we trace out the measurement device

$$\hat{\rho}_{\text{red}} = \sum_k \langle b_k | \hat{\rho} | b_k \rangle,$$

where  $|b_k\rangle$  is a complete basis for the measurement device. Using that

$$\sum_k \langle b_k | A \rangle \langle B | b_k \rangle = \sum_k \langle B | b_k \rangle \langle b_k | A \rangle = \langle B | A \rangle,$$

for any states  $|A\rangle$  and  $|B\rangle$  of the measurement device, we get the reduced qubit density matrix (written as a matrix in the 0/1 basis)

$$\rho_{\text{red}}(t) = \begin{pmatrix} |\alpha|^2 & \alpha\beta^*\langle e_1(t)|e_0(t)\rangle \\ \alpha^*\beta\langle e_0(t)|e_1(t)\rangle & |\beta|^2 \end{pmatrix},$$

where we used the normalization  $\langle e_0(t)|e_0(t)\rangle = \langle e_1(t)|e_1(t)\rangle = 1$ . Now, for a good detector the two states  $|e_0(t)\rangle$  and  $|e_1(t)\rangle$  should develop into different states, otherwise there is no way we can detect the qubit state by measuring the detector state. It's interesting to note that when the states are completely different, i.e. orthogonal  $\langle e_0(t)|e_1(t)\rangle = 0$ , the qubit is completely dephased and we can tell it's state with certainty.

## 2 Read-out in circuit cavity QED

We'll now see how read-out can be implemented in reality. Starting from the dispersive Jaynes-Cummings Hamiltonian

$$\hat{H}_{\text{disp}} = -\frac{1}{2} \left( \hbar\omega_0 + \frac{g^2}{\hbar\Delta} \right) \sigma_z + \left( \hbar\omega - \frac{g^2}{\hbar\Delta} \sigma_z \right) \hat{a}^\dagger \hat{a},$$

we see that the cavity is coupled to the qubit in the qubit eigenbasis. As we discussed before, there is a shift of the cavity resonance frequency, which depends on the qubit state

$$\omega_{\text{eff}} = \omega - \frac{g^2}{\hbar^2\Delta} \sigma_z.$$

By opening up the cavity we can probe the resonance frequency by sending near resonant microwaves and then detect the reflected or transmitted signal. The conceptually simplest measurement is to look at the transmitted signal. Sending microwaves off resonance almost all microwaves will be reflected, while on resonance all microwaves will be transmitted.

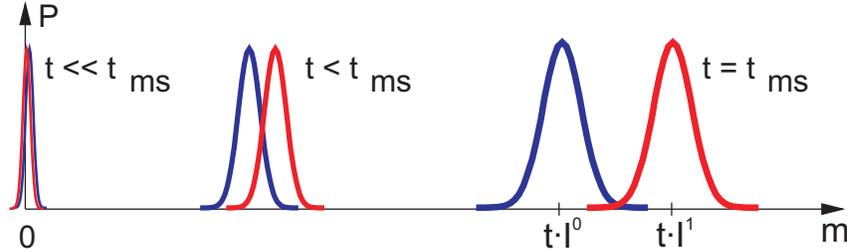


Figure 1: The probability distributions  $P$  for the number of photons  $m$  transmitted through the cavity. The two peaks corresponding to different qubit states separate due to the difference in average transparency and broaden due to photon shot-noise. The average photon number at time  $t$  is given by  $tI^{0,1}$ , where  $I^0$  and  $I^1$  are the photon currents transmitted through the cavity when the qubit is in state  $|0\rangle$  and state  $|1\rangle$ , respectively.

The qubit state can be detected by probing the cavity with microwaves of frequency  $\omega + \frac{g^2}{\hbar^2\Delta}$ , i.e. on resonance with the cavity when the qubit is in its excited state. When the qubit is in its ground state, some photons will be reflected. Looking at the number of photons transmitted through the cavity there will be two peaks corresponding to the two different qubit states. They separate due to the difference in average current and broaden due to shot-noise (see Fig. 1).

The number of photons in a coherent state has a Poisson distribution with average  $\bar{n}$  and variance  $\sqrt{\bar{n}}$ . So one has to measure for a time  $t_{ms}$  before the two distributions corresponding to the two different qubit states have separated, and the qubit state can be determined with certainty.

## 2.1 Back-action: Dephasing, quantum efficiency and if non-QND also mixing

Looking from the qubit perspective, it is the fluctuations of the photon number in the cavity which dephases the qubit, through the coupling term

$$\frac{g^2}{\hbar\Delta}\hat{a}^\dagger\hat{a}\sigma_z,$$

giving rise to a measurement induced dephasing rate  $\Gamma_\varphi$ .

The general relation between the distinguishability of qubit states and measurement induced dephasing translates into the inequality

$$\eta = (t_{ms}\Gamma_\varphi)^{-1} \leq 1,$$

where  $\eta$  is the so-called quantum efficiency. This is an interesting concept in optimizing qubit readout, because if it's less than unity some information about the qubit is lost on the way to the detector.

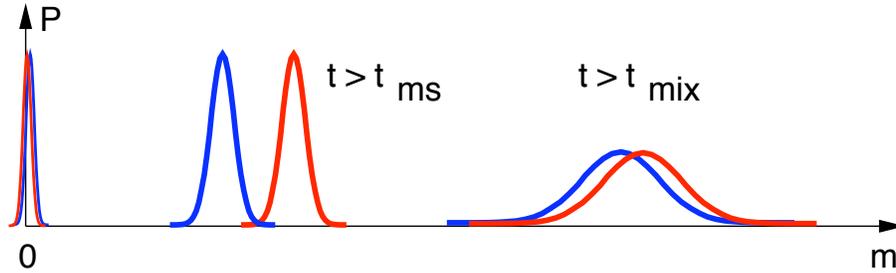


Figure 2: If not perfectly QND, the two distributions will eventually merge into one after the time  $t_{\text{mix}} = T_1$ .

If the read-out coupling has a transverse component it is not perfectly QND, e.g. if one probes the cavity too hard, so that the dispersive Hamiltonian is not a good approximation. Then there will be a finite a measurement induce mixing time  $T_1$ . After the mixing time the information about the initial population of the qubit will be erased (see Fig. 2). The signal to noise ratio (SNR) is given by ratio between the mixing time and measurement time

$$\frac{T_1}{t_{ms}}. \quad (2)$$