

Preface

Scaffolding

Reacting to criticism concerning the lack of motivation in his writings, Gauss remarked that architects of great cathedrals do not obscure the beauty of their work by leaving the scaffolding in place after the construction has been completed. His philosophy epitomized the formal presentation and teaching of mathematics throughout the nineteenth and twentieth centuries, and it is still commonly found in mid-to-upper-level mathematics textbooks. The inherent efficiency and natural beauty of mathematics are compromised by straying too far from Gauss's viewpoint. But, as with most things in life, appreciation is generally preceded by some understanding seasoned with a bit of maturity, and in mathematics this comes from seeing some of the scaffolding.

Purpose, Gap, and Challenge

The purpose of this text is to present the contemporary theory and applications of linear algebra to university students studying mathematics, engineering, or applied science at the postcalculus level. Because linear algebra is usually encountered between basic problem solving courses such as calculus or differential equations and more advanced courses that require students to cope with mathematical rigors, the challenge in teaching applied linear algebra is to expose some of the scaffolding while conditioning students to appreciate the utility and beauty of the subject. Effectively meeting this challenge and bridging the inherent gaps between basic and more advanced mathematics are primary goals of this book.

Rigor and Formalism

To reveal portions of the scaffolding, narratives, examples, and summaries are used in place of the formal definition–theorem–proof development. But while well-chosen examples can be more effective in promoting understanding than rigorous proofs, and while precious classroom minutes cannot be squandered on theoretical details, I believe that all scientifically oriented students should be exposed to some degree of mathematical thought, logic, and rigor. And if logic and rigor are to reside anywhere, they have to be in the textbook. So even when logic and rigor are not the primary thrust, they are always available. Formal definition–theorem–proof designations are not used, but definitions, theorems, and proofs nevertheless exist, and they become evident as a student's maturity increases. A significant effort is made to present a linear development that avoids forward references, circular arguments, and dependence on prior knowledge of the subject. This results in some inefficiencies—e.g., the matrix 2-norm is presented

before eigenvalues or singular values are thoroughly discussed. To compensate, I try to provide enough “wiggle room” so that an instructor can temper the inefficiencies by tailoring the approach to the students’ prior background.

Comprehensiveness and Flexibility

A rather comprehensive treatment of linear algebra and its applications is presented and, consequently, the book is not meant to be devoured cover-to-cover in a typical one-semester course. However, the presentation is structured to provide flexibility in topic selection so that the text can be easily adapted to meet the demands of different course outlines without suffering breaks in continuity. Each section contains basic material paired with straightforward explanations, examples, and exercises. But every section also contains a degree of depth coupled with thought-provoking examples and exercises that can take interested students to a higher level. The exercises are formulated not only to make a student think about material from a current section, but they are designed also to pave the way for ideas in future sections in a smooth and often transparent manner. The text accommodates a variety of presentation levels by allowing instructors to select sections, discussions, examples, and exercises of appropriate sophistication. For example, traditional one-semester undergraduate courses can be taught from the basic material in Chapter 1 (Linear Equations); Chapter 2 (Rectangular Systems and Echelon Forms); Chapter 3 (Matrix Algebra); Chapter 4 (Vector Spaces); Chapter 5 (Norms, Inner Products, and Orthogonality); Chapter 6 (Determinants); and Chapter 7 (Eigenvalues and Eigenvectors). The level of the course and the degree of rigor are controlled by the selection and depth of coverage in the latter sections of Chapters 4, 5, and 7. An upper-level course might consist of a quick review of Chapters 1, 2, and 3 followed by a more in-depth treatment of Chapters 4, 5, and 7. For courses containing advanced undergraduate or graduate students, the focus can be on material in the latter sections of Chapters 4, 5, 7, and Chapter 8 (Perron–Frobenius Theory of Nonnegative Matrices). A rich two-semester course can be taught by using the text in its entirety.

What Does “Applied” Mean?

Most people agree that linear algebra is at the heart of applied science, but there are divergent views concerning what “applied linear algebra” really means; the academician’s perspective is not always the same as that of the practitioner. In a poll conducted by SIAM in preparation for one of the triannual SIAM conferences on applied linear algebra, a diverse group of internationally recognized scientific corporations and government laboratories was asked how linear algebra finds application in their missions. The overwhelming response was that the primary use of linear algebra in applied industrial and laboratory work involves the development, analysis, and implementation of numerical algorithms along with some discrete and statistical modeling. The applications in this book tend to reflect this realization. While most of the popular “academic” applications are included, and “applications” to other areas of mathematics are honestly treated,

there is an emphasis on numerical issues designed to prepare students to use linear algebra in scientific environments outside the classroom.

Computing Projects

Computing projects help solidify concepts, and I include many exercises that can be incorporated into a laboratory setting. But my goal is to write a mathematics text that can last, so I don't muddy the development by marrying the material to a particular computer package or language. I am old enough to remember what happened to the FORTRAN- and APL-based calculus and linear algebra texts that came to market in the 1970s. I provide instructors with a flexible environment that allows for an ancillary computing laboratory in which any number of popular packages and lab manuals can be used in conjunction with the material in the text.

History

Finally, I believe that revealing only the scaffolding without teaching something about the scientific architects who erected it deprives students of an important part of their mathematical heritage. It also tends to dehumanize mathematics, which is the epitome of human endeavor. Consequently, I make an effort to say things (sometimes very human things that are not always complimentary) about the lives of the people who contributed to the development and applications of linear algebra. But, as I came to realize, this is a perilous task because writing history is frequently an interpretation of facts rather than a statement of facts. I considered documenting the sources of the historical remarks to help mitigate the inevitable challenges, but it soon became apparent that the sheer volume required to do so would skew the direction and flavor of the text. I can only assure the reader that I made an effort to be as honest as possible, and I tried to corroborate "facts." Nevertheless, there were times when interpretations had to be made, and these were no doubt influenced by my own views and experiences.

Supplements

Included with this text is a solutions manual and a CD-ROM. The solutions manual contains the solutions for each exercise given in the book. The solutions are constructed to be an integral part of the learning process. Rather than just providing answers, the solutions often contain details and discussions that are intended to stimulate thought and motivate material in the following sections. The CD, produced by Vickie Kearns and the people at SIAM, contains the entire book along with the solutions manual in PDF format. This electronic version of the text is completely searchable and linked. With a click of the mouse a student can jump to a referenced page, equation, theorem, definition, or proof, and then jump back to the sentence containing the reference, thereby making learning quite efficient. In addition, the CD contains material that extends historical remarks in the book and brings them to life with a large selection of

portraits, pictures, attractive graphics, and additional anecdotes. The supporting Internet site at MatrixAnalysis.com contains updates, errata, new material, and additional supplements as they become available.

SIAM

I thank the SIAM organization and the people who constitute it (the infrastructure as well as the general membership) for allowing me the honor of publishing my book under their name. I am dedicated to the goals, philosophy, and ideals of SIAM, and there is no other company or organization in the world that I would rather have publish this book. In particular, I am most thankful to Vickie Kearn, publisher at SIAM, for the confidence, vision, and dedication she has continually provided, and I am grateful for her patience that allowed me to write the book that I wanted to write. The talented people on the SIAM staff went far above and beyond the call of ordinary duty to make this project special. This group includes Lois Sellers (art and cover design), Michelle Montgomery and Kathleen LeBlanc (promotion and marketing), Marianne Will and Deborah Poulson (copy for CD-ROM biographies), Laura Helfrich and David Comdico (design and layout of the CD-ROM), Kelly Cuomo (linking the CD-ROM), and Kelly Thomas (managing editor for the book). Special thanks goes to Jean Anderson for her eagle-sharp editor's eye.

Acknowledgments

This book evolved over a period of several years through many different courses populated by hundreds of undergraduate and graduate students. To all my students and colleagues who have offered suggestions, corrections, criticisms, or just moral support, I offer my heartfelt thanks, and I hope to see as many of you as possible at some point in the future so that I can convey my feelings to you in person. I am particularly indebted to my students for conversations and suggestions that led to several improvements—special thanks goes to Michele Benzi. All writers are influenced by people who have written before them, and for me these writers include Gil Strang, Jim Ortega, Gene Golub, Charlie Van Loan, Leonid Mirsky, Ben Noble, Roger Horn, Charlie Johnson, Peter Lancaster, Paul Halmos, Franz Hohn, Richard Bellman, Nick Rose, and Pete Stewart—thanks for lighting the path. I want to offer particular thanks to Richard J. Painter and Franklin A. Graybill, two exceptionally fine teachers, for giving a rough Colorado farm boy a chance to pursue his dreams. Finally, neither this book nor anything else I have done in my career would have been possible without the love, help, and unwavering support from Bethany, my friend, partner, and wife. Her multiple readings of the manuscript and suggestions were invaluable. I dedicate this book to Bethany and our children, Martin and Holly, to our granddaughters, Margaret and Allison, and to the memory of my parents, Carl and Louise Meyer.

Carl D. Meyer
April 19, 2004