2.5 Coordinates

Definition. In 3-dimensional geometry, positions are represented by points (x, y, z). In physics, we are interested in *events* which have both time and position (t, x, y, z). The collection of all possible events is *spacetime*.

Definition. With an event (t, x, y, z) in spacetime we associate the units of cm with coordinates x, y, z. In addition, we express t (time) in terms of cm by multiplying it by c. (In fact, many texts use coordinates (ct, x, y, z) for events.) These common units (cm for us) are called *geometric units*.

Note. We express velocities in dimensionless units by dividing them by c. So for velocity v (in cm/sec, say) we associate the dimensionless velocity $\beta = v/c$. Notice that under this convention, the speed of light is 1.

Note. In an inertial frame S, we can imagine a grid laid out with a clock at each point of the grid. The clocks can by synchronized (see page 118 for details). When we mention that an object is *observed* in frame S, we mean that all of its parts are measured simultaneously (using the synchronized clocks). This can be quite different from what an observer at a point actually *sees*.

Note. From now on, when we consider two inertial frames S and S'

moving uniformly relative to each other, we adopt the conventions:

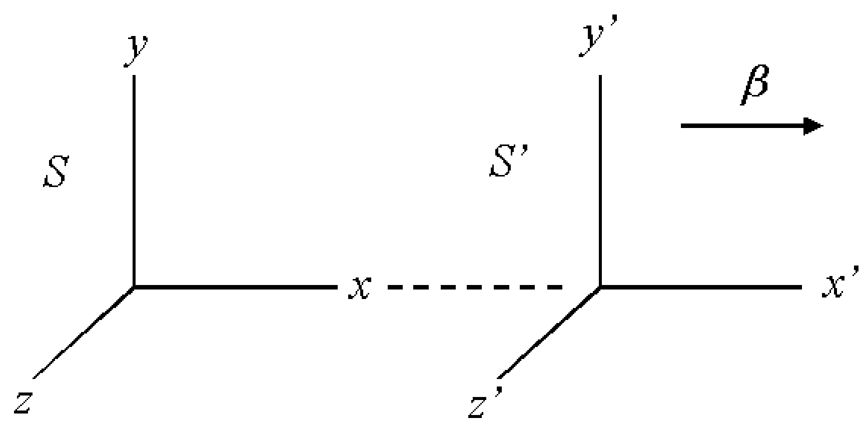
1. The $x-$ and $x'-$axes (and their positive directions) coincide.

2. Relative to S, S' is moving in the positive x direction with velocity β.

3. The $y-$ and $y'-$axes are always parallel.

4. The $z-$ and $z'-$axes are always parallel.

We call S the *laboratory frame* and S' the *rocket frame*:



Assumptions. We assume space is *homogeneous* and *isotropic*, that is, space appears the same at all points (on a sufficiently large scale) and appears the same in all directions.

Lemma. Suppose two inertial frames S and S' move uniformly relative to each other. Then lengths perpendicular to the direction of motion are the same for observers in both frames (that is, under our convention, there is no length contraction or expansion in the y or z directions).

Proof. Suppose there is a right circular cylinder C of radius R (as measured in S) with its axis along the $x-$axis. Similarly, suppose there is a right circular cylinder C' of radius R' (as measured in S') with its axis lying along the $x'-$axis. Suppose the cylinders are the same radius when "at rest." Since space is assumed to be isotropic, each observer will see a circular cylinder in the other frame (or else, there would be directional asymmetry to space). Now suppose the lab observer (S) measures a smaller radius $r < R$ for cylinder C'. Then he will see cylinder C' pass through the interior of his cylinder C (see Figure II-7, page 120). Now if two points are coincident (at the same place) in one inertial frame, then they must be coincident in another inertial frame (they are, after all, at the same place). So if the lab observer sees $\mathcal{C}'$ inside $\mathcal{C}$, then the rocket observer must see this as well. However, by the Principle of Relativity (P_1), the rocket observer must see C inside C'. This contradiction yields $r \geq R$. Similarly, there is a contradiction if we assume $r > R$. Therefore, $r = R$ and both observers see C and C' as cylinders with radius R. Therefore, there is no length change in the y or z directions. ∎

Note. In the next section, we'll see that things are much different in the direction of motion.