MATHEMATICAL METHODS: PROBLEM SET 2

October 16th, 2012

**** 1.** Cast the differential operator

$$L = (1 + x^2)D^2 + xD + sin(x)$$

into *self-adjoint* form.

**** 2.** Construct the Green's function for solving the differential operator

$$L = D^2 + D - 2$$

on the interval $[0, \pi/2]$.

**** 3.** Hence solve the differential equation

$$(D^2 + D - 2)y = e^x$$

Leave some of the solution as an integral.

***** 4.** Solve the differential equation

$$(x^2D^2 + xD + 1)y = 0$$

Hint: consider a change of variable $t = \ln x$.

****** 5.** Find any one solution, i.e. a *particular solution*, to the differential equation

$$(x^2D^2 + xD + 1)y = x^2(x + 1)$$

*** 6.** Define a power series $p(z)$ and define its radius of convergence R. Show that the radius of convergence R of

$$p(z) = \sum_{n=0}^{\infty} n!z^n$$

is zero (i.e. $p(z)$ diverges everywhere).

*** 7.** Show that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

has radius of convergence 1.

**** 8.** Show that

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots$$

has radius of convergence 1.

**** 9.** Show that

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

has radius of convergence infinity (i.e. it converges everywhere).

**** 10.** Use Taylor's theorem in the form

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots$$

to find the power series expansion of

$$\frac{1}{1+x^2}$$