



NUI MAYNOOTH  
Ollscoil na hÉireann Má Nuad

OLLSCOIL NA hÉIREANN MÁ NUAD

THE NATIONAL UNIVERSITY OF IRELAND MAYNOOTH

MATHEMATICAL PHYSICS

**Year 3**

**Semester 1**

2011–2012

**Quantum Mechanics**

**MP363**

Professor P. H. Damgaard, Professor D. M. Heffernan, Professor C. Nash.

Time allowed:  $1\frac{1}{2}$  hours

Answer **two** questions

**All questions** carry equal marks

1.

Show how the Planck radiation formula

$$I(\nu, T) = \frac{2\nu^2}{c^2} \frac{h\nu}{\exp(\frac{h\nu}{kT}) - 1}$$

reduces to the Rayleigh-Jeans formula for small frequencies and explain the significance of the two formulae.

Describe the model proposed by Bohr for the Hydrogen atom and include two of its shortcomings. Find the energies of the model and find also the radius of the second Bohr orbit.

*Note:*  $e = 1.602 \times 10^{-19} \text{ coul.}$ ,  $h = 6.63 \times 10^{-34} \text{ J s}$ ,  $m = 9.109 \times 10^{-31} \text{ kg}$ ,  $\epsilon_0 = 8.85 \times 10^{-12} \text{ volt - metre/coul.}$ ,  $c = 3 \times 10^8 \text{ m s}^{-1}$ .

2.

What is meant by an observable in quantum mechanics? Explain the measurement and probability laws of quantum mechanics and show how one ensures that transition probabilities lie between 0 and 1.

A physical system has Hamiltonian  $H$  and corresponding energy eigenstates  $\psi_i$  which obey

$$H\psi_i = i\hbar \frac{\partial \psi_i}{\partial t} = E_i \psi_i$$

If this system has a wave function  $\psi(x, y, z, t)$  normalised so that  $\langle \psi | \psi \rangle = 1$ . Show that

$$\frac{\partial}{\partial t} \langle \psi | \psi \rangle = 0$$

so that probability is conserved.

3.

An electron is confined by an infinite square well whose potential  $V(x)$  is given by

$$V(x) = \begin{cases} \infty & \text{if } -\infty < x \leq 0 \\ 0 & \text{if } 0 < x < a \\ \infty & \text{if } a \leq x < \infty \end{cases}$$

where  $a$  is some positive constant.

Write down the Hamiltonian  $H$  and solve the associated Schrödinger equation. Show that the energies  $E_n$  of the system are quantised and obtain an expression for  $E_n$ .

If the width  $a$  is given by  $a = 6 \times 10^{-10} \text{ m}$ , calculate

- (i) The normalised wave functions for the two lowest energy states.
- (ii) The wavelength  $\lambda$  of an electromagnetic wave which could excite the electron from the lowest energy state to the second energy state.

*Note:*  $c = 3 \times 10^8 \text{ m s}^{-1}$ ,  $h = 6.6260638 \times 10^{-34} \text{ J s}$ ,  $m = 9.109 \times 10^{-31} \text{ kg}$  (*electron mass*).