

MP463 - Fall 2011  
Problem Set 1  
Angular Momentum I

1. Write down the commutation relation for the components of the angular momentum.
2. Write down the operators for components of the electron spin in the matrix representation using the basis  $\mathcal{B} = \{|+\rangle, |-\rangle\}$  given by the eigenvectors of  $S_z$ , and calculate their eigenvalues and eigenvectors.
3. Calculate the operators  $S_+$  and  $S_-$  in the the matrix representation using the basis  $\mathcal{B} = \{|+\rangle, |-\rangle\}$  given by the eigenvectors of  $S_z$ , and determine whether these operators are self-adjoint, unitary or projection operators.
4. Let  $\vec{L} = \vec{R} \times \vec{P}$  be the orbital angular momentum operator of a system.
  - (i) Express the components of this operator in the coordinate representation using canonical quantization,
  - (ii) Calculate  $[L_i, L_j] = ?$  in this representation (the indices  $i, j = x, y, z$ ).
5. Let  $\vec{L} = \vec{R} \times \vec{P}$  be the orbital angular momentum operator of a system. Calculate the commutation relations using the canonical commutation relations:
  - (i)  $[L_i, \vec{P}^2] = ?$ ,
  - (ii)  $[L_i, \vec{R}^2] = ?$ ,
  - (iii)  $[L_i, \vec{R} \cdot \vec{P}] = ?$where the index  $i = x, y, z$ .

6. Let  $\vec{L} = \vec{R} \times \vec{P}$  be the orbital angular momentum operator of a system. Verify the commutation relations
- (i)  $[L_i, R_j] = i\hbar\epsilon_{ijk}R_k$ ,
  - (ii)  $[L_i, P_j] = i\hbar\epsilon_{ijk}P_k$ .
7. Consider a quantum system of angular momentum  $j = 3/2$ :
- (i) Write down the general formulas for the action of the rising and lowering operators onto the standard basis elements, i.e.  $J_+|j, m\rangle = ?$ , and  $J_-|j, m\rangle = ?$ .
  - (ii) Calculate the matrix elements of the operators  $J_-$  and  $J_+$ .
  - (iii) Calculate  $J_x$  and  $J_y$  in the matrix representation given by the standard basis.
8. Consider a quantum system of angular momentum  $j = 1$ .
- (i) What are the eigenvalues of the operators  $J^2$  and  $J_z$ .
  - (ii) Write down a general state of the system in the basis given by the eigenvectors of  $J^2$  and  $J_z$ .
9. Let a particle be in a state  $|l, m\rangle$  with a sharp values of  $L^2$  and  $L_z$ .
- (i) Find the mean value  $\langle L_{\vec{u}} \rangle$  of the angular momentum in the direction  $\vec{u}$ .
  - (ii) Find the mean values of  $\langle L_x^2 \rangle$  and  $\langle L_y^2 \rangle$ .
10. Consider a state of a quantum system with the angular momentum  $l = 1$   $|\psi\rangle = \alpha|1, -1\rangle + \beta|1, 0\rangle + \gamma|1, 1\rangle$ .
- (i) Calculate the mean values  $\langle L_z \rangle$  and  $\langle L^2 \rangle$  for the system in this state.
  - (ii) Find the mean values of  $\langle L_x^2 \rangle$  and  $\langle L_y^2 \rangle$ .