

MP463

Problem Set 4

Spin. Addition of the angular momentum.

1. Consider a deuterium atom, i.e. a system of the nucleus with the spin $I = 1$ and an electron. The electronic angular momentum is $\vec{J} = \vec{L} + \vec{S}$ where \vec{L} is the orbital angular momentum and \vec{S} is the spin. The total angular momentum of the atom is $\vec{F} = \vec{I} + \vec{J}$ where \vec{I} is the nuclear spin. The eigenvalues of \vec{J}^2 and \vec{F}^2 are $J(J + 1)\hbar^2$ and $F(F + 1)\hbar^2$ respectively.
 - (i) What are the possible values of the quantum numbers J and F for a deuterium atom in the $1s$ ground state?
 - (ii) What are the possible values of the quantum numbers J and F for a deuterium atom in the $2p$ excited state?
2. The hyperfine structure of an atom results from the coupling of the electronic angular momentum \vec{J} with the nuclear momentum \vec{I} . The relevant azimuthal quantum numbers for the ground state cesium atoms are $J = 1/2$ and $I = 7/2$. What hyperfine states, defined by the total atomic angular momentum, correspond to the cesium ground state and what are the degeneracies of these hyperfine states.
3. Consider the system of one electron and one proton with the spin angular momenta $s_1 = 1/2$ and $s_2 = 1/2$.
 - (i) What are the possible values of the total angular momentum S ;
 - (ii) using the theory of the addition of the angular momentum, calculate all the basis vectors $\{|S = 1, M\rangle\}$ of the subspace $\mathcal{E}(S = 1)$ in terms of the basis $\{|1/2, 1/2; m_1, m_2\rangle\}$;
 - (iii) calculate the remaining basis state $\{|S = 0, M = 0\rangle\}$ in terms of the basis $\{|1/2, 1/2; m_1, m_2\rangle\}$.

(See the lecture notes for the solution.)

4. (i) Express a general quantum state $|\psi\rangle$ of an electron in the spinor notation using the representation given by the basis $\mathcal{B} = \{|\mathbf{r}, \epsilon\rangle\}$ where \mathbf{r} is the position of the electron and ϵ refers to its spin using the appropriate completeness relation, and
 - (ii) express the operators \hat{S}_- and \hat{Z} in the spinor notation.
 - (iii) express the Hamiltonian for the spin-orbit coupling $\hat{H} = \xi \hat{S} \cdot \hat{L}$ in the spinor notation.
5. What are the eigenvalues and eigenvectors of the operators \vec{J}^2 and J_z of the total angular momentum of the system of two particles with the angular momenta $j_1 = 1$ and $j_2 = 1$.
6. Calculate the Clebsch-Gordan coefficients for the total angular momentum of the particle with the orbital angular momentum $l = 1$ and the spin $s = 1/2$.