

# MP463

## Problem Set 5

### Perturbation theory

#### Fine and hyperfine structure of the hydrogen atom

1. Consider the perturbative solution of the eigenvalue problem for the Hamiltonian  $H(\lambda) = H_0 + \lambda\hat{W}$ . Expand the eigenvalues and eigenvectors in terms of  $\lambda$ , and formulate the relevant general equations for the  $0^{th}$ ,  $1^{st}$ ,  $2^{nd}$  and  $q^{th}$  order.
2. Consider a small perturbation of a non-degenerate energy level. Derive the first order correction to the eigenvalue and eigenvector using the perturbation theory.
3. Consider a particle of mass  $m$  in one-dimensional infinite potential well of width  $a$ , i.e.  $V(x) = 0$  for  $0 \leq x \leq a$ , and  $V(x) = \infty$  for  $x > a$ . The particle is subject to perturbation of the form

$$W(x) = a\omega_0\delta(x - \frac{a}{2}) \quad (1)$$

where  $\delta(x - \frac{a}{2})$  is the  $\delta$ -function. The unperturbed system has the energy eigenvalues and eigenvectors given as

$$\psi_n^0(x) = \sqrt{\frac{2}{a}} \sin(\frac{\pi nx}{a}) \quad (2)$$

$$E_n^0 = \frac{\pi^2 \hbar^2 n^2}{2ma^2} \quad (3)$$

Calculate the changes in the energy level of the particle in the first order of  $\omega_0$ .

4. Calculate the effect of the spin orbit coupling

$$W_{SO} = \frac{1}{2m_e^2 c^2} \frac{1}{R} \frac{dV(R)}{dR} \vec{L} \cdot \vec{S} \quad (4)$$

onto the 2s shell of the hydrogen atom.

5. Calculate how the spin-orbit coupling splits the 2p subshell of the hydrogen atom. The radial part of the spin orbit matrix elements is given as  $\xi_{2p} = \frac{1}{48\hbar^2} m_e c^2 \alpha^4$ . What will be the degree of degeneracy of the split levels.
6. The hyperfine structure comes from the interaction between the magnetic moment of the electron with the magnetic moment of the nuclei. For 1s level of the hydrogen atom, it has a simple form

$$W_{hf} = A \vec{I} \cdot \vec{S} \quad (5)$$

where  $\vec{I}$  is the spin of the nuclei and  $\vec{S}$  is the spin of the electron and  $A$  is a constant. Calculate the splitting of the degeneracy of the 1s state due to the hyperfine interaction.