

MP463

Problem Set 6

Time-dependent perturbation theory

Quantum theory of the system of identical particles.

1. Consider a physical system with the Hamiltonian H_0 whose eigenvalue equation is $H_0|\phi_n\rangle = E_n|\phi_n\rangle$. At time $t = 0$ a time-dependent perturbation $W(t)$ is applied, so the new Hamiltonian is obtained

$$H(t) = H_0 + \hat{W}(t) = H_0 + \lambda W(t) \quad (1)$$

where $\lambda \ll 1$ and $W(t=0) = 0$. Using the time-dependent perturbation theory, we can approximate the state $|\psi(t)\rangle$ of the system at a later time $t > 0$. We express it as a linear combination of the eigenstates of H_0

$$|\psi(t)\rangle = \sum_n c_n(t) |\phi_n\rangle \quad (2)$$

where $c_n(t) = \langle \phi_n | \psi(t) \rangle$, and for small perturbation we can write it as

$$c_n(t) = b_n(t) e^{-iE_n t / \hbar} \quad (3)$$

where $b_n(t)$ is a slowly varying function of time. Substituted into the Schrödinger equation it gives

$$i\hbar \frac{d}{dt} b_n(t) = \lambda \sum_k e^{i\omega_{nk}t} W_{nk}(t) b_k(t) \quad (4)$$

where we have introduced the Bohr frequency $\omega_{nk} = (E_n - E_k)/\hbar$. Assume that the initial state of the system is $|\psi(t=0)\rangle = |\phi_i\rangle$. Using the time-dependent perturbation theory, derive the state $|\psi(t)\rangle$ of the system at a later time $t > 0$ to the first order.

2. Derive the transition probability $\mathcal{P}_{if}(t; \omega)$ in a two-level atom induced by sinusoidal perturbation, and show its resonant character.

3. Write down the Fermi golden rule and explain in words its individual terms.
4. Explain what are bosons and fermions.
5. Symmetrize and antisymmetrize a state of three particles.
6. Explain what the Pauli exclusion principle is, what type of particles it is relevant to, and demonstrate how it works on the example of the system of two identical relevant particles.