

## Aims and Objectives Quantum Physics I Session 15

### QUANTUM MEASUREMENT OF SUPERPOSITION STATES

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#### **Aims (What I intend to do)**

- 1) To look at the consequences of the third postulate of quantum mechanics.
- 2) To define what is meant by 'expectation value' of a measurement on a quantum system.

#### **Objectives (What you should be able to do after completing the lecture and worksheet)**

- 1) To be able to describe the possible outcomes of a measurement on a quantum system in a superposition state.
- 2) To be able to calculate the probability of different outcomes of a measurement on a system once the superposition state of the system has been established, and to be able to calculate the expectation value of the measurement on that system.

## Quantum Physics 1 PHY2002 Worksheet 15

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**Task 1.** Go over your lecture notes and consult sections 4.3 of Rae if you wish.

**Task 2.** A particle in an infinite square well is prepared in a state described by the wavefunction:

$$\begin{aligned}\Psi &= ax && \text{for } 0 < x < \frac{L}{2} \\ &= a(L-x) && \text{for } \frac{L}{2} < x < L \\ &= 0 && \text{elsewhere}\end{aligned}$$

It is straightforward to show that a value of  $a$  which normalizes  $\Psi$  is  $\sqrt{\frac{12}{L^3}}$  – try this.

a) Show that this wavefunction can be written as the following superposition of particle-in-a-box energy eigenfunctions (which as we know are of the form

$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right):$$

$$\Psi = \sum_{n \text{ odd}} \frac{\sqrt{96}}{n^2 \pi^2} \sin\left(\frac{n\pi x}{L}\right)$$

(Mathematically this is simply the Fourier sine series for  $\Psi$ )

- b) Write down the possible outcomes of a measurement of the energy of this particle, and calculate the probabilities for the two most probable outcomes.
- c) Estimate the expectation value of the energy, giving an answer that is correct to two significant figures.

**Task 3.** Follow through this proof about expectation values.

Show that  $\langle \hat{A} \rangle = \int \Psi^* \hat{A} \Psi dx = \sum_n |c_n|^2 a_n$

Proof: We write the wavefunction and its complex conjugate as linear superpositions of the eigenfunctions,

$$\begin{aligned}\int \Psi^* \hat{A} \Psi dx &= \int \sum_m c_m^* \psi_m^* \hat{A} \left( \sum_n c_n \psi_n \right) dx \\ &= \int \sum_m c_m^* \psi_m^* \left( \sum_n c_n a_n \psi_n \right) dx \\ &= \sum_n |c_n|^2 a_n\end{aligned}$$

where, in the last step, we have used the orthonormality of  $\{\psi_n\}$ .