

Aims and Objectives Quantum Physics I (PHY2002) Session 2

INTERPRETING THE WAVEFUNCTION

Aims (What I intend to do)

- 1) To highlight some of the counter intuitive aspects of Physics.
- 2) To look at how we can make sense of quantum interference, for example Young's double slits experiment with electrons.
- 3) To introduce the concept of the wavefunction as a wave way of describing a particle, both free and subject to a potential.

Objectives (What you should be able to do after completing the lecture and worksheet)

- 1) To be able to describe an interference experiment that cannot be explained using classical Physics.
- 2) To be able to discuss why measuring which slit an electron goes through in a Young's double slit experiment destroys the interference pattern.
- 3) To be able to explain the relationship between the wavefunction $\Psi(x, t)$ of a particle, and the probability of finding it between x and $x + dx$.

Quantum Physics 1 PHY2002 Worksheet 2

- Task 1.** Go over your lecture notes and READ Chapter 37 of Vol 1 of the Feynman Lectures on Physics. It has an EXCELLENT discussion of the double slit experiment.
- Task 2.** A Neodymium (Nd^{3+}) laser operates at a wavelength of 1.06×10^{-6} m. If the laser is operated in a pulsed mode, emitting pulses of duration 3×10^{-11} s, what is the minimum spread in (a) frequency and (b) wavelength of the laser beam?
- Task 3.** A uniform beam of monochromatic light of wavelength 4130 \AA and intensity $3.00 \times 10^{-14} \text{ W/m}^2$ is incident normally on a $10 \times 10 \text{ cm}$ screen. Assume that the light is completely absorbed by the screen, each photon producing a measurable flash (we don't have this technology yet – but its on its way!) find the average photon flux on any 1.0 cm^2 of surface. Comment on your findings – does each 1.0 cm^2 register the same number of photons?
- Task 4.** Comparison of classical and quantum predications. Consider the measurement of the position of a particle at the 'screen' in a double slit experiment. Describe what would happen,
a) on the basis of a classical particle picture,
b) on the basis of a classical wave picture,
c) on the basis of the quantum picture.
- Task 5.** Read some background for the next session, on wavepackets. Young and Freedman has a short section you could try – use the index to find the relevant section and read it.

Solution to task 3 of Worksheet 1.

We are looking for a wave equation. We start with our solution which is,

$$\Psi_f(x,t) = A \exp[i(kx - \omega t)]$$

Differentiating once w.r.t. time we find,

$$\frac{\partial \Psi_f}{\partial t} = -i\omega \Psi_f$$

Differentiating twice w.r.t. x we find,

$$\frac{\partial^2 \Psi_f}{\partial x^2} = -k^2 \Psi_f$$

Combining these two equations by eliminating $\Psi_f(x,t)$, we find,

$$\frac{i}{\omega} \frac{\partial \Psi_f}{\partial t} = \frac{-1}{k^2} \frac{\partial^2 \Psi_f}{\partial x^2}$$

Now, using $k = p/\hbar$ and $\omega = E/\hbar$ we obtain,

$$\frac{i\hbar}{E} \frac{\partial \Psi_f}{\partial t} = \frac{-\hbar^2}{p^2} \frac{\partial^2 \Psi_f}{\partial x^2} \quad (\text{equation 10 from the lecture})$$

If we now make use of the fact that $E = p^2/2m$, then we find

$$i\hbar \frac{\partial \Psi_f}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \Psi_f}{\partial x^2}$$

As required.

Solution to task 4 of Worksheet 1.

Here we have to take equation the equation,

$$\frac{i\hbar}{E} \frac{\partial \Psi_f}{\partial t} = \frac{-\hbar^2}{p^2} \frac{\partial^2 \Psi_f}{\partial x^2}$$

and make use of $E = p^2/2m + V(x)$ rather than $E = p^2/2m$. If we simply substitute our new value for E into equation 10 we obtain,

$$\begin{aligned} i\hbar \frac{\partial \Psi}{\partial t} &= \frac{-\hbar^2 \left[\frac{p^2}{2m} + V(x) \right] \partial^2 \Psi}{p^2 \partial x^2} \\ &= \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} - \frac{\hbar^2 V(x)}{p^2} \frac{\partial^2 \Psi}{\partial x^2} \end{aligned}$$

(We have changed the notation for the wavefunction from Ψ_f to Ψ because our particle is no longer free, it is constrained by the potential $V(x)$). For reasons that will become clear later in the module (hindsight is a wonderful thing) it only makes sense to modify the second of the terms on the right hand side. Now,

$$\Psi = A \exp[i(kx - \omega t)].$$

Differentiating twice w.r.t. x and recalling that $p = \hbar k$,

$$\frac{\partial^2 \Psi}{\partial x^2} = -k^2 \Psi,$$

so that,

$$\begin{aligned} \frac{-\hbar^2 V(x)}{p^2} \frac{\partial^2 \Psi}{\partial x^2} &= \frac{-\hbar^2 V(x)}{\hbar^2 k^2} \cdot -k^2 \Psi \\ &= V(x) \Psi \end{aligned}$$

Putting it all back together we thus find,

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right] \Psi$$

- the 1-dimensional time-dependent Schrödinger equation as required (admittedly it required some 'post' knowledge, but we will find out why when we look at quantum mechanical operators).