

Photoelectric effect

Observed 1888; explanation, Einstein 1905. A negatively charged metal plate loses charge slowly if illuminated with light, while a positive charge remains. Light of

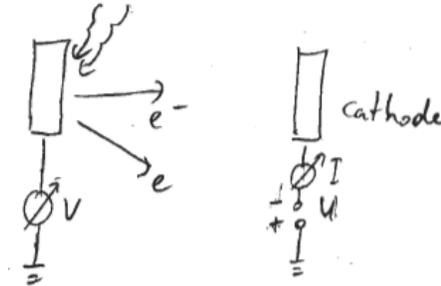


Figure I: Photoelectric effect. Electrons are released from a metal plate illuminated by monochromatic light. The energy of the released electrons is measured.

sufficiently short wavelength releases electrons, the electron current is proportional to the intensity of the light. Light below a certain cutoff frequency ν_0 does not release any electrons, no matter how high the intensity. Energy of released electrons can be measured by determining voltage V that prevents them from reaching cathode. A linear relation between the electron's kinetic energy W_{kin} and the frequency ν of the incident light is observed. The kinetic energy of the electrons does not depend on the intensity of the incident light.

Classical prediction

Incident power per unit area is given by intensity of light, and independent of frequency, thus electron's kinetic energy should be proportional to intensity (independent of ν), and there should be no cutoff frequency.

Einstein's explanation, 1905

Nobel prize, 1921. Einstein assigned physical reality to Planck's mathematical constructs (quanta of light):

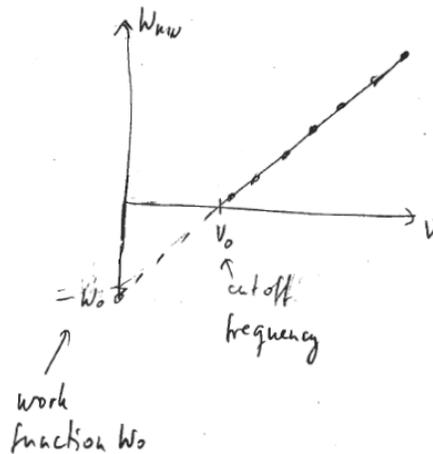


Figure II: The value of the cutoff frequency ν_0 depends on the metal. The slope does not.

1. Light consists of smallest energy units (quanta, photons) of energy $E = h\nu$.
2. An electron is bound to the metal with a binding energy W_0 .
3. Each electron is ejected by a single photon.

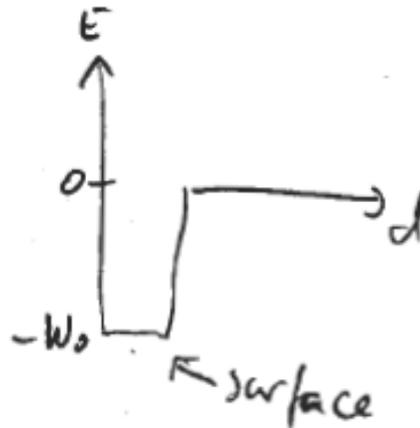


Figure III: Work function

These three assumptions can explain all of the observed features:

- Energy conservation in the process requires that

$$h\nu = W_0 + W_{\text{KIN}}$$

- Linear dependence of light frequency ν on electron's kinetic energy W_{KIN} with intercept $-W_0$.
- The slope of W_{KIN} is given by Planck's constant h .

- No photoelectrons can be released below cutoff frequency $\nu_0 = \frac{W_0}{h}$, the value of which is metal-dependent. W_0 is typically a few eV.
- Photoelectric current is proportional to the photon arrival rate, i.e. to the light intensity.

Einstein's explanation, in combination with experiments, also implied the **wave-particle duality** and the **stochastic nature** of the quantization hypothesis: When

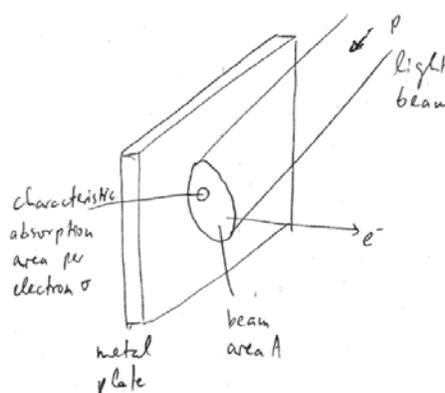


Figure IV: Photoelectric effect and wave-particle duality.

we lower the incident power to a level P , corresponding to a photon arrival rate $R = \frac{P}{h\nu}$, the first photoelectron is observed on average after a time $t_0 = \frac{1}{R}$. This is very surprising since the electron in an atom only occupies a small area $\sigma \ll A$. One might think that $\sigma = \pi r^2$, where $r \approx 1 \text{ \AA}$ is the atom radius. However, this is wrong because the oscillating electron represents an (optimally matched) dipole antenna, so $\sigma \sim \lambda^2 \sim (1 \mu\text{m})^2 \gg \pi(1 \text{ \AA})^2$ (by some 8 orders of magnitude). Nevertheless, $\sigma/A \ll 1$ ($\sigma = 10^{-8} \text{ cm}^2$, $A=1 \text{ cm}^2$, so it is quite impossible for enough light energy to have accumulated on average within the area σ after a time t_0). We must conclude that the full photon energy is located (randomly) somewhere within the beam area A to a spot size less or equal to σ : stochastic location of photons within beam, particle-like nature of photons. Nevertheless, the same light still behaves like a wave, e.g. exhibits diffraction, interference etc.: **wave-particle duality**.

In the photoelectric effect photons produce electrons whose kinetic energy depends on the photon frequency due to energy conservation. We now consider the reverse process, namely, the production of electromagnetic waves (photons) by electron impact. Energy conservation now determines the photon spectrum: x-rays.

X-ray production

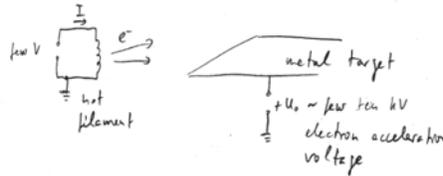


Figure V: X-ray production.

Electrons are accelerated in an electric field and gain a kinetic energy $W_{\text{KIN}} = qV_0$ that can be converted into electromagnetic radiation (photons) as they hit (decelerate in) the target. The observed spectrum of the emitted radiation is: λ_{min} is independent

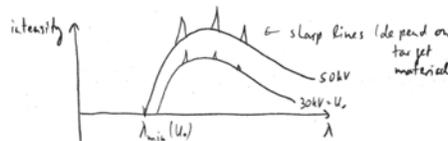


Figure VI: Spectrum of emitted X-rays.

of target material and limited by energy conservation: electron's total KE is converted into a single x-ray photon.

$$q_e V_0 = h\nu_{\text{max}} = h \frac{c}{\lambda_{\text{min}}} \quad (6-1)$$

Short-wavelength cutoff of x-rays:

$$\lambda_{\text{min}} = \frac{hc}{q_e V_0} = \frac{1.24 \text{ \AA}}{V_0 / 10 \text{ kV}} \quad (6-2)$$

X-ray emission is due to the deceleration of electrons in the field of the nucleus. Classically, since the electron can turn around very sharply within a very short time Δt , we expect a continuous spectrum extending to very high frequencies $\Delta = \frac{1}{\Delta t}$. Instead, we observe a cutoff due to energy conservation: all of the electron's KE is converted into a single photon. The superimposed line spectrum is due to electronic transitions in atoms (strongly bound inner-shell electrons). X-ray spectra can be measured using crystals as diffraction gratings.



Figure VII: German: Bremsstrahlung “braking emission” of an electron in the field of a nucleus.

Double-slit experiment, electrons as waves

To observe wave behavior for particles, we need a situation where a single classical path can no longer be ascribed to the particle. One example is single-slit diffraction, another even more striking example is double-slit interference: The interference pat-



Figure VIII: Interference pattern for double slit. The interference pattern persists even if there is only one electron in the apparatus at any given time.

tern is observed even if the beam is sufficiently attenuated so that there is only one electron in the apparatus at any given time: This indivisible electron passes through slit 1 **and** slit 2. Any measurement that allows one to determine which slit the electron actually passed through destroys the interference pattern: Blocking one of the slits obviously destroys the interference, but what about more subtle methods, such as optical observation (light scattering)?

Which-path measurements and destruction of interference

To optically resolve which slit the electron passed through we need short-wavelength light with $\lambda_p < d$. But then from Compton scattering we know that the momentum

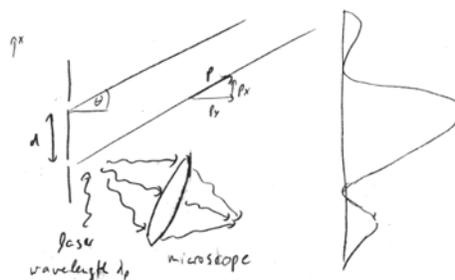


Figure IX: The interference disappears when the electron scatters light so that it is possible to determine (by optical imaging) which slit it passed through.

of the electron will be changed by an amount between 0 (photon scattered in forward direction, no momentum transferred onto electron) and $2\hbar k_p$ (photon scattered backward). Thus the unknown (unpredictable) x -momentum change of the electron is of order $\Delta p_x \sim 2\hbar k_p$. There is also y -momentum change, but it is less relevant. On the other hand, the x -momentum corresponding to constructive interference on the first maximum ($d \sin \theta_1 = \lambda_{\text{dB}}$) is $p_x = p \sin \theta_1 = p \frac{\lambda_{\text{dB}}}{d} = \frac{h}{\lambda_{\text{dB}}} \frac{\lambda_{\text{dB}}}{d} = \frac{h}{d}$. Since the uncertainty in momentum due to photon scattering is $\Delta p_x \sim 2\hbar \frac{2\pi}{\lambda_p} = \frac{h}{\lambda_p}$ and $\lambda_p \ll d$ in order to observe which slit the electron passed through, we must conclude that the interference pattern will be completely smeared out due to the momentum uncertainty Δp_x associated with spatial observation through photon scattering. As already in the single-slit case, the localization of a particle, be it by means of an aperture (slit) or photon scattering, acts back on the momentum and smears it out. The momentum spread, in turn, destroys the interference pattern. This is true for any type of measurement you can devise that tells you which slit the electron passed through.

Which-path information

Observation of the path actually taken by the electron in any particular realization of the experiment) **destroys the interference pattern**. If we increase the wavelength of the light, i.e. lower the optical resolution of the microscope, we recover the interference pattern when $\lambda > d$, i.e. when we can no longer resolve which slit the electron passed through. What happened if instead we lower the intensity of the light until, on average, only half of the electrons scatter a photon?

The contrast of the interference pattern will be reduced to $\frac{1}{2}$. However, there is an event-by-event correlation between interference and scattering: Post-selection of only those electron arrivals where no photon was observed produce full contrast, post selection of those trials where a photon was scattered by the electron produces no

contrast at all. We say that if the electron scatters a photon, the direction of the scattered photon becomes correlated (entangled) with the electron's path. Correlations ("entanglement") between different parts of a system constitute an essential feature of **QM** and of the measurement process. The latter involves averaging over ("tracing over") those parts and degrees of freedom of the measurement apparatus that were not observed.

If measurement ("which-path information") destroys the interference pattern, when does the measurement actually occur?

- at the moment when electron scatters photon
- at the moment when photon hits the photodetector
- at the moment when we learn about the result (*hear click*)