

Quantum Mechanics

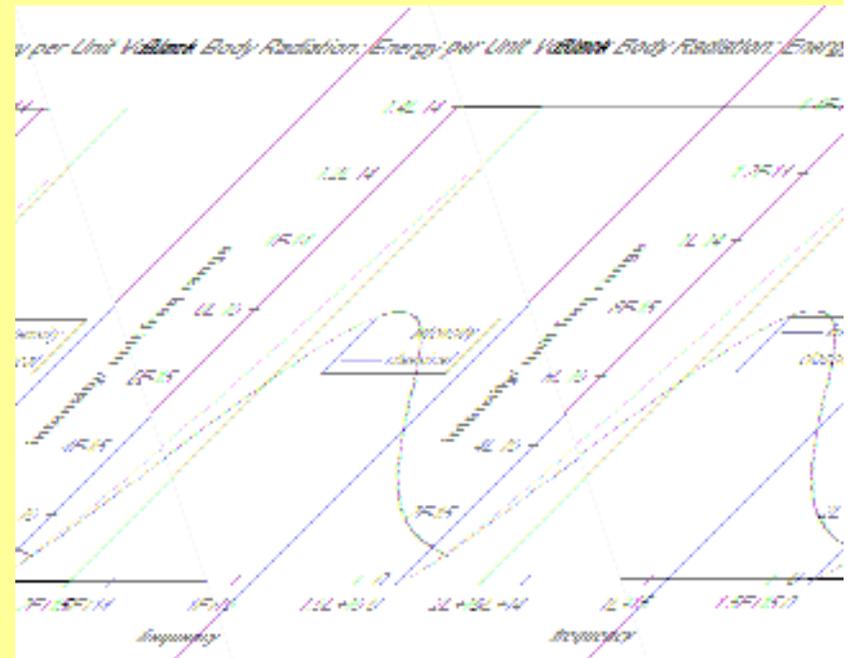
Small things are weird

## The Quantum Mechanics View

- **All matter (particles) has wave-like properties**
  - so-called particle-wave *duality*
- **Particle-waves are described in a probabilistic manner**
  - electron doesn't whiz around the nucleus, it has a probability distribution describing where it might be found
  - allows for seemingly impossible “quantum tunneling”
- **Some properties come in dual packages: can't know both simultaneously to arbitrary precision**
  - called the Heisenberg Uncertainty Principle
  - not simply a matter of measurement precision
  - position/momentum and energy/time are example pairs
- **The act of “measurement” fundamentally alters the system**
  - called entanglement: information exchange alters a particle's state

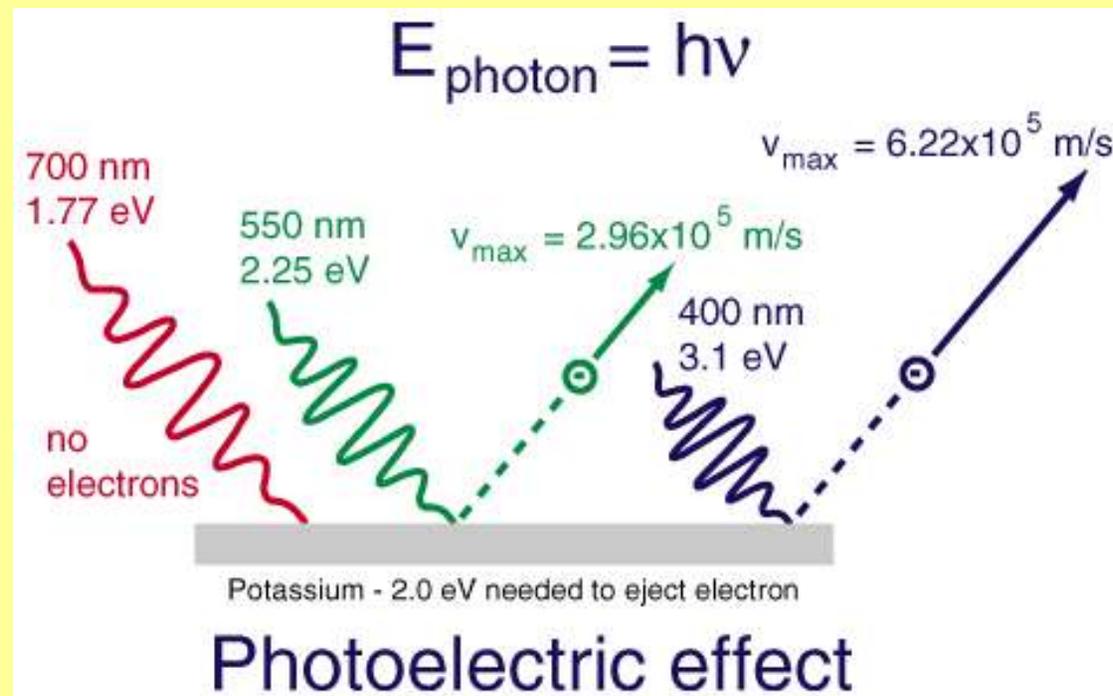
## Crises in physics that demanded Q.M.

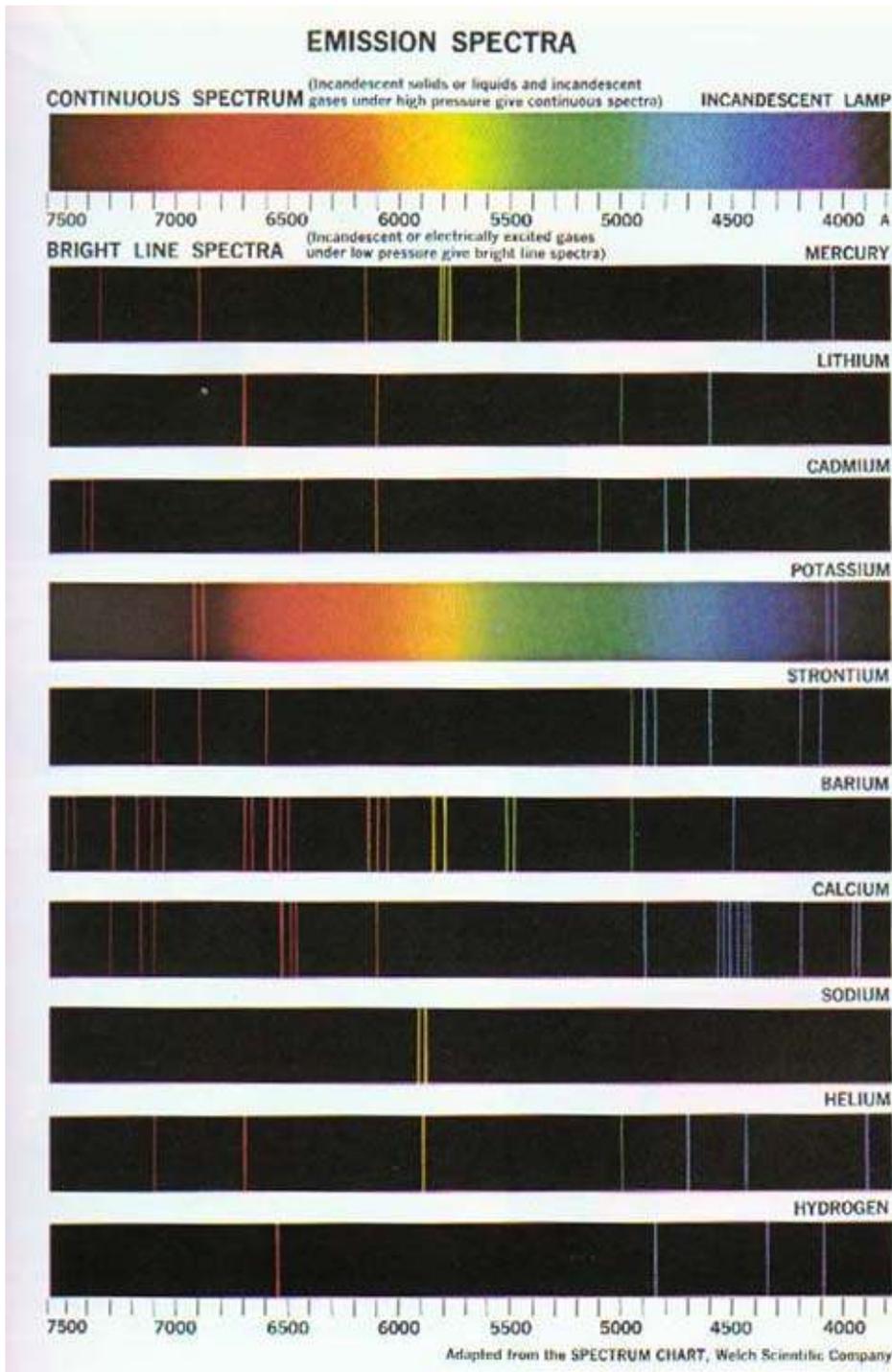
- **Why don't atoms disintegrate in nanoseconds?**
  - if electron is “orbiting”, it's accelerating (wiggling)
  - wiggling charges emit electromagnetic radiation (energy)
  - loss of energy would cause prompt decay of orbit
- **Why don't hot objects emit more ultraviolet light than they do?**
  - classical theory suggested a “UV catastrophe,” leading to obviously nonsensical infinite energy radiating from hot body
  - Max Planck solved this problem by postulating light quanta (now often called the father of quantum mechanics)



## Pre-quantum problems, cont.

- Why was red light incapable of knocking electrons out of certain materials, no matter how bright
  - yet blue light could readily do so even at modest intensities
  - called the photoelectric effect
  - Einstein explained in terms of photons, and won Nobel Prize





## Problems, cont.

- What caused spectra of atoms to contain discrete “lines”
  - it was apparent that only a small set of optical frequencies (wavelengths) could be emitted or absorbed by atoms
- Each atom has a distinct “fingerprint”
- Light only comes off at very specific wavelengths
  - or frequencies
  - or energies
- Note that hydrogen (bottom), with only one electron and one proton, emits several wavelengths

## The victory of the weird theory

- **Without Quantum Mechanics, we could never have designed and built:**
  - semiconductor devices
    - computers, cell phones, etc.
  - lasers
    - CD/DVD players, bar-code scanners, surgical applications
  - MRI (magnetic resonance imaging) technology
  - nuclear reactors
  - atomic clocks (e.g., GPS navigation)
- **Physicists didn't embrace quantum mechanics because it was gnarly, novel, or weird**
  - it's simply that the #!&@ thing worked so well

## Let's start with photon energy

- Light is *quantized* into packets called *photons*
- Photons have associated:
  - frequency,  $\nu$  (nu)
  - wavelength,  $\lambda$  ( $\lambda\nu = c$ )
  - speed,  $c$  (always)
  - energy:  $E = h\nu$ 
    - higher frequency photons  $\rightarrow$  higher energy  $\rightarrow$  more damaging
  - momentum:  $p = h\nu/c$
- The constant,  $h$ , is Planck's constant
  - has *tiny* value of:  $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$

## How come *I've* never seen a photon?

- **Sunny day (outdoors):**
  - $10^{15}$  photons per second enter eye (2 mm pupil)
- **Moonlit night (outdoors):**
  - $5 \times 10^{10}$  photons/sec (6 mm pupil)
- **Moonless night (clear, starry sky)**
  - $10^8$  photons/sec (6 mm pupil)
- **Light from dimmest naked eye star (mag 6.5):**
  - 1000 photons/sec entering eye
  - integration time of eye is about 1/8 sec  $\rightarrow$  100 photon threshold signal level

## Quantum Wavelength

- Every particle or system of particles *can* be defined in quantum mechanical terms
  - and therefore have wave-like properties
- The quantum wavelength of an object is:
  - $$\lambda = h/p \quad (p \text{ is momentum})$$
  - called the de Broglie wavelength
- typical macroscopic objects
  - masses  $\sim$  kg; velocities  $\sim$  m/s  $\rightarrow p \approx 1$  kg·m/s
  - $\lambda \approx 10^{-34}$  meters (too small to matter in macro environment!!)
- typical “quantum” objects:
  - electron ( $10^{-30}$  kg) at thermal velocity ( $10^5$  m/s)  $\rightarrow \lambda \approx 10^{-8}$  m
  - so  $\lambda$  is 100 times larger than an atom: very relevant to an electron!

# The Uncertainty Principle

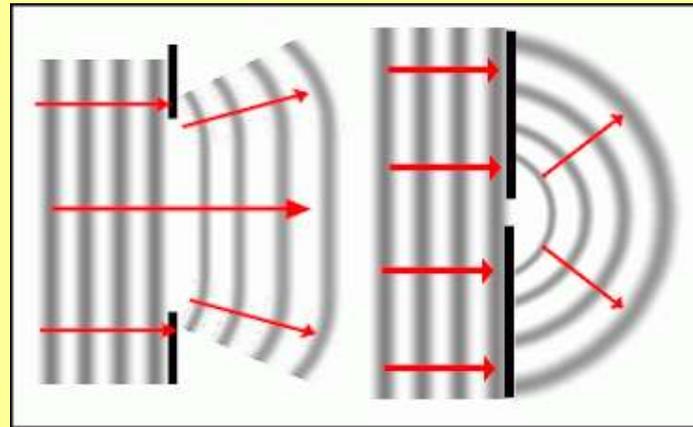
- **The process of measurement involves interaction**
  - this interaction necessarily “touches” the subject
  - by “touch,” we could mean by a photon of light
- **The more precisely we want to know where something is, the “harder” we have to measure it**
  - so we end up giving it a kick
- **So we must unavoidably alter the velocity of the particle under study**
  - thus changing its momentum
- **If  $\Delta x$  is the position uncertainty, and  $\Delta p$  is the momentum uncertainty, then inevitably,**

$$\Delta x \Delta p \geq h/2\pi$$

## Example: Diffraction

- Light emerging from a tiny hole or slit will diverge (diffract)
- We know its position very well (in at least one dimension)
  - so we give up knowledge of momentum in that dimension—thus the spread

large opening: greater position uncertainty results in smaller momentum uncertainty, which translates to less of a spread angle



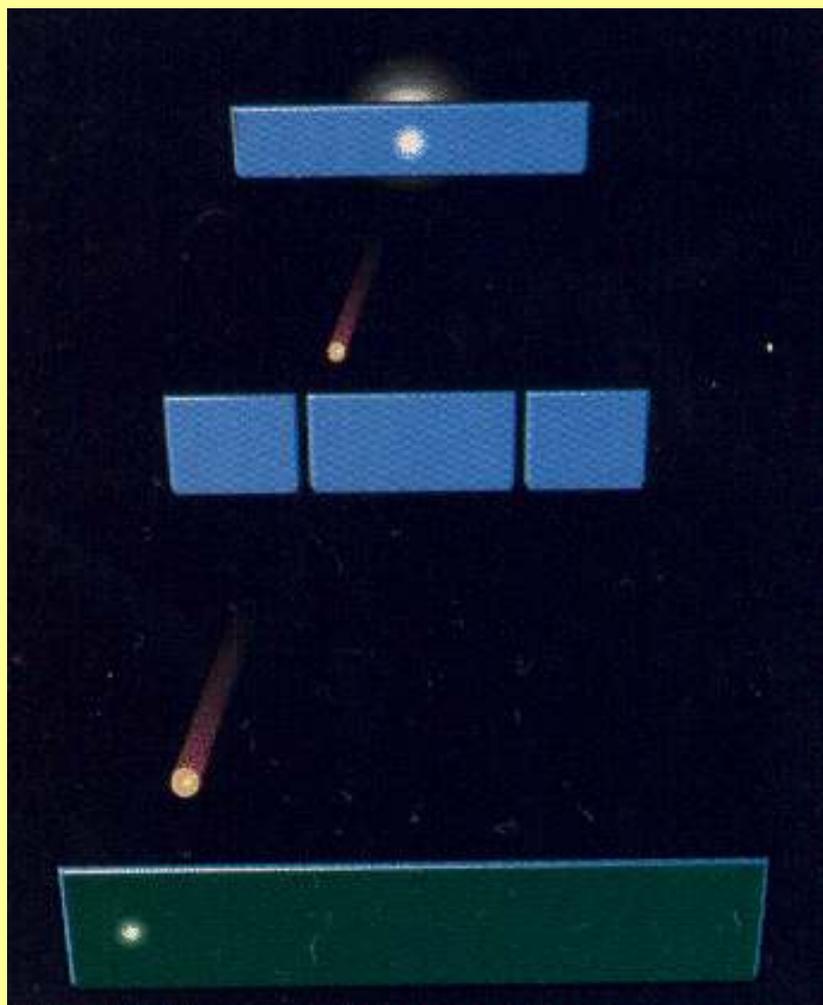
small opening: less position uncertainty results in larger momentum uncertainty, which translates to more of a spread angle

$$\text{angle} \approx \Delta p/p \approx h/p\Delta x \approx h\lambda/h\Delta x = \lambda/\Delta x$$

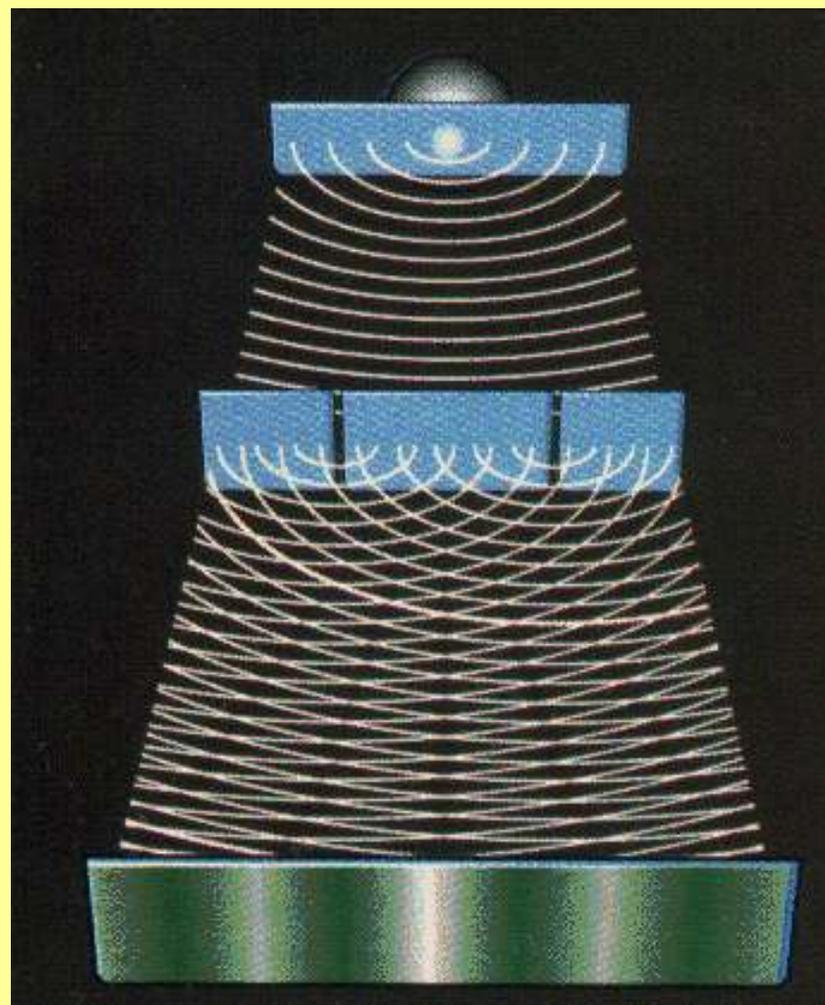
## Diffraction in Our Everyday World

- **Squint and things get fuzzy**
  - opposite behavior from particle-based pinhole camera
- **Eye floaters**
  - look at bright, uniform source through tiniest pinhole you can make—you'll see slowly moving specks with rings around them—*diffraction rings*
- **Shadow between thumb and forefinger**
  - appears to connect before actual touch
- **Streaked street-lights through windshield**
  - point toward center of wiper arc: diffraction grating formed by micro-grooves in windshield from wipers
  - same as color/streaks off CD

# The Double Slit Experiment



particle?



wave?

## Results

- The pattern on the screen is an interference pattern characteristic of waves
- So light is a wave, not particulate
- But repeat the experiment one photon at a time
- Over time, the photons *only land on the interference peaks*, not in the troughs
  - consider the fact that they also pile up in the middle!
  - pure ballistic particles would land in one of two spots

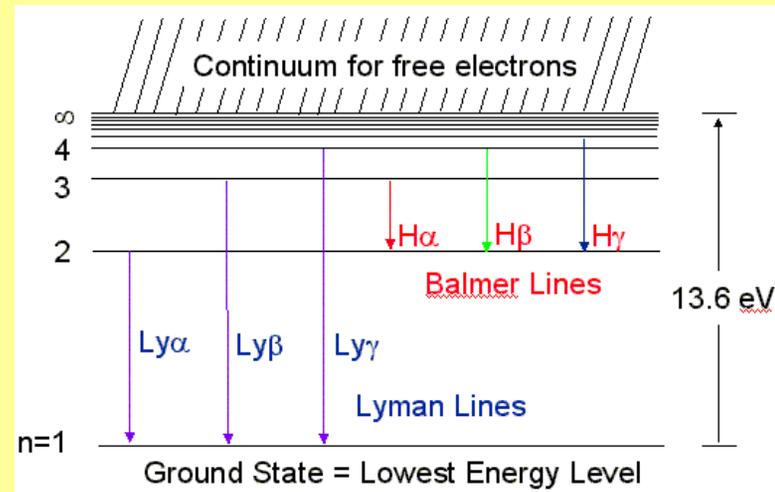
## Wave or Particle? Neither; Both; take your pick

- Non-intuitive combination of wavelike *and* particle-like
- Appears to behave in wavelike manner. But with low intensity, see the interference pattern build up out of individual photons, arriving one at a time.
- How does the photon *know* about “the other” slit?
  - Actually, it’s impossible to *simultaneously* observe interference *and* know which slit the photon came through
  - Photon “sees”, or “feels-out” *both* paths simultaneously!
- Speak of wave-part describing *probability distribution* of where individual photons may land

## The hydrogen atom

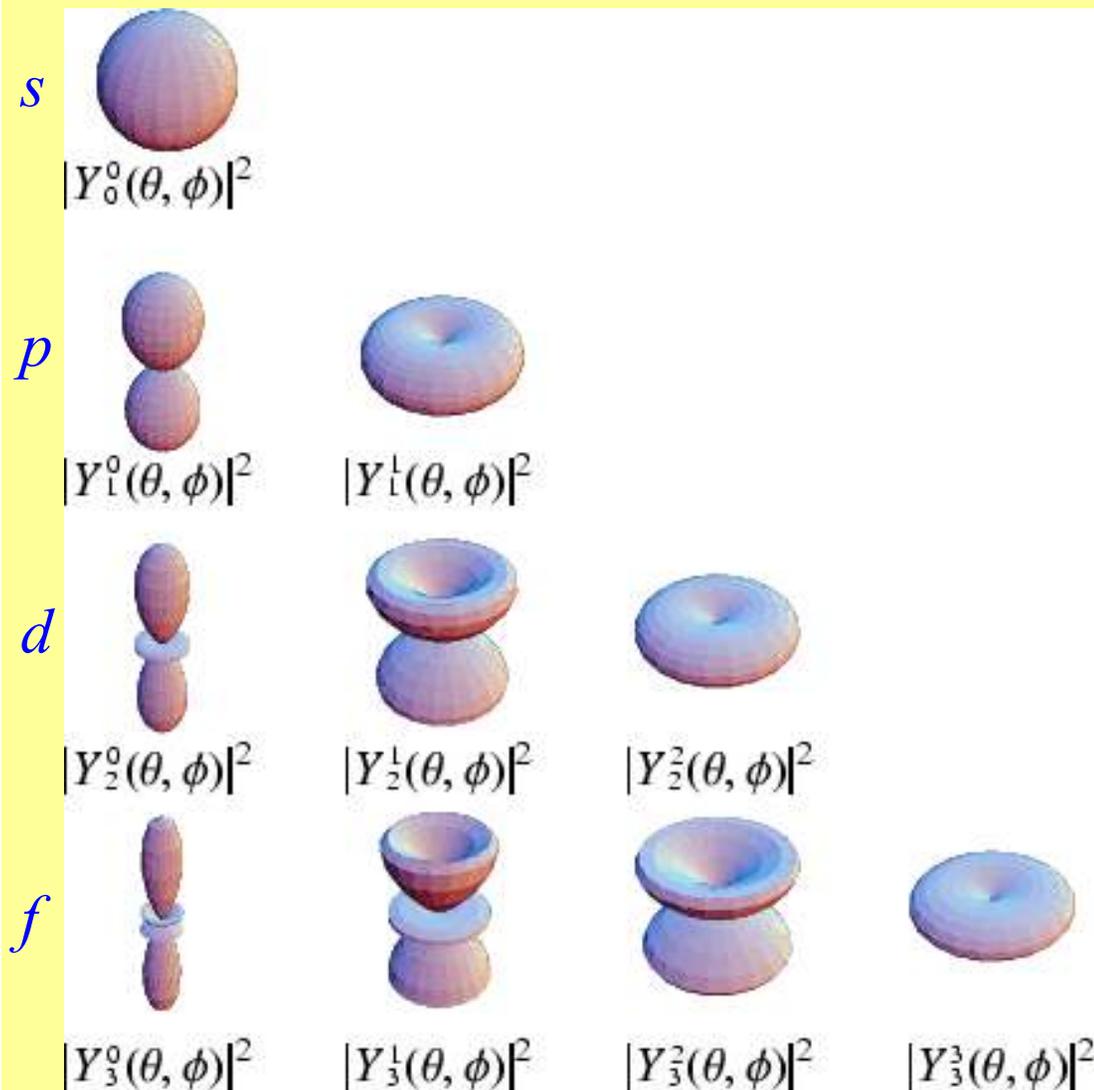
- When the mathematical machinery of quantum mechanics is turned to the hydrogen atom, the solutions yield energy levels in exact agreement with the optical spectrum
  - Emergent picture is one of probability distributions describing where electrons can be

The energy levels of hydrogen match the observed spectra, and fall out of the mathematics of quantum mechanics



- **Probability distributions are static**
  - electron is not thought to whiz around atom: it's in a “stationary state” of probability
- **Separate functions describe the radial and angular pattern**
  - <http://hyperphysics.phy-astr.gsu.edu/hbase/hydwf.html>

## The angular part of the story



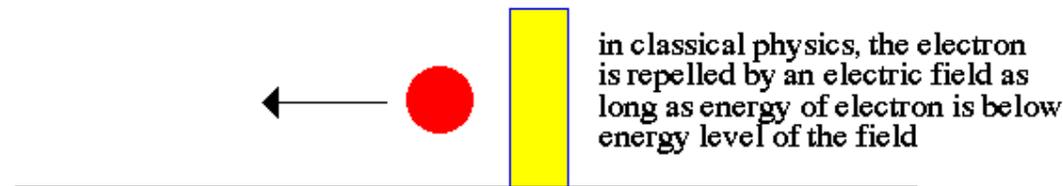
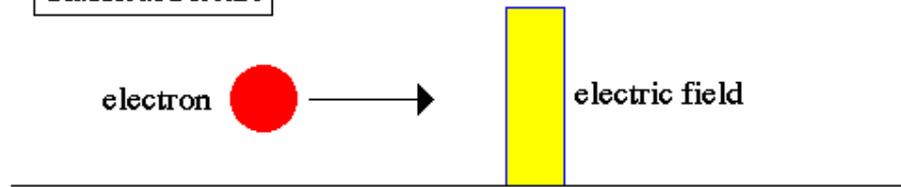
These plots describe the directions in which one is likely to find an electron. They are denoted with quantum numbers  $\ell$  and  $m$ , with  $\ell$  as the subscript and  $m$  as the superscript.

The  $s$  state ( $\ell=0, m=0$ ) is spherically symmetric: equal probability of finding in all directions.

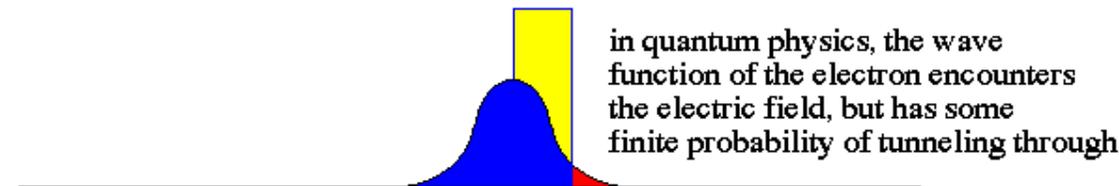
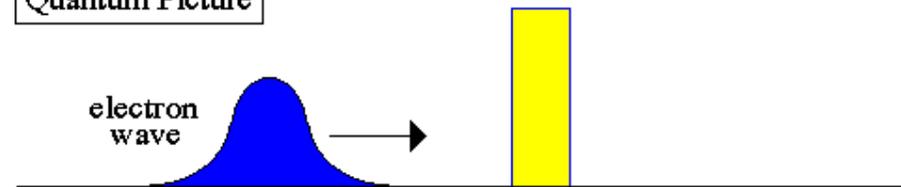
The  $p$  state can be most likely to find at the poles (and not at all at the equator) in the case of  $(1,0)$ , and exactly the opposite situation in the  $(1,1)$  state.

Quantum Tunneling

Classical Picture



Quantum Picture



electron always repelled

electron usually repelled, but will occasionally pop out on the other side of the barrier, even though it does not have enough energy to do so classically

Why do we not see tunneling in our daily lives?

If the wall is much thicker than the quantum wavelength, tunneling becomes improbable

## Assignments

- **References:**
  - Brian Greene's *The Elegant Universe* has an excellent description/analogy of the quantum solution to the ultraviolet catastrophe (among other quantum things)
  - Chapter 31 isn't half bad: read it for fun, even!
- **Assignments:**
  - Read Hewitt chapters 30 & 31 (Quantum & Light)
  - Read Hewitt pp. 566–572 on diffraction & interference
  - HW 7 due 3/10: 26.E.3, 26.E.4, 26.E.7, 26.E.9, 26.E.10, 26.E.11, 26.E.14, 26.E.38, 26.P.4, 31.E.4, 31.E.9, 30.E.42