

Quantum mechanics teaching at Heriot-Watt

Erika Andersson, Heriot-Watt University

(Scottish year N is equivalent to English year $N-1$. Appropriately qualified students enter directly into year 2.)

An issue is the weak mathematical skills of students. This has caused some staff members to partly rethink the way quantum mechanics is taught, while other staff members continue with a more traditional approach. In short, material on two-level systems and an introduction to quantum information, entanglement and quantum measurements has been introduced in years 3 and 4. The changes have been possible within the current core curriculum. The “more traditional” material is also covered, partly in other courses. A full solution of the Schrödinger equation for the hydrogen atom, for example, is covered in the 3rd year course Mathematical Physics 2, in connection with differential equations.

Student feedback has been excellent, with students feeling that they well can handle the newly introduced material. It has also inspired students to pursue BSc, Masters and PhD projects in areas related to quantum mechanics (not necessarily quantum information science). Exam results have also been encouraging in that the students actually do seem to master the new material better than some of the more traditional and mathematically more difficult material.

An important aim is that students should be armed with knowledge of how quantum mechanics is important for our daily lives (lasers, semiconductors, MRI, atomic clocks for GPS...). All through, examples are given e.g. of how the formalism for two-level systems applies to many different physical situations, such as double-well potentials, molecules, atoms interacting with a laser field resonant only with two levels, photon polarisation, and also qubits in quantum information – which of course always have to be realized by some kind of physical two-level systems.

While it is important to arm the students with the basic quantum mechanics knowledge they need to tackle further areas (solid state, materials science, nuclear physics etc.), it is also important to introduce material that excites and enthuses students, as this leads to effective learning. According to the survey of graduates, presented at the meeting in London on 7 Dec, it seems that soft skills such as problem solving, report writing etc. are more useful, and the knowledge of physics topics less so. But the soft skills are necessarily acquired while studying those “less useful” hard physics topics. It would be interesting to ask the graduates which physics topics they think most benefited e.g. their problem-solving skills. Quantum mechanics may have a role to play here, since its many counter-intuitive aspects challenges students to “think outside the box”. A topic that enthuses students is also likely to stimulate more active and inquisitive problem-solving, and hence may be beneficial for this reason. Firm conclusions are of course not possible to draw without actually doing such a survey. Nevertheless, for example, talking about the “quantum bomb detector” lead to a lively discussion, and to students drawing their own correct conclusions about what constitutes an “observation” in quantum mechanics. After tutorial problems on how one would represent a spatial 50/50 beam splitter and a

polarising beam splitter with a 2×2 and 4×4 matrix, respectively, a student at his own initiative worked out how a the 4×4 matrix for a 50/50 spatial beam splitter acting on polarisation states would look.

Lecture notes (80 pages) are handed out to students and available on a course web site. In these notes, detailed references for further reading are made, mostly to chapters in “Quantum Mechanics” by S. Gasiorowicz, and in the online lecture notes by James Cresser, available at <http://web.science.mq.edu.au/~jcresser/Phys304.html>. Not all students buy a book. The online lecture notes are therefore a useful resource. A list of other useful standard books on quantum mechanics is also given.

We also make extensive use of the many excellent interactive animations, available online: Physics Education Technology (PhET) by the Educational Physics Research group at Boulder, Colorado (Carl Wieman), available at <http://phet.colorado.edu/>. Quantum mechanics animations by Antje Kohnle, St Andrews, available at <http://www.st-andrews.ac.uk/~qmanim/>. These are especially useful when discussing time evolution, wave functions, tunnelling, eigenfunctions for different potentials, and superpositions of eigenfunctions. Some tutorial problems requires students to find out “what will happen” using an interactive animation, and then to explain what they see e.g. by referring to uncertainty relations.

4th year quantum mechanics at HW now comprises the following:

Partly the following material has been seen by the students in some form, and it is reviewed and deepened:

Two-slit experiment leading to a wave-particle picture, uncertainty relations, Fourier relation between position and momentum space, plane waves, wave packets.

“Derivation” of the Schrödinger equation (SE) by referring to plane waves (as in Gasiorowicz). Time-dependent and time-independent SE, how wave packets evolve.

Solving the SE in 1D square potentials.

States, operators, commutators, eigenvalues, eigenfunctions (examples of eigenfunctions include those of the hydrogen atom). The time-evolution operator, and time evolution of expectation values. Connection between wave function and bra-ket notation; from now on, use bra/ket notation.

Two-level systems, including their time evolution (e.g. two-level atom, double well).

Different types of operators, Hermitian, unitary, and their use. Going between matrix and bra-ket representation for Hamiltonians for 2- and 3-level systems, interpreting them.

New material:

The harmonic oscillator using creation and annihilation operators. Number states.

Superpositions of number states and their time evolution. Coherent states.

Light as a quantum mechanical harmonic oscillator.

Two-level atom resonantly coupled to a light field, time evolution (solve SE in zero- and one-excitation subspaces). How and why this is different from exponential decay.

Spin $\frac{1}{2}$ and angular momentum (brief mention since this has been covered in another course in connection with spectroscopy).

Quantum measurements, Schrödinger's cat, entanglement. Decoherence as "resolution" of the cat paradox (qualitative explanation only). "Supercorrelation" of entangled states. Introduction to quantum information science: the qubit as a two-level system, no cloning, quantum key distribution. The Deutsch-Jozsa algorithm.

Connections to current research mentioned in lectures and tutorial problems includes: Interference of large organic molecules (Vienna). Semiconductors. Solid-state devices such as quantum cascade lasers. "Quantum bomb detector" (interferometer, the matrix representation of beam splitters). STIRAP (through solving for eigenstates of 3-level system). Lasers, cavity QED, nanomechanical oscillators, atomic clocks. Preparing entangled states using cavity QED. Experiments that investigate the boundary between classical and quantum mechanics.

Material which is not included in the exam, but mentioned in the lecture notes for the course, with more interested and able students in mind:

Derivation of the CHSH-Bell inequality.

Time-independent and time-dependent perturbation theory and Fermi's Golden rule (further covered in year 5 for students on Masters courses).

Many-particle systems, Bose and Fermi statistics in terms of creation and annihilation operators (Mandel dip for 50/50 beam splitter).

Density operators – introduced through considering a situation where we do know that one of two states was prepared, with some prior probabilities, but not which of them.

Material taught at Heriot-Watt in earlier years:

Year 1: Very brief qualitative introduction to quantum mechanics and particle physics. Wave-particle duality, elementary particles.

Year 2: Photonics and Quantum Mechanics course. Fairly traditional approach at the moment for the quantum mechanics part: Black-body radiation, photoelectric effect, Bohr model, spectra, wave-particle duality, electron diffraction, uncertainty principle.

Year 3: Quantum Theory and Spectroscopy course. Most of the standard syllabus, including bra-ket notation, and an introduction to quantum information. As the name suggests, a relatively large amount of material related to spectroscopy. Within a course on Dynamics and Statistical Physics, some quantum statistics is discussed. The course Physical Mathematics 2 treats differential equations, with solving the Schrödinger equation for the hydrogen atom and the 3D quantum-mechanical harmonic oscillator as examples. Mathematically, this is quite advanced for many students. Fourier transforms and delta functions are also introduced here.